1. D
2. B
3. B
4. A
5. B
6. A
7. E
8. D
9. C
10. C
11. C
12. A
13. E
14. B
15. D
16. C
17. C
18. B
19. A
20. B
21. C
22. C
23. D
24. A
25. C
26. A
27. B
28. C
29. D
30. C

1. D It is helpful to notice that $\sin 75°$ is the same as $\cos 15°$ and $\sin 105°$ is the same as $\cos 15°$, so the expression becomes $\sin 15° \cos 15° \cos 15°$. The double-angle formula can be applied to the first two terms, leaving $\frac{\sin 30°}{2}\cos 15°$. The half-angle formula can be applied as $\cos 15° = \sqrt{\frac{1+\cos 30°}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{6}+\sqrt{2}}{4}$. Therefore, the answer is $\frac{\sqrt{6}+\sqrt{2}}{16}$.
2. B The area Spots can travel in without wrapping around a wall of the barn is in the shape of a circular sector. Since each vertex of the hexagonal barn has interior angle $120°$, the circular sector with center at a vertex has angle $240°$ and radius 6. The area of the sector is $\frac{2}{3} \times 6^2\pi = 24\pi$. Spots can also go past the two walls extending from the vertex he is leashed to. This forms two circular sectors of radius 1 with angle $60°$, which have area $2 \times \frac{1}{6} \times 1^2\pi = \frac{\pi}{3}$. Summing the two areas gives the answer $\frac{73}{3}\pi$.
3. B This can be visualized as a triangle with sides 5, $\sqrt{x^2-16}$, and $\sqrt{x^2+9}$. The angle referenced for $\sin^{-1}\frac{5}{\sqrt{x^2+9}}$ is the angle opposite from the side of length 5. Therefore, the cotangent of that angle would be $\frac{\sqrt{x^2-16}}{5}$.
4. A Angela's revenue can be represented by $(9+3x)(1.50-0.10x)$, where x is the number of ten cent decreases of the price. Expanding results in $-0.3x^2+3.6x+13.5$, which is a parabola. The maximum of this parabola occurs at $\frac{-b}{2a}$, which is at $x = 6$. Therefore, the maximum revenue occurs at $\$1.50 - 0.10 \times 6 = \$0.90$.
5. B The volume of the triangular pryramid will be $\frac{1}{6}$ of the volume of the parallelepiped formed by the three vectors. The volume of the parallelepiped is the magnitude of the scalar triple product of the three vectors, which is given by solving the magnitude of $| < 4, -2,\ 5 > \cdot (< 3, 10, -7 > \times < -2, 3, 9 >)|$.
6. A The term in the first row and second column of a rotation matrix for any point rotated θ degrees counterclockwise is $-\sin\theta$. In this case $\theta = 60°$, so the desired term is $-\frac{\sqrt{3}}{2}$.
7. E The equation can be rewritten as $2^{2x} + 3 * 2^x - 4 = 0$. Set $2^x = y$ and the equation becomes $y^2 + 3y - 4 = 0$. Factoring gives $(y+4)(y-1) = 0$ which has solutions $y = 1$ and $y = -4$. However, 2^x cannot be negative, so the only solution is $2^x = 1$ and $x = 0$, giving a product of 0.
8. D Spitting up the fraction gives you $25x\sin x + \frac{16}{x\sin x}$. AM-GM can be applied giving $\frac{25x\sin x + \frac{16}{x\sin x}}{2} \geq \sqrt{25x\sin x * \frac{16}{x\sin x}} = \sqrt{25 * 16} = 20$. Therefore, the minimum value is 40.

   *Note that minimizing value $\frac{5}{4}$ is achievable on the given domain, as $x\sin x = 0$ at $x = 0$ and $x\sin x = \frac{\pi}{2} > \frac{5}{4}$ at $x = \frac{\pi}{2}$..

9. C The answer will be $\lim_{x\to 0} \frac{\sin 2x}{6x\left(\cos\frac{x}{2}\right)^2 - 3x}$. The denominator factors: $3x(2\left(\cos\frac{x}{2}\right)^2 - 1)$ which can be simplified using double angle formula to $3x\cos x$. The numerator of the fraction can be expanded out to $2\sin x\cos x$. Simplifying the fraction gives $\frac{2\sin x}{3x}$. Using the fact that $\lim_{x\to\infty} \frac{\sin x}{x} = 1$, the expression we are looking for is $\frac{2}{3}$.

10. C The space diagonal of a cube with side length s has length $s\sqrt{3}$. In this case, $s\sqrt{3} = 4$ so $s = \frac{4\sqrt{3}}{3}$. The surface area of the cube is $6s^2 = 32$, which is equal to $4\pi r^2$, or the surface area of a sphere with radius r. Solving gives that $r = \frac{2\sqrt{2}}{\sqrt{\pi}}$. The volume of the sphere is $\frac{4}{3}\pi r^3$, which when plugged in gives an answer of $\frac{64\sqrt{2}}{3\sqrt{\pi}}$.

11. C The bridge can be thought of as a downward-facing parabola with vertex (0, 15). The equation would then be $(y - 15) = ax^2$, where a is a constant. Plugging in the point given, (5, 5), results in $a = -\frac{2}{5}$. The width of the bridge is twice the x-intercept. Plugging in 0 for y results in the x-intercept being $\frac{5\sqrt{6}}{2}$. Twice the x-intercept is $5\sqrt{6}$.

12. A The x-coordinate of the bug is $2 - \frac{1}{2} + \frac{1}{8} - \cdots$ and the y-coordinate of the bug is $1 - \frac{1}{4} + \frac{1}{16} - \cdots$. They are both geometric series with common ratio $-\frac{1}{4}$. The x-coordinate becomes $\frac{2}{1-(-\frac{1}{4})} = \frac{8}{5}$ and the y-coordinate becomes $\frac{1}{1-(-\frac{1}{4})} = \frac{4}{5}$. Using the distance formula gives that the bug is $\frac{4\sqrt{5}}{5}$ meters away from its starting point.
*Alternatively in the complex plane, the distance it travels is $\frac{2}{1-\frac{i}{2}}$ which has magnitude $\frac{2}{\frac{\sqrt{5}}{2}} = \frac{4\sqrt{5}}{5}$

13. E The cable is the hypotenuse of a right triangle that has a leg of length 20 yards opposite of the 30 degree angle of elevation. Since the ratio of the sides of a 30-60-90 triangle is $1 - \sqrt{3} - 2$, the hypotenuse has length 40 yards. However, the question is asking for the length in feet, so the answer is 120 inches.

14. B The equation for the population can be represented as $350e^{rt}$. After 7 hours, the population triples, so $350e^{7r} = 1050$. Taking the natural log of both sides gives that $r = \frac{\ln 3}{7}$. To find the time it takes to double, the equation $350e^{\frac{\ln 3}{7}t} = 700$ must be solved for t. Taking the natural log of both sides and simplifying yields $t = \frac{7\ln 2}{\ln 3}$.

15. D The class average can be represented as $7n + 2$. From the information given, the equation $23(7n + 2) + w = 24(7n + 4)$ must be true, where w is Wanda's score. Simplifying yields $w = 7n + 50$. Out of the answer choices, only 99 fits the criteria.

16. C Setting the two equations equal gives $2 - 5\cos\theta = \cos 2\theta = 2(\cos\theta)^2 - 1$. Solving the quadratic for $\cos\theta$ gives $\cos\theta = \frac{1}{2}$ (since -3 is extraneous for cosine). The smallest θ for which this occurs is $\frac{\pi}{3}$.

17. C Her first four rolls do not affect the probability of her rolling a five. There is a $\frac{1}{6}$ chance of rolling a five out of the six possibilities, so it will take her 6 more rolls to get a five. Therefore, she will roll the die 4 + 6 = 10 times in total.

18. B The initial volume of the ice cream is $\frac{1}{3}\pi * 4^2 * 6 = 32\pi$ inches cubed. After 3 seconds, 5π cubic inches have melted. The remaining volume is 27π. After melting, the ratio of the radius to the height is 2:3. If the radius of the new ice cream is r, the equation $\frac{1}{3}\pi * r^2 * \frac{3}{2}r = 27\pi$ must be true. Solving gives $r = 3\sqrt[3]{2}$, so $h = \frac{9\sqrt[3]{2}}{2}$.

19. A Using the information given, the equation is $d = k\frac{m}{h}$, where d is the distance traveled, k is a constant, m is his math test score, and h is his height. From the initial information, $35 = k\frac{84}{6}$, so $k = \frac{5}{2}$. The desired quantity is d when $m = 91$ and $h = \frac{21}{4}$. Solving $d = \frac{5}{2} * \frac{91}{\frac{21}{4}}$ results in $d = \frac{130}{3}$ feet.

20. B There is a $\frac{1}{8}$ chance they guess the correct answer, which gives them 3 points, and a $\frac{7}{8}$ chance they guess the incorrect answer, which gives them -4 points. The student will not leave any blank. Their expected score will be $32 * \left(\frac{1}{8} * 3 + \frac{7}{8} * (-4)\right) = -100$.

21. C The cosine addition formula gives $\cos(x+y) = \cos x \cos y - \sin x \sin y$. It is given that $\sin x = \frac{2}{3}$, so $\cos x = -\frac{\sqrt{5}}{3}$ because x is between $\frac{\pi}{2}$ and π, so $\cos x$ has to be negative. It is also given that $\tan y = \frac{9}{40}$ and y is between π and $\frac{3\pi}{2}$, so $\sin y = -\frac{9}{41}$ and $\cos y = -\frac{40}{41}$. Plugging back in, $\cos(x+y) = \frac{18 + 40\sqrt{5}}{123}$.

22. C The projection of Dana's vector onto Hannah's will be $\frac{<1,2,-7> \cdot <5,3,-4>}{|<1,2,-7>|^2} < 1,2,-7 >$ $= \frac{39}{(3\sqrt{6})^2} < 1,2,-7 > = < \frac{13}{18}, \frac{13}{9}, -\frac{91}{18} >$.

23. D The equation for compounded interest is $A = Pe^{rt}$. Using the information given, it follows that $861 = Pe^{0.05}$ so $P = \frac{861}{e^{0.05}}$. The equation to be solved is then $943 = \frac{861}{e^{0.05}} e^{0.05t}$. Taking the natural log of both sides gives $t = \frac{\ln\left(\frac{23}{21}\right)}{0.05} + 1$.

24. A The angle of rotation needed comes from the equation $\cot 2\theta = \frac{A-C}{B}$, where A, B, and C come from the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Plugging in results in $\cot 2\theta = \frac{2-1}{1}$, so $2\theta = \frac{\pi}{4}$ and $\theta = \frac{\pi}{8}$.

25. C One of the markers must be the same, and there are $\binom{3}{1}$ ways to pick one of the three original markers. The other two must be taken from the 12 markers that were not chosen the first time, so there are $\binom{12}{2}$ ways to choose the non-original markers. There is a total of $\binom{15}{3}$ ways to choose 3 markers from the 15 total markers, so the probability is $\frac{\binom{3}{1}\binom{12}{2}}{\binom{15}{3}} = \frac{198}{455}$.

26. A Say r is in the form $acis(\theta)$ and the first term in the sequence is b. Since $b = 2$ and $br^6 = 128$, $r^6 = a^6 cis(6\theta) = 64$. a must be 2 and $cis(6\theta) = 1$. Therefore, 6θ must equal $2\pi k$, where k is a constant. The shortest distance between two

possibilities of r would be between two consecutive values of k. One instance of this would be to take is $6\theta = 0$ and $6\theta = 2\pi$, resulting in $r = 2cis(0)$ and $r = 2cis(\frac{\pi}{3})$. The distance between the two is the distance between a triangle with two sides of length 2 and an angle of $60°$ between the two sides, which is 2.

27. B After 75 minutes, Bird A is 20 miles away from the starting point because $d = rt \rightarrow d = 16 * \frac{5}{4} = 20$. Bird B is 6 miles away from the starting point because $d = rt \rightarrow d = 8 * \frac{3}{4} = 6$. To find the distance between the two, apply Law of Cosines to a triangle with sides 6, 20, and angle 60°. $6^2 + 20^2 - 2 * 6 * 20 * \cos 60° = 316$, so the distance is $2\sqrt{79}$.

28. C The graphs may have intersections on the interval $[-20\pi, 20\pi]$ as that is where the range of the linear function is on $[-1, 1]$. There are 2 intersections per period of the sine graph, with the exception that there are only 3 intersections on the 2 periods spanning $[-2\pi, 2\pi]$ as 0 accounts for both sides.

Putting it together, there are 20 periods at 2 intersection each, with one double counted at 0, making it 39 intersections in total.

29. D This is a derangements questions where n = 6. Using PIE, we get

$$6!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right) = 720 * \frac{265}{720} = 265.$$

30. C Using the symmetry between 2,3,5, we can use states to simplify the problem.
*Note that n is some positive integer for the rest of the solution.
Let $a_0 =$ the expected number of rolls necessary given the current product is n^2
Let $a_1 =$ the expected number of rolls necessary given the current product is $2/3/5 * n^2$
Let $a_2 =$ the expected number of rolls necessary given the current product is $6/10/15 * n^2$
Let $a_3 =$ the expected number of rolls necessary given the current product is $30n^2$
Note that we are solving for a_0.

If the current product $= n^2$
If we roll a 0,1,4 the product is a perfect square. If we roll a 2,3,5 we move on to a_1

$$a_0 = 1 + a_1 \cdot \frac{3}{6}$$

If the current product $= 2/3/5 * n^2$
WLOG let the current product be $2n^2$
If we roll a 0,2 the product is a perfect square. If we roll a 1,4 we still have a_1. If we roll a 3,5 we move on to a_2

$$a_1 = 1 + a_1 \cdot \frac{2}{6} + a_2 \cdot \frac{2}{6}$$

If the current product $= 6/10/15 * n^2$
WLOG let the current product be $6n^2$
If we roll a 0 the product is a perfect square. If we roll a 2,3 we move on to a_1. If we roll a 1,4 we still have a_2. If we roll a 5 we move on to a_3

$$a_2 = 1 + a_1 \cdot \frac{2}{6} + a_2 \cdot \frac{2}{6} + a_3 \cdot \frac{1}{6}$$

If the current product $= 30n^2$
If we roll a 0 the product is a perfect square. If we roll a 2,3,5 we move on to a_2. If we roll a 1,4 we still have a_3.

$$a_3 = 1 + a_2 \cdot \frac{3}{6} + a_3 \cdot \frac{2}{6}$$

Solving this gives

$$a_0 = \frac{35}{12} \to 47$$