The answer choice E. NOTA indicates that 'none of these answers' are correct. Assume appropriate units. Good luck and have fun!

1. Violet's typing accuracy, a, is related to her speed, s, through the function $a(s) = \frac{20}{23+\sqrt{s}}$. What is her accuracy when she types with speed $s = 4$, expressed as a percentage?

A. 80% B. 85% C. 90% D. 95% E. NOTA

2. At time $0 < t < 1$, the probability that Taki remembers Mitsuha's name is given by the function $f(t) = 1 - \sin(t)\cos(t)\tan(t)\sec(t)\csc(\mathrm{t})\cot(t)$. What is the probability that Taki remembers Mitsuha's name at time $t = \pi/2023$?

A. 0 B. $1/3$ C. $1/2$ D. 1 E. NOTA

3. Haru plants a flower garden whose boundary is given by the parametric equations $x = 5\cos(t)$ and $y = 3\sin(t)$ $(0 \le t < 2\pi)$. What is the area of her garden? (This concept may be useful in a later question.)

A. π B. $\frac{15\pi}{2}$ C. 15π D. 30π E. NOTA

4. While playing tag, Thoma and Lannion reach a fork in the road with two paths that form a $60°$ angle. To avoid both getting caught, they each take one path and run 8 and 5 meters, respectively. What is the resulting distance between them?

A. 144 B. 49 C. 12 D. 7 E. NOTA

5. Don and Gilda are pushing a box with force vectors $\vec{u}$ and $\vec{v}$ such that $|\vec{u}| = 20$ and $|\vec{v}| = 23$. If the angle between their force vectors is $\frac{\pi}{4}$, then what is the value of $|\vec{u} \cdot \vec{v}| - |\vec{u} \times \vec{v}|$?

A. $-230\sqrt{2}$ B. 0 C. $230\sqrt{2}$ D. 460 E. NOTA

6. Hina and Hodaka are making money through their Sunshine Girl services. If their cumulative earnings after the nth day are $n^3 + n + 1$, then how much money did they make on their 5th day?

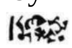
A. 62 B. 92 C. 130 D. 131 E. NOTA

7. Living the life as a Chuunibyou, Rikka wants to present Yuuta with a cute hand-drawn heart. Which of the following polar graphs should she draw to surprise Yuuta with a cardioid?

A. $r = 3 + 3\sin(\theta)$ B. $r = 4 + 3\cos(\theta)$
C. $r = 3\cos(3\theta)$ D. $r^2 = 9\sin(2\theta)$ E. NOTA

8. Emma is skipping stones in Grace Field Pond. The probability of a stone not sinking on its nth skip is $\sin\left(\frac{\pi}{n+1}\right)$. What is the probability her stone skips at least 3 times?

A. $\sqrt{2}/2$ B. $\sqrt{3}/2$ C. $\sqrt{6}/4$ D. 1 E. NOTA

9. In the Seven Walls, space and can be modeled by the matrix $\begin{bmatrix} x & 2-y \\ 2+y & 4x \end{bmatrix}$ for real numbers x and y. As Ray gets close,  tries to interfere by inverting space (i.e. inverting the matrix). If S is the set of "safe" pairs (x, y) such that  cannot invert space, find the area enclosed by the graph of S.

A. 0 B. $\frac{\pi}{2}$ C. π D. 2π E. NOTA

10. The strength of Shigeo's psychic powers varies over time and can be modeled by the equation $S(t) = \sqrt{t + \sqrt{2t - 1}}$. Similarly, the strength of Teruki's psychic powers can be modeled by the equation $T(t) = \sqrt{t - \sqrt{2t - 1}}$. Assuming psychic power is additive, which of the following is closest to the absolute difference between Shigeo's and Teruki's psychic powers at time $t = 23/20$?

A. $1/2$ B. 1 C. $3/2$ D. 2 E. $5/2$

11. Legosi has a circular spinner centered at $(0,0)$ in the coordinate plane. If he flicks the spinner with his right hand, he creates enough impulse to spin the arrow counterclockwise from $(1,0)$ to $\left(-\frac{3}{5},\frac{4}{5}\right)$. When he flicks it with his left hand, he creates enough impulse to spin the arrow counterclockwise from $(1,0)$ to $\left(\frac{5}{13},\frac{12}{13}\right)$. If he takes the spinner, initially at $(1,0)$, flicks it counterclockwise with his right hand, and then flicks it counterclockwise with his left hand, find the x-coordinate of the location the arrow ends up.

A. $\frac{33}{65}$ B. $-\frac{33}{65}$ C. $\frac{63}{65}$ D. $-\frac{63}{65}$ E. NOTA

12. Biske is collecting jewels. She finds one in the shape of the set of all points (x, y, z) that satisfy $x^2 + y^2 + z^2 \leq 1$ and $|x| + |y| + |z| \geq 1$. What is the volume of Biske's jewel?

A. $\frac{4}{3}$ B. $\frac{4\pi}{3}$ C. $\frac{4}{3}(\pi - 1)$ D. $\frac{4}{3}(\pi + 1)$ E. NOTA

13. The Heritage seniors and Saathvik are doing a challenge in November of 2023. On the nth day of November, the number of people that fail this challenge and become "fallen warriors" is the number of positive factors of n. Given that November 1st is a Sunday, on what day of the week did 6 people fail on the same day for the first time?

A. Tuesday B. Wednesday C. Thursday D. Friday E. NOTA

14. Jae throws a ball, and the height of this ball above the ground as a function of time t is $h(t) = -16t^2 + 240t + 10$. What is the maximum height the ball reaches?

A. 15 B. 910 C. 15/2 D. 1820 E. NOTA

15. An object is hung from a spring and set into oscillation. In physics, it is known that the height of this object above the ground (in cm) is given by $h(t) = A\cos(Bt) + C$ at t seconds after it is released. The object is released at its highest position of 100cm, falls all the way down to 40cm above the ground, and returns back to the original position for the first time after 6 seconds. Find the height of the object above the ground (in cm) 11 seconds after the release.

A. 70 B. 75 C. 80 D. 85 E. NOTA

16. In economics, the federal reserve sets the required reserve ratio, r, which designates that the bank must hold a proportion r of the checkable deposits they receive as "required reserves." The rest of the money, named the "excess reserves," may be lent out. Suppose that the federal reserve sets the required reserve ratio to be 0.1, and a bank finds itself with $1,000 in excess reserves. This bank lends out its excess reserves of $1,000, which becomes a checkable bank deposit elsewhere. The bank that receives the $1,000 deposit keeps 10% as required reserves and lends the remaining 90%, which again becomes a checkable deposit elsewhere, and so on. With infinite time, find the maximum possible increase in checkable deposits throughout the banking system, including the initial $1,000.

A. $\$0$ B. $\$9,000$ C. $\$10,000$ D. $\$11,000$ E. NOTA

17. In chemistry, the Gay-Lussac Law states that at constant volume, pressure varies directly with absolute temperature (in Kelvin). Refer to the following unit conversions:
- Kelvin = Celsius + 273 (temperature)
- 1 atm = 760 torr (pressure)

Suppose an empty watermelon has a volume of 2023 mL and experiences a pressure of 3.5 atm at 12°C. Find the pressure in torr if the watermelon was heated to 99°C (the watermelon does not change volume due to heating).

A. 2023 B. 3472 C. 3742 D. 3745 E. NOTA

18. In chemistry, the half-life of a radioactive substance, denoted as $t_{1/2}$, is the amount of time it takes for exactly half the substance to decay. Suppose that we run an experiment in which a container initially has 16 atoms of a radioactive element. After 9 days, we observe that there are 2 atoms of the radioactive element left in the container. Find the half-life (in days) of this radioactive element.

A. 1 B. 2 C. 3 D. 4 E. NOTA

19. Connor is practicing a bunch of matrix operations starting with a 3×3 matrix with determinant 3. On this matrix, he performs a scalar multiplication by $\frac{1}{2}$. On the resultant matrix, he adds three times the second row to the third row. Then, he switches the first column with the third column and the second row with the first row. Next, he multiplies each element in the third row by 1/3. After that, he multiplies each element of the matrix by -1. Finally, he inverts it. What is the determinant of the resultant matrix?

A. -2 B. 2 C. -8 D. 8 E. NOTA

20. Vlad has a 4th degree polynomial with real coefficients and leading coefficient 1. Two of its roots are $1+2i$ and $1-\sqrt{2}$, and the sum of the roots of this polynomial is $4-\sqrt{2}$. Find the sum of the coefficients of Vlad's polynomial.

A. 0 B. $\sqrt{2}$ C. $2\sqrt{2}$ D. $4-\sqrt{2}$ E. NOTA

21. Shizuku randomly draws two chords in a circle. Find the probability that they intersect.

A. $1/3$ B. $1/2$ C. $2/3$ D. 1 E. NOTA

22. Chrollo arranges 6 evenly spaced points around a circle and draws 2 distinct line segments connecting pairs of these points. Find the probability that these segments intersect inside the circle.

A. $1/3$ B. $1/7$ C. $1/10$ D. $1/42$ E. NOTA

23. A group of expert bank robbers know that the police will arrive within 2 minutes and attempt to finish their job at any given bank before the police arrive. In response, the police decide to guess a bank and patrol there for 2 minutes (the same amount of time the robbers would stay at the bank). When put on the Cartesian plane, the locations of banks are lattice points (x, y) such that $0 \le x \le 5$ and $0 \le y \le 5$. The robbers start with bank $(0,0)$, and the police start with bank $(5,5)$. Every 2 minutes, the robbers move to the bank 1 unit up or right while the police officers move to the bank 1 unit left or down. What is the probability that the police officers will catch the robbers at the same bank?

A. $63/256$ B. $1/4$ C. $65/256$ D. $63/128$ E. NOTA

24. Mitsuha is standing at a focus of Itomori volcano, defined as the ellipse $\frac{x^2}{49}+\frac{y^2}{33}=1$. She runs in a straight line to a point on the ellipse's perimeter, then straight back to the other focus and stops. There is a twilight zone, defined as the circle $x^2+(y-3)^2=4$, which sends her irreversibly into the future upon contact. What is the maximum distance Mitsuha can run while in the future?

A. 11 B. 12 C. 15 D. 16 E. NOTA

25. Megumin is interested in finding all numbers that have a remainder of 2 when divided by 5, and a remainder of 5 when divided by 7. Megumin is a three-year-old, so she can only count positive numbers, and up to and including all three-digit numbers. What is the median of all the numbers she found?

A. 356 B. 402 C. 456 D. 502 E. NOTA

26. In table tennis, a game is won by being the first player to reach 11 points and be at least 2 points ahead of the opponent. If both players have won 10 points, then the first player to get a 2 point lead wins the game. If players tie at or after 10 points it is called a "deuce." Suppose Alex and Jae are playing a game of table tennis. Each would normally have an equal chance of winning a point, but if they reach a deuce, Alex starts using his non-dominant hand for the rest of the match, and he then only has a $\frac{1}{10}$ chance of scoring any given point. What is the probability that Alex beats Jae in a game of table tennis given that they reach a deuce?

A. $1/2$ B. $1/41$ C. $1/82$ D. $1/164$ E. NOTA

27. In China, the number 4 is considered an unlucky number. Vincent does not like the number 4, so he invents *Vincent Numbers*, which are numbers made up of 4's and 5's such that no two adjacent digits are both 4. How many 10 digit *Vincent Numbers* are there? (Note that *Vincent Numbers* do not need to include both 4 and 5. 4 and 5 are both 1-digit *Vincent Numbers*.)

A. 144 B. 169 C. 72 D. 196 E. NOTA

28. In the framework of the previous question, define V_n as the number of n digit *Vincent Numbers*. Evaluate $\sum_{n=1}^{\infty} \frac{V_n}{3^n}$.

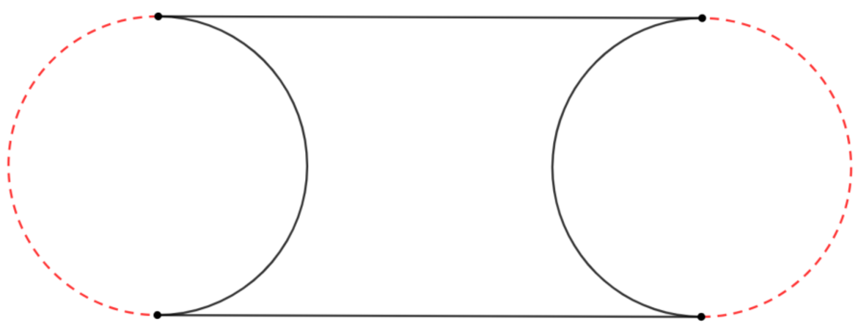
A. $7/3$ B. $7/5$ C. 1 D. $14/3$ E. NOTA

29. Hisoka traces out the graph of $|z-4|+|z+4|=10$ in the Argand plane with eccentricity e_1. For every point z in the graph, he then applies the transformation $z \to z^2$ using his Bungee Gum^tm. If the transformed points form a new graph with eccentricity e_2, then the value of $\frac{e_2}{e_1}$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

A. 2 B. 13 C. 25 D. 27 E. NOTA

30. Norman is flipping 2023 of his magical Λ7214 coins, each circular with diameter 1, to random positions on a finite table (but of sufficient size) such that no two coins overlap. Find the expected total length of *connected* perimeter among his coins. (see definition below)

Define a point P on the edge of a coin C_1 to be *connected* if there exists a point Q on the edge of any other coin C_2 such that the line segment between the two points does not pass through the interior of C_1 or C_2 For example, in the diagram below shows the situation for 2 coins, where the two segments connecting the circles are the exterior tangents. The points in the solid arcs (but not the solid tangent segments) are *connected* whereas the points on the dotted arcs are not.



A. 2022π B. $\frac{2023\pi}{2}$ C. π D. 2023π E. NOTA