Alpha School Bowl
Test #
Question #0

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Question #0

Let X be the binary representation of a number such that

$$23_{20} + 20_{23} + 23_{2023} + 2023_4 = X_2$$

How many times does the digit 1 appear in X?

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Solution:

$23_{20} + 20_{23} + 23_{2023} + 2023_4 = 2(20) + 3 + 2(23) + 2(2023) + 3 + 2(64) + 2(4) + 3 = 43 + 46 + 4049 + 139 = 4277 = 4096 + 181 = 4096 + 128 + 32 + 16 + 4 + 1 = 1000010110101_2$ which has $\boxed{6}$ ones.

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Question #1

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Question #1

For all parts, $\theta = \text{arccsc}\left(\frac{25}{7}\right)$ and $\alpha = \arccos\left(-\frac{3}{5}\right)$.

Let $A = \sin(\theta)$

Let $B = \tan(\theta)$

Let $\text{C} = \cos(2\alpha)$

Let $D = \tan(\theta - \alpha)$

Find $A + 6B + C + 11D$.

Solution:

Note that both θ is in $Q1, \alpha$ is in $Q2$.
$A = \sin(\theta) = \frac{7}{25}$.
$B = \tan(\theta) = \frac{1}{\cot(\theta)} = \frac{1}{\sqrt{\csc^2(\theta) - 1}} = \frac{7}{\sqrt{625-49}} = \frac{7}{24}$.
$C = \cos(2\alpha) = 2\cos^2(\alpha) - 1 = \frac{18}{25} - 1 = -\frac{7}{25}$.
We will need $\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{\sqrt{1-\frac{9}{25}}}{\frac{3}{5}} = \frac{4}{3}$. $D = \tan(\theta - \alpha) = \frac{\tan(\theta)-\tan(\alpha)}{1+\tan(\theta)\tan(\alpha)} = \frac{\frac{7}{24}+\frac{4}{3}}{1-\frac{28}{72}} = \frac{\frac{13}{8}}{\frac{11}{18}} = \frac{117}{44}$.
The final answer is

$$\frac{7}{25} + \frac{7}{4} - \frac{7}{25} + \frac{117}{4} = \boxed{31}$$

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Question #2

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Question #2

Let A be the sum of the real solutions to the relation $x = \log_2(x + \log_2(x + \log_2(x + \cdots \log_2(x)) \ldots))$

Let B be the number of values of θ in $[0, \pi)$ for which

$$\frac{-1+\sqrt{3}}{2} = \frac{\cos(2023\theta)}{1 + \frac{\cos(2023\theta)}{1 + \frac{\cos(2023\theta)}{1 + \cdots}}}$$

Find $A + B$.

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Solution:

$x = \log_2(x + \log_2(x + \log_2(x + \cdots))) = \log_2(x + x) = \log_2(2x) \to 2^x = 2x \to x = 1{,}2 \to A = 3.$

$\frac{-1+\sqrt{3}}{2} = \frac{\cos(2023\theta)}{1+\frac{\cos(2023\theta)}{1+\frac{\cos(2023\theta)}{1+\cdots}}} = \frac{\cos(2023\theta)}{1+\left(\frac{-1+\sqrt{3}}{2}\right)} = \frac{\cos(2023\theta)}{\frac{1+\sqrt{3}}{2}} \to \cos(2023\theta) = \left(\frac{1+\sqrt{3}}{2}\right)\left(\frac{-1+\sqrt{3}}{2}\right) = \frac{1}{2}.$

This equivalent to asking for the number of solutions of

$$\cos(x) = \frac{1}{2}, x \in [0, 2023\pi)$$

This equation has a solution at $Q1{,}4$ or one solution every π. Thus, there are 2023 solutions.

The final answer is

$$3 + 2023 = \boxed{2026}$$

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Question #3

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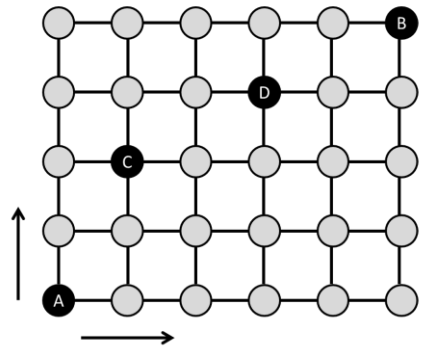
Question #3



Tiger is moving along the above lattice from point A to point B using only steps upwards and to the right.

Let R be the total number of paths to get from point A to point B.

Let E be the total number of paths to get from point A to point B that passes through point D.

If a path is chosen uniformly at random, let N be the probability that the path goes through point D given that the chosen path goes through point C.

Find $R + EN$.

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Solution:

There will be nine total moves, of which four must be upward, so

$$R = \binom{9}{4} = \frac{9!}{4!\,5!} = \frac{9*8*7*6}{4*3*2} = 63*2 = 126$$

To pass through point D, first Tiger must go six moves of which three are upward, and then three moves of which one is upward. So

$$E = \binom{6}{3}\binom{3}{1} = 20*3 = 60$$

$$N = P(D|C) = \frac{P(C\&D)}{P(C)} = \frac{\binom{3}{1}\binom{3}{1}\binom{3}{1}}{\binom{3}{1}\binom{6}{2}} = \frac{27}{45} = \frac{3}{5}$$

The final answer is

$$126 + (60)\left(\frac{3}{5}\right) = 126 + 36 = \boxed{162}$$

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Question #4

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Question #4

Note that the domain of all variables in this question are all real numbers.

Let A be the maximum value of $6\sin(2023x) + 8\cos(2023x)$

Let B be the maximum value of $2023 + 18x - 3x^2$

Let C be the maximum value of xyz given that $20x + 2y + 3z = 6$ and $x, y, z > 0$

Let D be the maximum value of $3x + 6y + 22z$ given that $x^2 + y^2 + z^2 = 25$

Find $(A + B + D)C$.

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Solution:

For A, the amplitude is $\sqrt{6^2 + 8^2} = 10$.

For B, the maximum occurs at the vertex which is $-\frac{b}{2a} = -\frac{18}{-6} = 3$. That makes the maximum value $2023 + 18 * 3 - 27 = 2050$

For C, we use AM-GM so $(xyz)^{\frac{1}{3}} \leq \frac{x+y+z}{3} \rightarrow 20x * 2y * 3z \leq \frac{(20x+2y+3z)^3}{3^3} = 8 \rightarrow xyz \leq \frac{1}{15}$

For D, we use Cauchy-Schwartz so $\boldsymbol{u} \cdot \boldsymbol{v} \leq \|\boldsymbol{u}\|\|\boldsymbol{v}\| \rightarrow 3x + 6y + 22z =< x, y, z > \cdot < 3,6,22 > \leq \sqrt{x^2 + y^2 + z^2}\sqrt{3^2 + 6^2 + 22^2} = 5 * 23 = 115$

So, the answer is

$$\frac{10 + 2050 + 115}{15} = \boxed{145}$$

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Question #5

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Question #5

#5 Alpha School Bowl
MAΘ National Convention 2023

The polynomial $f(x) = 7x^3 - 119x^2 + 289x - 2023$ has distinct roots r_1, r_2, and r_3

Let $A = r_1 + r_2 + r_3$

Let $B = r_1 \cdot r_2 \cdot r_3$

Let $C = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

Let $D = r_1^2 + r_2^2 + r_3^2$

Find $AC + \frac{D}{B}$.

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Solution:

$$A = -\frac{-119}{7} = 17$$

$$B = -\frac{-2023}{7} = 289$$

$$C = \frac{r_1r_2 + r_1r_3 + r_2r_3}{r_1r_2r_3} = \frac{289}{2023} = \frac{1}{7}$$

$(r_1 + r_2 + r_3)^2 = r_1^2 + r_2^2 + r_3^2 + 2(r_1r_2 + r_1r_3 + r_2r_3) \rightarrow$

$$D = r_1^2 + r_2^2 + r_3^2 = (17)^2 - 2\left(\frac{289}{7}\right) = 289 \cdot \left(\frac{5}{7}\right)$$

The final answer is

$$(17)\left(\frac{1}{7}\right) + \frac{289\left(\frac{5}{7}\right)}{289} = \boxed{\frac{22}{7}}$$

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Question #6

Let A be the number of positive integral factors of 2023.

Let B be the number of positive integers less than 2023 that are relatively prime to 2023.

Let C be the remainder when 23^{3266} is divided by 2023.

Find $A + B + C$.

#6 Alpha School Bowl
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Solution:

$2023 = 7 * 17^2 \rightarrow A = 2 * 3 = 6$.

$\varphi(2023) = 7^0(7-1)17^1(17-1) = 6 * 17 * 16 = 1632 = B$.

Since $\gcd(23,2023) = 1$, we can write $23^{3266} \equiv (23^{1632})^2 23^2 \pmod{2023} \equiv (1)^2 23^2 \pmod{2023}$ by Euler's Totient Theorem. Therefore $C = 23^2 = 529$.

$$A + B + C = 6 + 1632 + 529 = \boxed{2167}$$

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Question #7

#7 Alpha School Bowl
MAΘ National Convention 2023

Let $A = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{2023}$

Let B be the smallest real part that a solution to $x^3 = 8i$ can have.

Let

$$C = \sum_{n=0}^{\infty} \frac{\sin\left(\frac{n\pi}{6}\right)}{\left(\sqrt{3}\right)^n}$$

Hint: $\sin(x) = Im(e^{ix})$

Let D be the sum of the 2023rd roots of unity which have non-zero imaginary parts.

Find $10(Ai + B + C + D)$, expressed a complex number in the form $a + bi$ where a, b are real.

#7 Alpha School Bowl
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Solution:

$A = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{2023} = \left(e^{-\frac{i\pi}{3}}\right)^{2023} = \left(e^{-\frac{i\pi}{3}}\right)^{2022} e^{-\frac{i\pi}{3}} = e^{-674\pi i} e^{-\frac{i\pi}{3}} = e^{-\frac{i\pi}{3}} = \frac{1}{2} - \frac{\sqrt{3}}{2}i.$

$x^3 = 8i = 8e^{\frac{\pi i}{2}} e^{2\pi i n} \rightarrow x = 2e^{\frac{\pi i}{6}} e^{\frac{2\pi i n}{3}} = \left\{2e^{\frac{\pi i}{6}}, 2e^{\frac{5\pi i}{6}}, 2e^{\frac{3\pi i}{2}}\right\}$. Of these, $2e^{\frac{5\pi i}{6}} = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$ has the smallest real part, so $B = -\sqrt{3}$

$C = \sum_{n=0}^{\infty} \frac{\sin\left(\frac{n\pi}{6}\right)}{\left(\sqrt{3}\right)^n} = Im\left(\sum_{n=0}^{\infty} \frac{e^{\frac{n\pi i}{6}}}{\left(\sqrt{3}\right)^n}\right) = Im\left(\sum_{n=0}^{\infty} \left(\frac{e^{\frac{\pi i}{6}}}{\sqrt{3}}\right)^n\right) = Im\left(\frac{1}{1 - \frac{e^{\frac{\pi i}{6}}}{\sqrt{3}}}\right) = Im\left(\frac{\sqrt{3}}{\sqrt{3} - \frac{\sqrt{3}}{2} - \frac{1}{2}i}\right) = Im\left(\frac{\sqrt{3}}{\frac{\sqrt{3}}{2} - \frac{1}{2}i}\right) =$
$Im\left(\sqrt{3}e^{\frac{\pi i}{6}}\right) = \sqrt{3}\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$

The solutions to $z^{2023} - 1 = 0$ obviously add to zero by Vieta's formula. It is also clear that $z = 1$ will be a solution to this equation, but not $z = -1$. Therefore, all solutions with non-zero imaginary parts will add to $D = -1$.

The final answer is

$$10\left(\frac{1}{2}i + \frac{\sqrt{3}}{2} - \sqrt{3} + \frac{\sqrt{3}}{2} - 1\right) = \boxed{-10 + 5i}$$

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Question #8

The values a_1 and a_3 are chosen uniformly at random with replacement from the set $\{\pm1, \pm2, \pm3\}$

Let A be the probability that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse. (In other words, when graphed on the Cartesian plane, the graph is an ellipse with positive area).

Let B be the probability that the area contained by the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is less than or equal to 2023π, given that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse.

Find $\frac{1}{A} + B$.

Solution:

A: To be an ellipse, the discriminant $B^2 - 4AC = 16 - 4a_1a_3 < 0 \rightarrow 4 < a_1a_3$. To avoid imaginary ellipses, we also need $\delta \cdot a_3 = \begin{vmatrix} a_1 & 2 & 0 \\ 2 & a_3 & 0 \\ 0 & 0 & -2023 \end{vmatrix} \cdot a_3 = -2023a_3(a_1a_3 - 4) < 0$. Since $a_1a_3 - 4 > 0$ this means $a_3 > 0 \rightarrow a_1 > 0$ since $0 < 4 < a_1a_3$. The only possible values that fit these criteria are $a_1 = a_3 = 3$, $a_1 = 2$ & $a_3 = 3$, or $a_1 = 3$ & $a_3 = 2$ The probability is therefore $\frac{3}{6^2} = \frac{1}{12} = A$.

B: The area enclosed by $Lx^2 + Mxy + Ny^2 = 1$ is $\frac{2\pi}{\sqrt{4LN-M^2}}$. Therefore the area in this curve is $\frac{2023\pi}{\sqrt{a_1a_3-4}}$. The maximum value this could ever attain is 2023π, so the area will always be less than or equal to 2023π and the probability is thus $1 = B$.

Final Answer: $\frac{1}{A} + B = 12 + 1 = \boxed{13}$.

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Question #9

#9 Alpha School Bowl
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If $\{a_n\}_{n=1}^{\infty}$ is an arithmetic sequence with a fifth term of 23 and a fifty-fifth term of 2023, let $A = a_1$.

If $\{b_n\}_{n=1}^{\infty}$ is a geometric sequence with a first term of 2023 and a fifth term of 7, let $B = b_3$.

The sequence has each term c_n defined by a cubic polynomial $P(n) = c_n$. Given $c_1 = 7, c_2 = 23, c_3 = 63, c_4 = 139$, let $C = P(6)$.

If $M_n = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n$, let $D = \det(M_{2023})$

Find $A + B + C + D$.

#9 Alpha School Bowl
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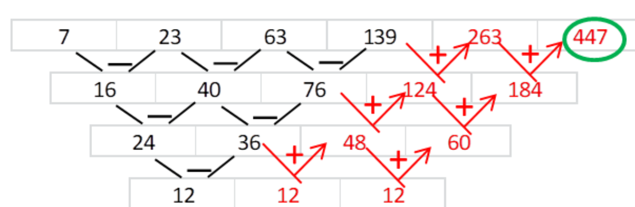
Solution:

If $\{a_n\}_{n=1}^{\infty}$ is an arithmetic sequence, then in general $a_n = a_1 + (n-1)d$. Therefore $a_5 = 23 = a_1 + 4d$ and $a_{55} = 2023 = a_1 + 54d$. Subtracting these equations gives $2000 = 50d \rightarrow d = 40 \rightarrow a_1 = 23 - 160 = -137 = A$

If $\{b_n\}_{n=1}^{\infty}$ is a geometric sequence with a first term of 2023 then $b_n = 2023r^{n-1}$. So $b_5 = 7 = 2023r^4 \rightarrow \frac{1}{289} = r^4 \rightarrow r = \frac{1}{\sqrt{17}} \rightarrow b_3 = 2023\left(\frac{1}{17}\right) = 119$



Polynomial sequences are defined via finite differences:
So $C = 447$
Since $\det(M^n) = (\det(M))^n = (-1)^n, D = (-1)^{2023} = -1$
The final answer is

$$-137 + 119 + 447 - 1 = \boxed{428}$$

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Question #10

#10 Alpha School Bowl
MAΘ National Convention 2023

Consider the following lines in three-dimensional Cartesian space:

$$\text{Line } \mathcal{L}_1: x+1 = \frac{y-1}{2} = \frac{z-3}{2}$$
$$\text{Line } \mathcal{L}_2: \frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{6}$$

Let D_1 be the minimum distance between the point (2,2,3) and $\mathcal{L}_1$.
$D_1^2 = \frac{m}{n}$ in simplest form, $A = m + n$.

Let D_2 be the minimum distance between $\mathcal{L}_1$ and $\mathcal{L}_2$.
$D_2^2 = \frac{m}{n}$ in simplest form. $B = m + n$.

Find $A + B$.

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Solution:

$\mathcal{L}_1$ goes through the point $P_1: (-1,1,3)$ and has directional vector $\overrightarrow{v_1} = \langle 1,2,2 \rangle$. $\mathcal{L}_2$ goes through the point $P_2(1,3,-2)$ and has directional vector $\overrightarrow{v_2} = \langle 2,3,6 \rangle$.

If we define $Q: (2,2,3)$ then the distance A is given by $d = \frac{\|\overrightarrow{v_1} \times \overrightarrow{P_1Q}\|}{\|\overrightarrow{v_1}\|} = \frac{\|\langle 1,2,2\rangle \times \langle -3,-1,0\rangle\|}{\|\langle 1,2,2\rangle\|} = \frac{1}{3}\left\| \begin{vmatrix} \hat{\imath} & j & k \\ 1 & 2 & 2 \\ -3 & -1 & 0 \end{vmatrix} \right\|$
$= \frac{\|\langle 2,-6,5\rangle\|}{3} = \frac{\sqrt{65}}{3}$. So $A^2 = \frac{65}{9}$

The distance between these lines is given by $d = \frac{|(\overrightarrow{v_1} \times \overrightarrow{v_2}) \cdot \overrightarrow{P_1P_2}|}{\|\overrightarrow{v_1} \times \overrightarrow{v_2}\|}$. $\overrightarrow{v_1} \times \overrightarrow{v_2} = \begin{vmatrix} \hat{\imath} & j & k \\ 1 & 2 & 2 \\ 2 & 3 & 6 \end{vmatrix} = \langle 6,-2,-1 \rangle$.
$\overrightarrow{P_1P_2} = \langle -2,-2,5 \rangle$. So $d = \frac{|-12+4-5|}{\sqrt{36+4+1}} = \frac{13}{\sqrt{41}}$ and $B^2 = \frac{169}{41}$

Therefore, the final answer is

$$65 + 9 + 169 + 41 = \boxed{284}$$

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Question #11

#11 Alpha School Bowl
MAΘ National Convention 2023

The below table gives a numerical value assign to each listed kind of plane curve:

Circle	10	Limaçon with Inner Loop	20	Lemniscate	8
Non-Circular Ellipse	11	Cardioid	3	Rose with n Petals	n
Hyperbola	4	Dimpled Limaçon	6	Line	1
Parabola	7	Convex Limaçon	2	Any Other Curve	0

Find the sum of the values of the polar graphs of the following twelve equations:

$r(\theta) = 2023\cos(\theta)$

$r(\theta) = 3\theta + 1$

$r(\theta) = 2023 + 2024\sin(\theta)$

$r(\theta) = 289\sin(3\theta)$

$r^2(\theta) = 289\sin(2\theta)$

$r(\theta) = \sec(\theta)\tan(\theta)$

$r(\theta) = -\csc(\theta)$

$r(\theta) = 7$

$r(\theta) = \sin^2\left(\frac{\theta}{2}\right)$

$\theta = \frac{\pi}{4}$

$r(\theta) = \frac{4}{2+\cos(\theta)}$

$r(\theta) = \sin(\theta)\cos(\theta)\cos(2\theta)\cos(4\theta)$

Values can be used more than once or not at all.

#11 Alpha School Bowl
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Solution:

$r(\theta) = 2023\cos(\theta)$
Circle: 10

$r(\theta) = 3\theta + 1$
Spiral: 0

$r(\theta) = 2023 + 2024\sin(\theta)$
$\frac{2023}{2024} < 1$ →Inner Loop: 20

$r(\theta) = 289\sin(3\theta)$
Rose with 3 petals: 3

$r^2(\theta) = 289\sin(2\theta)$
Lemniscate: 8

$r(\theta) = \sec(\theta)\tan(\theta)$
$r = \frac{\sin(\theta)}{\cos^2(\theta)} \to r^2\cos^2(\theta) = r\sin(\theta) \to$
$x^2 = y$ →Parabola: 7

$r(\theta) = -\csc(\theta)$
Line: 1

$r(\theta) = 7$
Circle: 10

$r(\theta) = \sin^2\left(\frac{\theta}{2}\right)$
$= \frac{1}{2} - \frac{1}{2}\cos(\theta)$ →Cardioid: 3

$\theta = \frac{\pi}{4}$
Line: 1

$r(\theta) = \frac{4}{2+\cos(\theta)}$
$\frac{4}{2+\cos(\theta)} = \frac{2}{1+0.5\cos(\theta)} \to e = 0.5$ →Ellipse: 11

$r(\theta) = \sin(\theta)\cos(\theta)\cos(2\theta)\cos(4\theta)$
$= \frac{1}{2}\sin(2\theta)\cos(2\theta)\cos(4\theta) =$
$\frac{1}{4}\sin(4\theta)\cos(4\theta) = \frac{1}{8}\sin(8\theta)$ →Rose with 16 petals: 16

The total is $10 + 0 + 20 + 3 + 8 + 7 + 1 + 10 + 3 + 1 + 11 + 16 = \boxed{90}$

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#12 Alpha School Bowl
MAΘ National Convention 2023

Let $A = \lim_{x \to 3} \frac{x^3 - 3x^2 + 4x - 12}{x^2 + x - 12}$

Let $B = \lim_{x \to \infty} \frac{x^2 - 3x^3 + 4x - 12}{x^2 + x^3 - 12}$

Let $C = \lim_{x \to 0} (1 + 2023x)^{\frac{2}{x}}$

Let $D = \lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3}$

Find $70A + B + \ln(C) + D$

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Solution:

$A = \lim_{x \to 3} \frac{x^3 - 3x^2 + 4x - 12}{x^2 + x - 12} = \lim_{x \to 3} \frac{(x-3)(x^2+4)}{(x-3)(x+4)} = \frac{13}{7}.$

$B = \lim_{x \to \infty} \frac{x^2 - 3x^3 + 4x - 12}{x^2 + x^3 - 12} = -3.$

$C = \lim_{x \to 0} (1 + 2023x)^{\frac{2}{x}} = \lim_{x \to \infty} \left(1 + \frac{2023}{x}\right)^{2x} = e^{4046}.$

$D = \lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3} = \lim_{x \to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3} = \lim_{x \to 9} \left(\sqrt{x} + 3\right) = 6.$

The final answer is $70\left(\frac{13}{7}\right) - 3 + 4046 + 6 = \boxed{4179}$

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#13 Alpha School Bowl
MAΘ National Convention 2023

Let $A = \sum_{n=1}^{K} 2023$

Let $B = \sum_{n=1}^{2023} n$

Let $C = \sum_{n=1}^{2023} n^2$

Let $D = \sum_{n=1}^{2023} n^3$

Find the smallest positive integer value of K so that $\gcd\left(A, \frac{BC}{D}\right) > 1$.

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Solution:

$A = \sum_{n=1}^{K} 2023 = 2023K = 7 \cdot 17^2 \cdot K$.

$B = \sum_{n=1}^{2023} n = \frac{2023(2024)}{2}$.

$C = \sum_{n=1}^{2023} n^2 = \frac{2023(2024)(4047)}{6}$.

$D = \sum_{n=1}^{2023} n^3 = \frac{2023^2 2024^2}{4}$.

Since $\frac{BC}{D} = \frac{\left(\frac{2023(2024)}{2}\right)\left(\frac{2023(2024)(4047)}{6}\right)}{\frac{2023^2 2024^2}{4}} = \frac{4047}{3} = 1349 = 19 * 71$, $K = \boxed{19}$ will be the smallest positive value of K so that A is not relatively prime to $\frac{BC}{D}$.

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#14 Alpha School Bowl
MAΘ National Convention 2023

The partial fraction decomposition of $\frac{25}{(x-2)^2(x^2+1)}$ is $\frac{Ax+B}{x^2+1}+\frac{C}{x-2}+\frac{D}{(x-2)^2}$.

Find $A+B+C+D$.

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Solution:

$\frac{25}{(x-2)^2(x^2+1)}=\frac{Ax+B}{x^2+1}+\frac{C}{x-2}+\frac{D}{(x-2)^2} \to 25=(Ax+B)(x-2)^2+C(x-2)(x^2+1)+D(x^2+1)$

When $x=2, 25=5D \to D=5$. When $x=0, 25=4B-2C+5 \to C=2B-10$. When $x=1,\ 25=A+B+(2B-10)(-1)(2)+5(2) \to 25=A-3B+30 \to -5=A-3B \to -35=7A-21B$. When $x=3,\ 25=3A+B+(2B-10)(1)(10)+5(10) \to 25=3A+21B-50 \to 75=3A+21B$.

Adding the last two together gives $40=10A \to A=4$ which gives $-5=4-3B \to B=3$ which gives $C=-4$. Adding them together results in

$$4+3-4+5=\boxed{8}$$

ANSWERS

0. 6
1. 31
2. 2026
3. 162
4. 145
5. $\frac{22}{7}$
6. 2167
7. $-10 + 5i$
8. 13
9. 428
10. 284
11. 90
12. 4179
13. 19
14. 8