1. E Luke is picking a number from a continuous distribution. The probability of picking any one specific number from such a distribution is 0.
2. B The possible values of $\sqrt{N}$ are 2 with probability $\frac{1}{3}$, 4 with probability $\frac{1}{4}$, and 3 with probability $\frac{5}{12}$. Multiplying the values by the probabilities and adding them up, we get $2 * \frac{1}{3} + 4 * \frac{1}{4} + 3 * \frac{5}{12} = \frac{35}{12}$
3. A There are 48 cards left in the deck, 44 of which are not aces nor kings. Thus the probability we seek is $\frac{\binom{44}{3}}{\binom{48}{3}} = \frac{3311}{4324}$.
4. B Since Buffy has not seen Andy's cards, we will assume they are still in the deck for the purposes of our calculation. There are 50 cards Buffy has not seen, so the denominator will be $\binom{50}{3}$. For the numerator, we count the number of successful outcomes, which is Buffy getting exactly one king, or exactly two kings (no aces in either case). There are $2 * \binom{44}{2}$ ways for Buffy to get exactly 1 king since there are two kings left in the deck and 44 non-ace, non-king cards. Similarly there are 44 ways to get exactly 2 kings and no aces. Thus the probability we seek is $\frac{2 * \binom{44}{2} + 44}{\binom{50}{3}} = \frac{121}{1225}$.
5. D We can square this equation (we'll worry about the left side being negative later) to get $1 + \sin(2\theta) < 1 + \frac{\sqrt{3}}{2} \rightarrow \sin(2\theta) < \frac{\sqrt{3}}{2}$. We will now consider the possible values of 2θ on the interval $[0,4\pi)$ and divide by 2. The possible values of 2θ are $\left(0,\frac{\pi}{3}\right) U \left(\frac{2\pi}{3},\frac{7\pi}{3}\right) U \left(\frac{8\pi}{3},4\pi\right]$, meaning that θ can be $\left(0,\frac{\pi}{6}\right) U \left(\frac{\pi}{3},\frac{7\pi}{6}\right) U \left(\frac{4\pi}{3},2\pi\right]$. However on the interval $[\frac{7\pi}{6},\frac{4\pi}{3}]$, $\sin(\theta) + \cos(\theta)$ is negative, meaning it satisfies the equation, so we include this interval as well. Thus only interval that doesn't work is $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$, meaning the desired probability is $1 - \frac{\frac{\pi}{3}-\frac{\pi}{6}}{2\pi} = \frac{11}{12}$.
6. D There are $\binom{50}{2}$ handshakes between the children and $\binom{40}{2}$ between the parents, $\binom{50}{2}$ + $\binom{40}{2}$ = 2005.
7. D A customer can buy three different flavors, in which case there are $\binom{5}{3} = 10$ possibilities, two different flavors, in which case there are $5 * 4 = 20$ possibilities, or 1 flavor, in which case there are 5 possibilities. This gives a total of 35 possibilities.
8. C The fact that it's the fourth and fifth ball is not particularly relevant; we can just treat them as the first and second ball. There is a $\frac{4}{10}$ probability that the first ball is yellow and a $\frac{3}{9}$ probability that the second ball is red given that the first ball is yellow. Thus the answer is $\frac{3}{9} * \frac{4}{10} = \frac{2}{15}$.
9. D We can think of the probabilities that Sarthak is on each vertex as a row vector. Initially, the vector is $(1\ \ 0\ \ 0\ \ 0)$. Now we multiply by the appropriate transition matrix 3 times for each of the 3 minutes. Row i, column j of the transition matrix

represents the probability of Sarthak going to vertex j at the end of the minute given that he starts at vertex i. Thus the matrix is $\begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{bmatrix}$. Multiplying the row vector by the matrix once gives $\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$, multiplying again gives $\begin{pmatrix} \frac{11}{36} & \frac{7}{36} & \frac{2}{9} & \frac{5}{18} \end{pmatrix}$, and when we multiply for the third time, we only need the last element of the vector since that represents the probability of being on D. Thus the answer is $\frac{5}{18}$.

10. D If we call the probability that $A \cap B \cap C$ occurs x, we can see that the probability that exactly 2 events occur is $(105 - 3x)\%$ by drawing a Venn diagram, so we have $105 - 3x = 2x \rightarrow x = 21$. Thus the answer is 21%.

11. B There are 3 possible red faces you could be looking at, but only 1 of them is black on the other side. Thus the answer is $\frac{1}{3}$.

12. B This is a geometric distribution in which we are basically asked to find the mean. The mean of a geometric distribution is $\frac{1}{p} = \frac{1}{25\%} = 4$.

13. A If we think about it from Helena's point of view, there are 39 people she could be partnered with, and 1 of those people is Jeffrey, so the probability we desire is $\frac{1}{39}$.

14. B If cosine is picked, $f(\theta) < 2$ with probability 1. If cosecant is picked, we have $\csc(\theta) < 2 \rightarrow \sin(\theta) > \frac{1}{2} \rightarrow \frac{\pi}{6} < \theta < \frac{\pi}{2}$, so $f(\theta) < 2$ with probability $\frac{\frac{\pi}{2} - \frac{\pi}{6}}{\frac{\pi}{2}} = \frac{2}{3}$. If cotangent is picked, we have $\cot(\theta) < 2 \rightarrow \tan(\theta) > \frac{1}{2} \rightarrow \arctan\left(\frac{1}{2}\right) < \theta < \frac{\pi}{2}$, giving us a probability of $\frac{\frac{\pi}{2} - \arctan\left(\frac{1}{2}\right)}{\frac{\pi}{2}} = 1 - \frac{2\arctan\left(\frac{1}{2}\right)}{\pi}$. The overall probability is the average of the ones calculated so far, which is $\frac{8}{9} - \frac{2\arctan\left(\frac{1}{2}\right)}{3\pi}$. Putting this in the form of the answer, since $\arctan\left(\frac{1}{2}\right) = \frac{\pi}{2} - \arctan(2)$, the answer is $\frac{8}{9} - \frac{2\left(\frac{\pi}{2} - \arctan(2)\right)}{3\pi} = \frac{5}{9} + \frac{2\arctan(2)}{3\pi}$

15. D Let a_n be the number of ways Archie can climb n steps. We can find a recursive relationship between a_n, a_{n-1}, and a_{n-2}. If the last stride Archie takes is 1 step, there are $2 * a_{n-1}$ possibilities because Archie climbs the first $n - 1$ steps (possibly doing a backflip on the seventh step) and either does a backflip or doesn't do a backflip at the top of the stairs. If the last stride Archie takes is 2 steps, there are $2 * a_{n-2}$ possibilities by a similar argument. Thus $a_n = 2a_{n-1} + 2a_{n-2}$. We can find $a_1 = 2$ and $a_2 = 6$ by listing out the possibilities, then build our way up to a_8 using the recursive relationship we found, which gives us a total of 2448 possibilities.

16. C The sum of the 1336 unfair dice has 10 possibilities for the remainder when it is divided by 10. If the remainder is x, then a roll of $10 - x$ or $20 - x$ will make the sum divisible by 10. Thus there are always two rolls from the fair die that will make

the total sum a multiple of 10, no matter what the sum of the first 1336 dice is. Thus the answer is $\frac{2}{20} = \frac{1}{10}$.

17. B We must have Ryan's face come up either four or five times. The probability of four times is $\binom{5}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right) = \frac{10}{243}$ and the probability of 5 times is $(\frac{1}{3})^5 = \frac{1}{243}$, so our final answer is the sum of these, which is $\frac{11}{243}$.

18. A There is a $\frac{1}{2}$ chance that Kejin will win the next point, meaning Amy loses, and a $\frac{1}{2}$ chance that Amy will win the next point, meaning they are tied. When they are tied, the probability of Amy winning is $\frac{1}{2}$ since Amy and Kejin have equal probabilities of winning each point. Thus she has a $\frac{1}{2}$ chance of winning a point that gives her a $\frac{1}{2}$ chance of winning the game, so her probability of winning is $\frac{1}{2} * \frac{1}{2} = \frac{1}{4}$.

19. B The probability of choosing the fair coin and getting this outcome is $\frac{2}{3} * \binom{4}{2} * \left(\frac{1}{2}\right)^4 = \frac{1}{4}$. The probability of choosing the unfair coin and getting this outcome is $\frac{1}{3} * \binom{4}{2} * \left(\frac{2}{5}\right)^2 * \left(\frac{3}{5}\right)^2 = \frac{72}{625}$. Thus the conditional probability we desire is $\frac{\frac{1}{4}}{\frac{1}{4}+\frac{72}{625}} = \frac{625}{913}$.

20. C $N = \frac{1}{37}$ since we initially assume the game is fair. For any bet size b, the expected profit is $\frac{1}{37} * 35b + \frac{36}{37} * (-b) = \frac{-b}{37}$, which is negative, thus any bet you place (even if you place multiple bets on the same spin) has a negative expected profit, meaning that no positive expected profit strategy exists, so I is false. In order for a bet to be profitable, the probability of the marble landing there would have to be greater than $\frac{1}{36}$. $\frac{1}{37} + \frac{1}{1400} < \frac{1}{36}$, so II is false, but $\frac{1}{37} + \frac{1}{1300} > \frac{1}{36}$, so III is true. Thus III is the only true statement.

21. C The only way for this to occur is if the song numbers alternate between odd and even. There are two possible arrangements of the parities: EOEO….O or OEOE….E. Once the parities are determined, there are $300!$ ways to arrange the evens and $300!$ ways to arrange the odds. There are $600!$ total song arrangements, so the probability is $\frac{2*(300!)^2}{600!} = \frac{2}{\binom{600}{300}}$.

22. A The formula for the number of ways to arrange keys on a keychain is $\frac{(n-1)!}{2}$ where n is the number of keys. Thus the answer is $\frac{(3-1)!}{2} = 1$.

23. A Since $\sin^2\theta + \cos^2\theta = 1$, the inequality we're given is the same as $1 < 1$, which is never true. Thus the desired probability is 0.

24. A The scalar projection of vector u onto v is $\frac{u \cdot v}{|v|} = \frac{20a}{\sqrt{14}}$. Thus $\frac{20a}{\sqrt{14}} < 30 \rightarrow a < \frac{3\sqrt{14}}{2}$. Thus our desired probability is $\frac{\frac{3\sqrt{14}}{2}}{10} = \frac{3\sqrt{14}}{20}$.

25. D Using algebra we get that $n * \binom{n-1}{2} = \frac{n(n-1)(n-2)}{2!}$. Notice that $\binom{n}{3} = \frac{n(n-1)(n-2)}{3!}$, so $n * \binom{n-1}{2} = 3 * \binom{n}{2}$. Thus our desired sum equals $3 * (\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \cdots \binom{32}{3})$ which equals $3 * \binom{33}{4}$ by the hockey stick identity. Thus our final answer is 122760.

26. C I, II, and IV are properties of $E[X]$, so they are true. III is not necessarily true and V is not true since $E[X+c] = E[X] + c$. Thus 3 are true.

27. C If we count not wearing a certain accessory as a possibility, there are 5 possibilities for bracelets, 4 for gloves, and 6 for baseball caps, for a total of $5 * 4 * 6 = 120$ possibilities. However, this counts the case in which we wear none of the accessories, which is not allowed, so the answer is $120 - 1 = 119$.

28. C The set containing the seventeenth roots of unity is the same as the set containing the cubes of the seventeenth roots of unity. Thus, we can just count the number of seventeenth roots of unity in the first quadrant. The arguments of the desired roots are $\frac{2\pi}{17}, \frac{4\pi}{17}, \frac{6\pi}{17}$ and $\frac{8\pi}{17}$. Thus 4 out of 17 of the roots are in the first quadrant, making the answer $\frac{4}{17}$.

29. A Our condition is $a^2 + b^2 + c^2 < 36$. If we think about this geometrically, The possible values of $a, b,$ and c form a cube with side length 10, and the condition forms a sphere of radius 6. The intersection between the sphere and cube is an eighth of a sphere of radius 6, so the desired probability is $\frac{\frac{1}{8} * \frac{4}{3}\pi * 6^3}{1000} = \frac{9\pi}{250}$

30. B Since $22.5\% = \frac{9}{40}$, The expected value of the points that Samuel gets from 1 question is $\frac{9}{40} * 5 = \frac{9}{8}$. Guessing on 3 questions will give him an expected gain of $\frac{3}{8}$ points. He already has $27 * 5 = 135$ points from the first 27 questions, so his expected score for the test is $135 + \frac{27}{8} = 138.375$.