1. X, Y, and Z are three independently occurring events. Their probabilities are 0.6, 0.3, and 0.1, respectively. Find the probability that at least one of the following scenarios is satisfied –

   I. Both Y and Z occur II. Neither X nor Y occur.

   A. 0.21 B. 0.24 C. 0.28 D. 0.31 E. NOTA

2. A coin is flipped 4 times. If at least one head is flipped, then what is the expected number of tails?

   A. $\frac{4}{3}$ B. $\frac{3}{2}$ C. $\frac{7}{4}$ D. 2 E. NOTA

3. How many distinct permutations are there of the 8 characters in the words "RACE CAR" if the space between the words cannot be placed in front of the first letter or behind the last letter? (For example, RACE CAR would be considered different to RAC ECAR)

   A. 3780 B. 5040 C. 630 D. 840 E. NOTA

4. The ratio of boys to girls at a party is 2 to 5. After a while, 11 girls leave, and 3 boys leave. The new ratio of girls to boys is 4 to 3. What was the original number of people at the party?

   A. 21 B. 28 C. 35 D. 42 E. NOTA

5. Terrence is giving his 3 imaginary friends pieces of candy to celebrate their friendship. Terrence has 11 identical pieces of candy to distribute amongst the friends. If each friend gets at least one piece of candy, then find the number of ways he can give out the pieces.

   A. 11 B. 45 C. 78 D. 165 E. NOTA

6. People A and B are getting new 10-digit telephone numbers. Person A's telephone number starts with 505, while person B's starts with 336. The last seven digits in A's telephone number will be four 5's and three 0's in random positions, while the last seven digits in person B's telephone number will consist of two 3's and five 6's in random positions. The two telephone numbers are independent of each other. Find the probability both A's and B's telephone number are palindromes.

   A. $\frac{4}{735}$ B. $\frac{2}{343}$ C. $\frac{3}{245}$ D. $\frac{1}{147}$ E. NOTA

7. Fred has 3 green balls and 2 red balls. He puts them into a hat. George chooses a ball at random. If the ball is red, he replaces it with a green ball. If it is green, he replaces with a red ball. Afterwards, Fred takes out a ball at random. What is the probability Fred takes out a red ball?

A. $\frac{9}{25}$ B. $\frac{2}{5}$ C. $\frac{11}{25}$ D. $\frac{12}{25}$ E. NOTA

8. Five numbers are taken at random without replacement from the integers 1 to 100. Find the probability they are taken out in ascending or descending order. (For example: 92, 65, 43, 37, 6)

A. $\frac{1}{45}$ B. $\frac{3}{160}$ C. $\frac{1}{50}$ D. $\frac{1}{60}$ E. NOTA

9. How many 3-digit palindromes are divisible by 11?

A. 8 B. 9 C. 10 D. 11 E. NOTA

10. Nine unit squares are laid out in a 3×3 grid. Three distinct unit squares are selected at random, and a dot is placed in the center of each. What is the probability that the 3 dots are collinear?

A. $\frac{3}{28}$ B. $\frac{1}{28}$ C. $\frac{1}{14}$ D. $\frac{2}{21}$ E. NOTA

11. A frog is jumping along a line of lily pads that are labeled 1, 2, 3, 4, and so on in increasing order. The frog starts on the lily pad labeled 1. Each time he jumps, he either moves 1 lily pad ahead or jumps 3 lily pads ahead, each with equal probability. The frog jumps 7 times. Find the probability it will land on the 8^{th} lily pad at some point during the 7 jumps.

A. $\frac{21}{128}$ B. $\frac{49}{128}$ C. $\frac{69}{128}$ D. $\frac{71}{128}$ E. NOTA

12. Value x is randomly selected from the interval $\left(-\frac{2\pi}{9}, \frac{5\pi}{7}\right)$. Which of the following functions has the highest probability to be a positive?

A. $\sec(x)$ B. $\csc(x)$ C. $\cot(x)$ D. x E. $-x$

13. 5 standard dice are rolled. Find the probability that all the numbers facing up are distinct from each other.

A. $\frac{1}{6}$ B. $\frac{5}{54}$ C. $\frac{2}{9}$ D. $\frac{5}{216}$ E. NOTA

14. A six-digit number is in the form $175A6B$, where A and B are digits. A is randomly assigned an integer value between 0 and 4, inclusive, while B is randomly assigned an integer value between 5 and 9, inclusive. What is the probability that the resulting six-digit number is not divisible by 3 or 4?

A. $\frac{12}{25}$ B. $\frac{13}{25}$ C. $\frac{14}{25}$ D. $\frac{16}{25}$ E. NOTA

15. 3 cards are taken at random without replacement from a standard deck of cards. Find the probability two of the cards are the same suit but not all three.

A. $\frac{196}{375}$ B. $\frac{129}{250}$ C. $\frac{167}{300}$ D. $\frac{234}{425}$ E. NOTA

16. Jim is going to a dart throwing contest. Contestants are supposed to bring a standard dart board with an outer ring radius of 7 units and an inner ring diameter of 6; however, Jim cheats and brings a modified board with the same radius for the outer ring but an inner ring radius of 5. At the competition Jim will always hit at a random position on the board. Hitting in the inner radius is 4 points and hitting in the outer ring but not in the inner ring is 1 point. Find the positive difference between the expected value of Jim's score after 1 dart throw on the modified board and the expected value of Jim's score after 1 dart throw on the standard board.

A. $\frac{33}{49}$ B. $\frac{1}{2}$ C. $\frac{48}{49}$ D. 1 E. NOTA

17. 2 random numbers from the set {1, 2, 3, 4, 5, 6} are chosen without replacement. The 2 numbers are multiplied together. Determine the probability they produce a multiple of 4 or 6.

A. $\frac{11}{15}$ B. $\frac{2}{5}$ C. $\frac{2}{3}$ D. $\frac{3}{5}$ E. NOTA

18. Nathan is at the plate in a baseball game. The pitcher has two possible pitches: fastball or curveball. Nathan has a 70% chance of hitting a fastball but only a 50% chance of hitting a curveball. The pitcher throws a fastball 60% of the time and a curveball 40% of the time, and makes his selection at random and independently for each pitch. Find the probability Nathan misses the next pitch.

A. $\frac{19}{50}$ B. $\frac{21}{50}$ C. $\frac{23}{50}$ D. $\frac{25}{50}$ E. NOTA

19. Two cards are taken out of a standard deck at random without replacement. What is the probability the cards are different suits, and neither are spades?

A. $\frac{6}{25}$ B. $\frac{3}{8}$ C. $\frac{13}{34}$ D. $\frac{9}{16}$ E. NOTA

20. The function $f(x) = \left\lfloor \frac{x^2-10x+31}{6} \right\rfloor$ represents the greatest integer that is less than or equal to $\frac{x^2-10x+31}{6}$. If x is an integer chosen at random between 1 through 10, inclusive, then what is the expected value of $f(x)$?

A. 1.8 B. 2 C. 2.5 D. 2.9 E. NOTA

21. A group of 20 students from a math competition were asked what phrase first came to their head after hearing the word "Mao". The results were 60% of the students said, "Mu Alpha Theta", 30% of the students answered "Mao Zedong", and the remaining answered something else, different from each other as well. Two different students are selected at random from this group of 20 students. Find the probability that they gave different answers.

A. $\frac{16}{25}$ B. $\frac{9}{25}$ C. $\frac{109}{190}$ D. $\frac{81}{190}$ E. NOTA

22. How many positive integral factors of 75600 are divisible by 3, 5, or both?

A. 90 B. 110 C. 100 D. 115 E. NOTA

23. Rupert is teaching a class of 8 students. Every day, at the end of class, he chooses 3 random students to form a Dum Dum team. Working together, this group of 3 gets the chance to win a grand prize—Dum Dum lollipops! After day n, it turns out every student in the class has been on a Dum Dum team with every other student at least once. What is the smallest possible value of n?

A. 10 B. 11 C. 12 D. 13 E. NOTA

24. How many three-digit numbers are there that contain both the digit 1 and the digit 2?

A. 52 B. 54 C. 62 D. 64 E. NOTA

25. Two friends Tom and Nolan are playing each other in a game. The game consists of rounds, where the winner of each round is given 1 point. There are no ties in a round. Since Nolan is a more experienced player, he has a 60% chance to beat Tom in any given round. What is the probability Tom reaches 2 points before Nolan does?

A. $\frac{2}{5}$ B. $\frac{3}{10}$ C. $\frac{8}{25}$ D. $\frac{24}{125}$ E. NOTA

26. An ant moves along the number line. The ant moves 1 unit every second, randomly selected between positive and negative directions. Find probability that it ends in the same place it started after 6 seconds.

A. $\frac{5}{16}$ B. $\frac{3}{8}$ C. $\frac{9}{32}$ D. $\frac{15}{64}$ E. NOTA

27. Jonathan wants to go to the beach during a 3-day weekend, but he doesn't want to be rained on. He will go if and only if it does not rain. The probabilities it rains on each of the three days are 0.6, 0.5, and 0.3. What is the probability Jonathan will go to the beach at least two days of three?

A. 0.45 B. 0.5 C. 0.55 D. 0.6 E. NOTA

28. Bill and Will love eating krill. They love krill so much that they decide to go to a seafood restaurant together. They order 7 pounds of krill in total and decide to share. For each of the first 6 pounds, they flip a coin to see who gets to eat that pound, heads being Bill gets to eat it and tails being Will gets to eat it. For the last pound, Will gets to eat it no matter what. Find the probability one of them eats at least 5 of the 7 pounds of krill.

A. $\frac{1}{2}$ B. $\frac{29}{64}$ C. $\frac{15}{32}$ D. $\frac{27}{64}$ E. NOTA

29. Event A has a $\frac{3}{5}$ probability of occurring and event B has a $\frac{1}{3}$ probability of occurring. Find is the probability of both events occurring.

A. $\frac{1}{3}$ B. $\frac{1}{5}$ C. $\frac{14}{15}$ D. Not enough information E. NOTA

30. There are N palindromic numbers that are 6, 7, 8, or 10 digits long. What is the sum of the digits of N?

A. 18 B. 27 C. 36 D. 81 E. NOTA