For all questions, NOTA means "None of the Above" and $i = \sqrt{-1}$. Additionally,

- For $z = a + bi,\ Re(z) = a, Im(z) = b,\ \overline{z} = a - bi$.
- $\sinh(x) = \frac{e^x - e^{-x}}{2}$ and $\cosh(x) = \frac{e^x + e^{-x}}{2}$.
- $\cos\theta + i\sin\theta$ will be abbreviated as $cis(\theta)$.
- An imaginary number has, z, has $Im(z) \neq 0$ while a pure imaginary number has both $Im(z) \neq 0$ and $Re(z) = 0$.
- The argument of a complex number falls within the principle argument of $(-\pi, \pi]$.

1. What is the resulting complex number when $4 + 2i$ is rotated 45 degrees clockwise about the origin?

   A. $3\sqrt{2} - \sqrt{2}i$ B. $\sqrt{2} + 3\sqrt{2}i$ C. $\sqrt{2} - 2i$ D. $\sqrt{2} + 2i$ E. NOTA

2. Define $sic(\theta) = sin(\theta) + icos(\theta)$. Which of the following is equal to $sic\left(\frac{\pi}{3}\right) \cdot cis\left(\frac{3\pi}{4}\right) \div sic\left(\frac{-5\pi}{6}\right)$?

   A. $sic\left(\frac{5\pi}{12}\right)$ B. $cis\left(\frac{5\pi}{12}\right)$ C. $sic\left(\frac{11\pi}{12}\right)$ D. $cis\left(\frac{11\pi}{12}\right)$ E. NOTA

3. In regular hexagon ALBERT, vertex L is located at (2, 4) and A is located on the y-axis while vertex B has a y-coordinate of 1. None of the vertices are located in the second, third or fourth quadrant. Find the y-coordinate of vertex A.

   A. $10 + 2\sqrt{3}$ B. 0 C. $6 - 3\sqrt{3}$ D. $4 + 2\sqrt{3}$ E. NOTA

4. For how many two-digit positive integers n is $i^n = -i$?

   A. 22 B. 23 C. 24 D. 25 E. NOTA

5. Which of the following is equal to $(cosh(1) + isinh(1))^2$

   A. $e^2 - \frac{1}{e^2}$ B. $\frac{e^2 + \frac{1}{e^2}}{2}$ C. $\frac{e^2 - \frac{1}{e^2}}{2}i$ D. $1 - \frac{e^2}{2}i$ E. NOTA

6. Consider a fifth-degree polynomial such that each coefficient is pure imaginary. What is the minimum number of real roots that this polynomial must have?

A. 1 B. 2 C. 4 D. 5 E. NOTA

7. Evaluate $-\sqrt{-3}\cdot\sqrt{-12}$

A. -6 B. 6 C. $-6i$ D. $6i$ E. NOTA

8. Evaluate $(2-w)(2-w^2)(2-w^3)(2-w^4)$ when $w = cis\left(\frac{-4\pi}{5}\right)$.

A. 15 B. 16 C. 31 D. 32 E. NOTA

9. Find the area formed in the Argand plane by the twentieth roots of unity.

Hint: $\sin\left(\frac{\pi}{10}\right) = \frac{-1+\sqrt{5}}{4}$.

A. $-5+5\sqrt{5}$ B. $5+5\sqrt{5}$ C. $\frac{5+5\sqrt{5}}{2}$ D. $\frac{-5+5\sqrt{5}}{2}$ E. NOTA

10. If $z = \sqrt{15 + 112i}$, Evaluate $|Re(z) - Im(z)|$

A. 15 B. 1 C. 7 D. 11 E. NOTA

For the next two questions, let $f(z)$ be the reflection of z across the graph of $Re(z) = Im(z)$ on the Argand plane where f is defined for all complex numbers that satisfy $1 < Re(z) < 10$.

11. Evaluate $(z + f(z))^{12}$ when $z = 2 - i$.

A. -64 B. 64 C. -4096 D. 4096 E. NOTA

12. Evaluate $\lim_{b\to\infty} \frac{f(a+bi)}{a+bi}$ where $1 < a < 10$.

A. i B. $-i$ C. $2-i$ D. $1+i$ E. NOTA

13. Evaluate $\frac{(1+\sqrt{3}i)^{1000}}{(1-i)^{2000}}$.

A. $\frac{-1}{2}-\frac{\sqrt{3}}{2}i$ B. $\frac{-1}{2}+\frac{\sqrt{3}}{2}i$ C. $\frac{1}{2}-\frac{\sqrt{3}}{2}i$ D. $\frac{1}{2^{1000}}$ E. NOTA

14. m and n are chosen randomly and without replacement from the set containing the twelfth roots of unity. What is the probability that $\frac{m}{n}$ is either a real number or pure imaginary number?

A. $\frac{3}{11}$ B. $\frac{2}{11}$ C. $\frac{1}{6}$ D. $\frac{1}{3}$ E. NOTA

15. $3+2i$ is a complex number with argument θ where $0<\theta<\frac{\pi}{2}$. Evaluate $cot(3\theta)$.

A. $\frac{9}{46}$ B. $\frac{46}{9}$ C. $-\frac{46}{9}$ D. $-\frac{9}{46}$ E. NOTA

16. Compute

$$\sum_{k=1}^{100} i^{3k+17}$$

A. 0 B. 25 C. -25 D. 50 E. NOTA

17. Which answer choice gives every true statement out of the following 3?

I. $cos(ix)=cosh(x)$
II. $sin(ix)=-isinh(x)$
III. $cos^4(x)-sin^4(x)=cosh^4(ix)-sinh^4(ix)$

A. I and II B. II and III C. I and III D. All 3 E. NOTA

18. What shape is $|z-2+i|=|Re(z)|$ in the Argand Plane?

A. parabola B. hyperbola C. circle D. line E. NOTA

19. What is the area of the figure defined by $|z-4i| + |z+2| = 8$ in the Argand Plane?

A. $8\sqrt{59}\pi$ B. $4\sqrt{11}\pi$ C. 4π D. $4\sqrt{7}\pi$ E. NOTA

20. What is the shortest distance between $6+3i$ and the graph of $Im(z) = 3Re(z) - 5$ in the Argand Plane?

A. $\frac{\sqrt{10}}{5}$ B. $\sqrt{10}$ C. $\frac{2\sqrt{5}}{3}$ D. $\frac{2\sqrt{5}}{15}$ E. NOTA

21. x^{2i} is equivalent to $cis(k)$ for all real x such that $x > 1$. Which of the following could be k?

A. $ln(x^2)$ B. $-ln(x^2)$ C. $2+ln(x)$ D. $2ln(x^2)$ E. NOTA

22. Evaluate $\left|\frac{7+24i}{4+3i} + 2cis\left(\frac{\pi}{6}\right) - |\sqrt{2}+i|\right|$

A. 6 B. $2\sqrt{5}$ C. 8 D. $4\sqrt{2}$ E. NOTA

23. Evaluate $e^{\sqrt{2}cis\left(\frac{-\pi}{4}\right)}$

A. $e^{\sqrt{2}}cis(-1)$ B. $e^{\sqrt{2}}cis(1)$ C. $ecis(1)$ D. $ecis(-1)$ E. NOTA

24. If $sin(z) - cos(z) = 4$, which of the following is a possible value of $|e^{2iz}|$?

A. $\sqrt{142}$ B. $\frac{15}{2}$ C. 15 D. $\sqrt{449}$ E. NOTA

25. $z = (cos(53°) + cos(37°) + isin(53°) + isin(37°))^2$. Find $\frac{z+\bar{z}}{2}$.

A. $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ B. $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ C. 0 D. 1 E. NOTA

26. $\sqrt{1+cos(\theta)}+i\sqrt{1-cos(\theta)}=\sqrt{2}e^{\frac{\pi i}{5}}$. Which of the following is a possible value of θ?

A. $\frac{\pi}{5}$ B. $\frac{2\pi}{5}$ C. $\frac{4\pi}{5}$ D. $\frac{\pi}{10}$ E. NOTA

27. Let $\theta = tan^{-1}\left(\frac{4}{3}\right)$. Evaluate $\sum_{n=1}^{\infty}\frac{cos(n\theta)}{2^{n-1}}$

A. $\frac{-1}{2}$ B. $\frac{2}{13}$ C. -1 D. $\frac{3}{5}$ E. NOTA

28. The polynomial x^3-4x^2+3x+1 has distinct real roots a, b, c. Let $P(x)$ be the polynomial with leading coefficient 1 and roots ai, bi, ci. What is $P(1)$?

A. $2-\mathrm{I}$ B. 3 C. $-2+5\mathrm{i}$ D. $-2-5\mathrm{i}$ E. NOTA

29. Point A is at $6+3i$, point B is at $-9+3i$, and point O is at 0. What is the measure of angle AOB in the Argand Plane?

A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. $\frac{3\pi}{4}$ D. $\frac{5\pi}{6}$ E. NOTA

30. How many distinct real roots does the polynomial $2x^4+8x^3-3x^2+8x-5$ have? (Hint: Yes, it factors)

A. 0 B. 1 C. 2 D. 4 E. NOTA