Unless otherwise specified, a die refers to a fair six-sided die, a standard deck of cards refers to a 52-card deck with cards ranging from Ace to King in four different suits (clubs, diamonds, hearts, and spades), $E[X]$ refers to the expected value of the random variable X, and $f(X)$ refers to the random variable that has a value of $f(x)$ when the value of X is x.

1. Luke picks a real number uniformly at random from the interval [0, 10]. What is the probability that he picks 4?

   A. $\frac{1}{4}$ B. $\frac{1}{10}$ C. $\frac{2}{5}$ D. Not enough information E. NOTA

2. James is trying to decide how many origami creations he will make today. He will make 4 with probability $\frac{1}{3}$, 16 with probability $\frac{1}{4}$, and 9 with probability $\frac{5}{12}$. If the random variable N represents the number of origami creations James will make today, what is $E[\sqrt{N}]$?

   A. 3 B. $\frac{35}{12}$ C. $\frac{\sqrt{327}}{6}$ D. $\frac{\sqrt{109}}{12}$ E. NOTA

For problems 3 and 4, consider the following scenario. Andy is playing Texas Hold 'em against Buffy, in which each player has two cards that the other cannot see. Andy holds a pair of aces while Buffy holds a pair of kings. On the flop, three new cards will be revealed from the top of the deck that both players can see and use for their hand. Assume a standard deck of cards is being used.

3. What is the probability that neither an ace nor a king comes up on the flop given all the information above?

   A. $\frac{3311}{4324}$ B. $\frac{3311}{4900}$ C. $\frac{3311}{4900}$ D. $\frac{4324}{5525}$ E. NOTA

4. From Buffy's point of view, what is the probability that at least one card on the flop will be a king, but none will be an ace? Assume that Buffy does not know and does not make any assumptions about the cards Andy holds.

   A. $\frac{473}{4900}$ B. $\frac{121}{1225}$ C. $\frac{121}{1081}$ D. $\frac{473}{4324}$ E. NOTA

5. The angle θ is chosen uniformly at random on the interval $[0, 2\pi)$. What is the probability that $\sin(\theta) + \cos(\theta) < \sqrt{\frac{2+\sqrt{3}}{2}}$?

A. $\frac{2}{3}$ B. $\frac{3}{4}$ C. $\frac{5}{6}$ D. $\frac{11}{12}$ E. NOTA

6. At a party, every child shake hand with every other child once. Every parent shake hand with every other parent once. There are no handshakes between parents and children. If there are 40 parents and 50 children, how many handshakes occurred?

A. 2075 B. 2045 C. 2055 D. 2005 E. NOTA

7. There are 5 ice cream flavors available at Sharay's Sweet Treats: Vanilla, Chocolate, Strawberry, Mint, and Rocky Road. Customers may choose to get the Cool Combo which consists of 3 scoops of ice cream that may be different flavors, but not necessarily. The order of the scoops in the Cool Combo is irrelevant. How many distinct Cool Combos can a customer buy. Assume that one scoop of ice cream can only be one flavor.

A. 10 B. 25 C. 30 D. 35 E. NOTA

8. An urn contains 4 yellow balls, 3 red balls, 3 green balls. If you take all the balls out of the urn one at a time, what is the probability that the fourth and fifth ball you take out will be a yellow ball and red ball in that order?

A. $\frac{3}{25}$ B. $\frac{6}{25}$ C. $\frac{2}{15}$ D. $\frac{4}{15}$ E. NOTA

9. Sarthak the snail is crawling along the edges of tetrahedron ABCD. He starts at vertex A, and every minute, he moves to another vertex. When Sarthak is on vertex A, B, or D, he has an equal probability of moving to any other vertex, but when he is on vertex C, he has a $\frac{1}{2}$ probability of moving to vertex D, a $\frac{1}{4}$ probability of moving to vertex A, and a $\frac{1}{4}$ probability of moving to vertex B. What is the probability that Sarthak is on vertex D after 3 minutes?

A. $\frac{11}{36}$ B. $\frac{7}{36}$ C. $\frac{2}{9}$ D. $\frac{5}{18}$ E. NOTA

10. The probability that both events A and B occur is 30%, that both events B and C occur is 40%, and both A and C occur is 35%. The probability that exactly two of events A, B, and C occur is twice the probability that all of A, B, and C occur. What is the probability That all of A, B, and C occur?

A. 26.25% B. 30% C. 52.5% D. 21% E. NOTA

11. You are given two special cards in an opaque envelope. One card is red on both sides while the other card is red on one side and black on the other side. You take one card out of the envelope without looking at it and randomly choose a face to place facing up on a table. A red face is showing. What is the probability that card is black on the other side?

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. NOTA

12. Mr. Frazer is shooting free throws. He has a 25% chance of making each shot. What is the expected number of attempts Mr. Frazer will take to make his first shot?

A. 3 B. 4 C. 5 D. 6 E. NOTA

13. Ms. Hurst is pairing people up to work on a biology lab. The class has 40 people, all of whom will be paired up. What is the probability that she will pair up the two students Jeffrey and Helena?

A. $\frac{1}{39}$ B. $\frac{1}{780}$ C. $\frac{1}{20}$ D. $\frac{1}{390}$ E. NOTA

14. John is the co-pilot of an airplane, so his favorite trigonometric functions are naturally those with the "co" prefix (cosecant, cotangent, and cosine). Eric, the pilot, randomly selects a value of θ on uniformly at random from the interval $\left(0, \frac{\pi}{2}\right)$ and John selects a function, f, from his favorite trigonometric functions, each with equal probability. The probability that $f(\theta) < 2$ can be expressed as $a + b\left(\frac{\arctan(2)}{\pi}\right)$ where a, b are rational. What is ab?

A. $\frac{8}{27}$ B. $\frac{10}{27}$ C. $\frac{4}{9}$ D. $\frac{5}{9}$ E. NOTA

15. Archie the acrobat is walking up a staircase. There are 8 steps he must climb. With each stride he takes, he can either climb either 1 or 2 steps. However, in between strides he also may do a backflip on the stairs (he is an acrobat after all). He does at most one backflip in between each pair of consecutive strides or at the top of the stairs, but not at the bottom. How many possible sequences of steps and backflips can Archie take. Examples of valid sequences include 1 step, backflip, 2 steps, 2 steps, 1 step, 2 steps or 2 steps, 2 steps, backflip, 2 steps, 1 step, backflip, 1 step.

A. 288 B. 896 C. 1936 D. 2448 E. NOTA

16. 1336 unfair icosahedral dice and 1 fair icosahedral die are rolled simultaneously. The unfair dice are weighted such that the probability of rolling the number n on any one of them is inversely proportional to n. What is the probability that the sum of the numbers shown on all 1337 dice is a multiple of 10?

A. $\frac{1}{2}$ B. $\frac{1}{5}$ C. $\frac{1}{10}$ D. $\frac{1}{20}$ E. NOTA

17. Jake has an unfair coin that has his face on one side and his brother Ryan's face on the other. When flipped, Jake's face comes up $\frac{2}{3}$ of the time. If Jake flips the coin 5 times, what is probability that Ryan's face will come at least 4 times?

A. $\frac{10}{243}$ B. $\frac{11}{243}$ C. $\frac{80}{243}$ D. $\frac{112}{243}$ E. NOTA

18. Amy and Kejin are playing tennis. They are evenly matched, so they each have a 50% chance of winning a single point. In a certain game, Kejin has the advantage, meaning he has one more point than Amy. If either player gains a two-point lead over the other, that player wins. What is the probability that Amy will win the game?

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{3}{8}$ D. $\frac{1}{2}$ E. NOTA

19. Sean has a fair coin and an unfair coin, which lands on tails 60% of the time. Sean picks one of the coins randomly, such that he has a $\frac{2}{3}$ chance of choosing the fair coin, and tosses it four times, getting two heads and two tails. He guesses that he picked the fair coin. What is the probability that he is correct?

A. $\frac{2}{3}$ B. $\frac{625}{913}$ C. $\frac{1152}{1777}$ D. $\frac{2859}{4207}$ E. NOTA

20. Robert is playing roulette at a casino. In roulette, a wheel is spun and a marble rolls randomly into 1 of 37 spots, numbered 0 through 36. The marble is equally likely to land in any of the 37 spots. Robert may place a bet on any number from 1 through 36. He receives a payout of 36 times his bet (for a profit of 35 times his bet) if it lands there but receives a payout of 0 (for a loss the size of his bet) otherwise. He may not place a bet that the marble will land on 0. He may place multiple independent bets of different sizes on different numbers for a single spin. Which of the following are true?

I. A strategy exists such that Robert has a positive expected profit
II. Assume that the game is rigged such that the marble lands on the spot numbered 36 with probability $\frac{1}{37}+\frac{1}{1400}$. Robert has a positive expected profit if he bets on number 36.
III. Assume that the game is rigged such that the marble lands on the spot numbered 36 with probability $\frac{1}{37}+\frac{1}{1300}$. Robert has a positive expected profit if he bets on number 36.

A. I, II, and III B. I only C. III only D. None E. NOTA

21. Ian is listening 600 song playlist. Each song has a unique integer associated with it from 1 to 600 inclusive. He plays the playlist in a random order in which songs cannot repeat. What is the probability that the order in which the songs play has the property that the sum of the numbers associated with any two songs played next to each other is always odd?

A. $\frac{300!}{600!}$ B. $\frac{2\cdot 300!}{600!}$ C. $\frac{2}{\binom{600}{300}}$ D. $\frac{(300!)^2}{600!}$ E. NOTA

22. A circular keychain has 3 keys on it. In how many distinguishable ways can the keys be arranged? Two arrangements are not considered distinct if one can be obtained from the other by rotating the key chain or flipping it over.

A. 1 B. 2 C. 3 D. 6 E. NOTA

23. An angle θ is chosen at random from an exponential distribution with mean 1. What is the probability that $\sin^2\theta+\cos^2\theta<1$?

A. 0 B. $\frac{1}{2}$ C. $\frac{1}{4}$ D. $\frac{1}{e}$ E. NOTA

24. Consider the projection of the vector $\langle a, 4a, 9a\rangle$ onto $\langle 1, -2, 3\rangle$ where a is a real number chosen uniformly at random from the interval $[0, 10]$.What is the probability that the magnitude of this projection is less than 30?

A. $\frac{3\sqrt{14}}{20}$ B. $\frac{3\sqrt{14}}{10}$ C. 1 D. $1 - \frac{\sqrt{14}}{4}$ E. NOTA

25. Evaluate $\sum_{n=3}^{32}\left(n \cdot \binom{n-1}{2}\right)$

A. 10660 B. 31980 C. 40960 D. 122760 E. NOTA

26. How many of the following are necessarily true of expected value where X and Y are random variables and c is a constant?

I. $E[X+Y] = E[X] + E[Y]$
II. $E[XY] = E[X]E[Y]$ when X and Y are independent
III. $E[XY] = E[X]E[Y]$ when X and Y are *not* independent
IV. $E[cX] = cE[X]$
V. $E[X+c] = E[X]$

A. 1 B. 2 C. 3 D. 4 E. NOTA

27. You have 4 different bracelets, 3 pairs of gloves, and 5 baseball caps (which are all considered accessories for this problem). You can wear at most 1 of each type of accessory (i.e. 1 bracelet, 1 pair of gloves, and 1 baseball cap), but you must wear at least 1 accessory. How many assortments of accessories can you wear given these constraints?

A. 59 B. 60 C. 119 D. 120 E. NOTA

28. If you pick a random seventeenth root of unity such that all seventeenth roots of unity have an equal probability of being selected, what is the probability that its cube will be in the first quadrant of the Argand plane (being on an axis does not count as being in a quadrant)?

A. $\frac{2}{17}$ B. $\frac{3}{17}$ C. $\frac{4}{17}$ D. $\frac{5}{17}$ E. NOTA

29. $a, b,$ and c are independently chosen uniformly at random from the interval $[0, 10]$. What is the probability that the magnitude of the vector $\langle a, b, c \rangle$ is less than 6?

A. $\frac{9\pi}{250}$ B. $\frac{9\pi}{125}$ C. $\frac{18\pi}{125}$ D. $\frac{36\pi}{125}$ E. NOTA

30. There are thirty seconds left in a Mu Alpha Theta test, and Samuel has not answered the last three questions! He guesses C on each of the last three questions. The answer to each question has a 10% probability of being E and equal probabilities of being any of the other four answer choices. Assuming he answers the first 27 questions correctly, no questions are thrown out, and the scoring system is 5 points for a correct answer, 1 point for a blank, and 0 points for an incorrect answer, what is the expected value of Samuel's score on the test?

A. 138.125 B. 138.375 C. 137.625 D. 137.875 E. NOTA