Alpha School Bowl
Test #822
Question #0

Alpha School Bowl
Test #822
Question #0

#0 Alpha School Bowl
MAΘ National Convention 2023

Let X be the binary representation of a number such that

$$23_{20} + 20_{23} + 23_{2023} + 2023_4 = X_2$$

How many times does the digit 1 appear in X?

#0 Alpha School Bowl
MAΘ National Convention 2023

Let X be the binary representation of a number such that

$$23_{20} + 20_{23} + 23_{2023} + 2023_4 = X_2$$

How many times does the digit 1 appear in X?

Alpha School Bowl
Test #822
Question #1

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Question #1

#1 Alpha School Bowl
MAΘ National Convention 2023

For all parts, $\theta = \text{arccsc}\left(\frac{25}{7}\right)$ and $\alpha = \arccos\left(-\frac{3}{5}\right)$

Let $A = \sin(\theta)$

Let $B = \tan(\theta)$

Let $\text{C} = \cos(2\alpha)$

Let $D = \tan(\theta - \alpha)$

Find $A + 6B + C + 11D$.

#1 Alpha School Bowl
MAΘ National Convention 2023

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Find $A + 6B + C + 11D$.

Alpha School Bowl
Test #822
Question #2

Alpha School Bowl
Test #822
Question #2

#2 Alpha School Bowl
MAΘ National Convention 2023

Let A be the sum of the real solutions to the relation $x = \log_2(x + \log_2(x + \log_2(x + \cdots \log_2(x)) \ldots))$

Let B be the number of values of θ in $[0, \pi)$ for which

$$\frac{-1+\sqrt{3}}{2} = \cfrac{\cos(2023\theta)}{1 + \cfrac{\cos(2023\theta)}{1 + \cfrac{\cos(2023\theta)}{1 + \cdots}}}$$

Find $A + B$.

#2 Alpha School Bowl
MAΘ National Convention 2023

Let A be the sum of the real solutions to the relation $x = \log_2(x + \log_2(x + \log_2(x + \cdots \log_2(x)) \ldots))$

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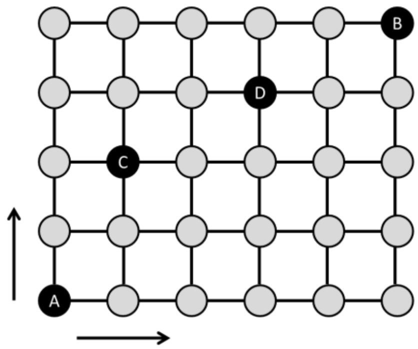
$$\frac{-1+\sqrt{3}}{2} = \cfrac{\cos(2023\theta)}{1 + \cfrac{\cos(2023\theta)}{1 + \cfrac{\cos(2023\theta)}{1 + \cdots}}}$$

Find $A + B$.

Alpha School Bowl
Test #822
Question #3

Alpha School Bowl
Test #822
Question #3

#3 Alpha School Bowl
MAΘ National Convention 2023



Tiger is moving along the above lattice from point A to point B using only steps upwards and to the right.

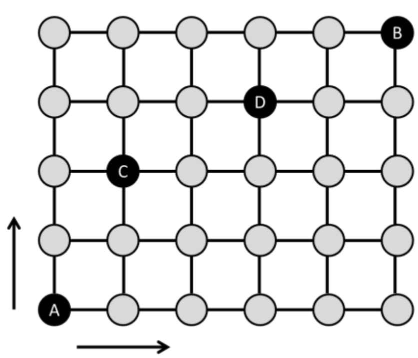
Let R be the total number of paths to get from point A to point B.

Let E be the total number of paths to get from point A to point B that passes through point D.

If a path is chosen uniformly at random, let N be the probability that the path goes through point D given that the chosen path goes through point C.

Find $R + EN$.

#3 Alpha School Bowl
MAΘ National Convention 2023



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Find $R + EN$.

Alpha School Bowl
Test #822
Question #4

Alpha School Bowl
Test #822
Question #4

#4 Alpha School Bowl
MAΘ National Convention 2023

Note that the domain of all variables in this question are all real numbers.

Let A be the maximum value of $6\sin(2023x) + 8\cos(2023x)$

Let B be the maximum value of $2023 + 18x - 3x^2$

Let C be the maximum value of xyz given that $20x + 2y + 3z = 6$ and $x, y, z > 0$

Let D be the maximum value of $3x + 6y + 22z$ given that $x^2 + y^2 + z^2 = 25$

Find $(A + B + D)C$.

#4 Alpha School Bowl
MAΘ National Convention 2023

Note that the domain of all variables in this question are all real numbers.

Let A be the maximum value of $6\sin(2023x) + 8\cos(2023x)$

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Let C be the maximum value of xyz given that $20x + 2y + 3z = 6$ and $x, y, z > 0$

Let D be the maximum value of $3x + 6y + 22z$ given that $x^2 + y^2 + z^2 = 25$

Find $(A + B + D)C$.

Alpha School Bowl
Test #822
Question #5

Alpha School Bowl
Test #822
Question #5

#5 Alpha School Bowl
MAΘ National Convention 2023

The polynomial $f(x) = 7x^3 - 119x^2 + 289x - 2023$ has distinct roots r_1, r_2, and r_3

Let $A = r_1 + r_2 + r_3$

Let $B = r_1 \cdot r_2 \cdot r_3$

Let $C = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

Let $D = r_1^2 + r_2^2 + r_3^2$

Find $AC + \frac{D}{B}$.

#5 Alpha School Bowl
MAΘ National Convention 2023

The polynomial $f(x) = 7x^3 - 119x^2 + 289x - 2023$ has distinct roots r_1, r_2, and r_3

Let $A = r_1 + r_2 + r_3$

Let $B = r_1 \cdot r_2 \cdot r_3$

Let $C = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

Let $D = r_1^2 + r_2^2 + r_3^2$

Find $AC + \frac{D}{B}$.

Alpha School Bowl
Test #822
Question #6

Alpha School Bowl
Test #822
Question #6

#6 Alpha School Bowl
MAΘ National Convention 2023

Let A be the number of positive integral factors of 2023.

Let B be the number of positive integers less than 2023 that are relatively prime to 2023.

Let C be the remainder when 23^{3266} is divided by 2023.

Find $A + B + C$.

#6 Alpha School Bowl
MAΘ National Convention 2023

Let A be the number of positive integral factors of 2023.

Let B be the number of positive integers less than 2023 that are relatively prime to 2023.

Let C be the remainder when 23^{3266} is divided by 2023.

Find $A + B + C$.

Alpha School Bowl
Test #822
Question #7

Alpha School Bowl
Test #822
Question #7

#7 Alpha School Bowl
MAΘ National Convention 2023

Let $A = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{2023}$

Let B be the smallest real part that a solution to $x^3 = 8i$ can have.

Let

$$C = \sum_{n=0}^{\infty} \frac{\sin\left(\frac{n\pi}{6}\right)}{\left(\sqrt{3}\right)^n}$$

Let D be the sum of the 2023rd roots of unity which have non-zero imaginary parts.

Find $10(Ai + B + C + D)$, expressed a complex number in the form $a + bi$ where a, b are real..

#7 Alpha School Bowl
MAΘ National Convention 2023

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Find $10(Ai + B + C + D)$, expressed a complex number in the form $a + bi$ where a, b are real.

Alpha School Bowl
Test #822
Question #8

Alpha School Bowl
Test #822
Question #8

#8 Alpha School Bowl
MAΘ National Convention 2023

The values a_1 and a_3 are chosen uniformly at random with replacement from the set $\{\pm 1, \pm 2, \pm 3\}$

Let A be the probability that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse.
(In other words, when graphed on the Cartesian plane, the graph is an ellipse with positive area).

Let B be the probability that the area contained by the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is less than or equal to 2023π, given that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse.

Find $\frac{1}{A} + B$.

#8 Alpha School Bowl
MAΘ National Convention 2023

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Let A be the probability that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse.
(In other words, when graphed on the Cartesian plane, the graph is an ellipse with positive area).

Let B be the probability that the area contained by the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is less than or equal to 2023π, given that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse.

Find $\frac{1}{A} + B$.

Alpha School Bowl
Test #822
Question #9

Alpha School Bowl
Test #822
Question #9

#9 Alpha School Bowl
MAΘ National Convention 2023

If $\{a_n\}_{n=1}^{\infty}$ is an arithmetic sequence with a fifth term of 23 and a fifty-fifth term of 2023, let $A = a_1$.

If $\{b_n\}_{n=1}^{\infty}$ is a real-valued geometric sequence with a first term of 2023 and a fifth term of 7, let $B = b_3$.

The sequence has each term c_n defined by a cubic polynomial $P(n) = c_n$. Given $c_1 = 7, c_2 = 23, c_3 = 63, c_4 = 139$, let $C = P(6)$.

If $M_n = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n$, let $D = \det(M_{2023})$

Find $A + B + C + D$.

#9 Alpha School Bowl
MAΘ National Convention 2023

If $\{a_n\}_{n=1}^{\infty}$ is an arithmetic sequence with a fifth term of 23 and a fifty-fifth term of 2023, let $A = a_1$.

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If $M_n = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n$, let $D = \det(M_{2023})$

Find $A + B + C + D$.

Alpha School Bowl
Test #822
Question #10

Alpha School Bowl
Test #822
Question #10

#10 Alpha School Bowl
MAΘ National Convention 2023

Consider the following lines in three-dimensional Cartesian space:

$$\text{Line } \mathcal{L}_1 : x + 1 = \frac{y-1}{2} = \frac{z-3}{2}$$
$$\text{Line } \mathcal{L}_2 : \frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{6}$$

Let D_1 be the minimum distance between the point (2,2,3) and $\mathcal{L}_1$.
$D_1^2 = \frac{m}{n}$ in simplest form, $A = m + n$.

Let D_2 be the minimum distance between $\mathcal{L}_1$ and $\mathcal{L}_2$.
$D_2^2 = \frac{m}{n}$ in simplest form. $B = m + n$.

Find $A + B$.

#10 Alpha School Bowl
MAΘ National Convention 2023

Consider the following lines in three-dimensional Cartesian space:

$$\text{Line } \mathcal{L}_1 : x + 1 = \frac{y-1}{2} = \frac{z-3}{2}$$
$$\text{Line } \mathcal{L}_2 : \frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{6}$$

Let D_1 be the minimum distance between the point (2,2,3) and $\mathcal{L}_1$.
$D_1^2 = \frac{m}{n}$ in simplest form, $A = m + n$.

Let D_2 be the minimum distance between $\mathcal{L}_1$ and $\mathcal{L}_2$.
$D_2^2 = \frac{m}{n}$ in simplest form. $B = m + n$.

Find $A + B$.

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Question #11

Alpha School Bowl
Test #822
Question #11

#11 Alpha School Bowl
MAΘ National Convention 2023

The below table gives a numerical value assign to each listed kind of plane curve:

Circle	10	Limaçon with Inner Loop	20	Lemniscate	8
Non-Circular Ellipse	11	Cardioid	3	Rose with n Petals	n
Hyperbola	4	Dimpled Limaçon	6	Line	1
Parabola	7	Convex Limaçon	2	Any Other Curve	0

Find the sum of the values of the polar graphs of the following twelve equations:

$r(\theta) = 2023\cos(\theta)$ $\quad$ $r(\theta) = 3\theta + 1$ $\quad$ $r(\theta) = 2023 + 2024\sin(\theta)$

$r(\theta) = 289\sin(3\theta)$ $\quad$ $r^2(\theta) = 289\sin(2\theta)$ $\quad$ $r(\theta) = \sec(\theta)\tan(\theta)$

$r(\theta) = -\csc(\theta)$ $\quad$ $r(\theta) = 7$ $\quad$ $r(\theta) = \sin^2\left(\frac{\theta}{2}\right)$

$\theta = \frac{\pi}{4}$ $\quad$ $r(\theta) = \frac{4}{2+\cos(\theta)}$ $\quad$ $r(\theta) = \sin(\theta)\cos(\theta)\cos(2\theta)\cos(4\theta)$

Values can be used more than once or not at all.

#11 Alpha School Bowl
MAΘ National Convention 2023

The below table gives a numerical value assign to each listed kind of plane curve:

Circle	10	Limaçon with Inner Loop	20	Lemniscate	8
Non-Circular Ellipse	11	Cardioid	3	Rose with n Petals	n
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Values can be used more than once or not at all.

Alpha School Bowl
Test #822
Question #12

Alpha School Bowl
Test #822
Question #12

#12 Alpha School Bowl
MAΘ National Convention 2023

Let $A = \lim_{x\to 3} \frac{x^3-3x^2+4x-12}{x^2+x-12}$

Let $B = \lim_{x\to\infty} \frac{x^2-3x^3+4x-12}{x^2+x^3-12}$

Let $C = \lim_{x\to 0}(1+2023x)^{\frac{2}{x}}$

Let $D = \lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$

Find $70A + B + \ln(C) + D$

#12 Alpha School Bowl
MAΘ National Convention 2023

Let $A = \lim_{x\to 3} \frac{x^3-3x^2+4x-12}{x^2+x-12}$

Let $B = \lim_{x\to\infty} \frac{x^2-3x^3+4x-12}{x^2+x^3-12}$

Let $C = \lim_{x\to 0}(1+2023x)^{\frac{2}{x}}$

Let $D = \lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$

Find $70A + B + \ln(C) + D$

Alpha School Bowl
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Question #13

Alpha School Bowl
Test #822
Question #13

#13 Alpha School Bowl
MAΘ National Convention 2023

Let $A = \sum_{n=1}^{K} 2023$

Let $B = \sum_{n=1}^{2023} n$

Let $C = \sum_{n=1}^{2023} n^2$

Let $D = \sum_{n=1}^{2023} n^3$

Find the smallest positive integer value of K so that $\gcd\left(A, \frac{BC}{D}\right) > 1$.

#13 Alpha School Bowl
MAΘ National Convention 2023

Let $A = \sum_{n=1}^{K} 2023$

Let $B = \sum_{n=1}^{2023} n$

Let $C = \sum_{n=1}^{2023} n^2$

Let $D = \sum_{n=1}^{2023} n^3$

Find the smallest positive integer value of K so that $\gcd\left(A, \frac{BC}{D}\right) > 1$.

Alpha School Bowl
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Question #14

Alpha School Bowl
Test #822
Question #14

#14 Alpha School Bowl
MAΘ National Convention 2023

The partial fraction decomposition of $\frac{25}{(x-2)^2(x^2+1)}$ is $\frac{Ax+B}{x^2+1}+\frac{C}{x-2}+\frac{D}{(x-2)^2}$.

Find $A+B+C+D$.

#14 Alpha School Bowl
MAΘ National Convention 2023

The partial fraction decomposition of $\frac{25}{(x-2)^2(x^2+1)}$ is $\frac{Ax+B}{x^2+1}+\frac{C}{x-2}+\frac{D}{(x-2)^2}$.

Find $A+B+C+D$.