Alpha ciphering solutions nationals 2023

0. $\left(\cos^2 x+\sin^2 x\right)\left(\cos^2 x-\sin^2 x\right)=\left(\cos^2 x-\sin^2 x\right)=\cos 2x \to \frac{2\pi}{2}=\pi$

1. You could set up a system of equations and solve for the explicit rule but it is faster to know that the differences of the differences of the differences are the same in a cubic sequence.

8 20 38 62

12 18 24

6 6

$6+24=30 \to 62+30=92 \to 126+92=218$

2.
$$4x^2+9y^2-16x+90y+205=0 \to 4\left(x^2-4x+4\right)+9\left(y^2+10y+25\right)=-205+16+225$$
$$\frac{(x-2)^2}{9}+\frac{(y+5)^2}{4}=1 \to a=3, b=2, c=\sqrt{5}$$
$$\frac{1}{2}bh=\frac{1}{2}\bullet\frac{2b^2}{a}(a+c)=b^2+\frac{cb^2}{a}=4+\frac{4\sqrt{5}}{3}=\frac{12+4\sqrt{5}}{3}$$

3.
$$x^2+18x+30=2\sqrt{x^2+18x+45} \to y=x^2+18x+45$$
$$(y-15)^2=4y \to y^2-34y+225=0 \to y=9,25$$
$$9=x^2+18x+45 \to x^2+18x+36=0 \to extraneous$$
$$25=x^2+18x+45 \to x^2+18x+20=0 \to \frac{c}{a}=20$$

4. Treat the 4000’s and 6000’s different than the 5000’s since they lead with an even and 5000’s don’t

$2\bullet 8\bullet 7\bullet 4+1\bullet 8\bullet 7\bullet 5=448+280=728$

5. rewrite: $x^{2023}-\left(x-\frac{1}{2}\right)^{2023}=0$. The x^{2023} terms will cancel so we need the next two after that to use sum of roots formula. $x^{2023}-\left(x-\frac{1}{2}\right)^{2023}=0\to\frac{-b}{a}=\frac{\frac{2023\bullet 2022}{2}\left(\frac{1}{2}\right)^2}{2023\bullet\frac{1}{2}}=\frac{1011}{2}$

6. $cos(a+b)=cos\,a\,cos\,b-sin\,a\,sin\,b=\frac{2}{5\sqrt{2}}\bullet\frac{\sqrt{97}}{10}-\frac{\sqrt{46}}{5\sqrt{2}}\bullet\frac{-\sqrt{3}}{10}$

$$\frac{\sqrt{2}\bullet\sqrt{97}+\sqrt{69}}{50}=\frac{\sqrt{69}+\sqrt{194}}{50}$$

7. $3x+b=x^2-4x+6\to x^2-7x+6-b=0\to D=0=49-4(6-b)$

$b=\frac{-25}{4}\to y=3x-\frac{25}{4}\to\frac{25}{12}$

8. $\frac{-1}{(k-2)(k-3)}=\frac{A}{k-2}+\frac{B}{k-3}\to-1=A(k-3)+B(k-2)\to A=1,B=-1$

$\frac{1}{k-2}-\frac{1}{k-3}=\left(\frac{1}{2}-1\right)+\left(\frac{1}{3}-\frac{1}{2}\right)+\left(\frac{1}{4}-\frac{1}{3}\right)+\ldots=-1$

9. Draw yourself a picture and set up similar triangles. Call the side of the square x.

$\frac{12}{5}=\frac{12-x}{\frac{x}{2}}\to 6x=60-5x\to x=\frac{60}{11}$

10.
$$\begin{matrix} 4 & -1 & 0 & 4 & -1 \\ -2 & 1 & 3 & -2 & 1 \\ i & j & k & i & j \end{matrix} \to 4k-3i-\left(12j+2k\right)=-3i+2k-12j \to 72$$

11.
$$2\log_4(x+1)+3\log_8(x-3)=\log_2 12$$
$$\log_2(x+1)+3\log_2(x-3)=\log_2 12$$
$$x^2-2x-3=12 \to \left(x-5\right)\left(x+3\right)=0 \to x=5$$

12.
$$\frac{\frac{1}{2}ab\sin 2c}{\frac{1}{2}ab\sin c}=\frac{\sin 2c}{\sin c}=\frac{2\sin c\cos c}{\sin c}=2\cos c=\frac{4\sqrt{2}}{3}$$

Is it too easy?

Answers:

0. π

1. 218

2. $4+\frac{4\sqrt{5}}{3}=\frac{12+4\sqrt{5}}{3}$

3. 20

4. 728

5. $\frac{1011}{2}$

6. $\frac{\sqrt{69}+\sqrt{194}}{50}$

7. $\frac{25}{12}$

8. -1

9. $\frac{60}{11}$

10. 72

11. 5

12. $\frac{4\sqrt{2}}{3}$