1. A To rotate 45 degrees clockwise, we will multiply $4+2i$ by $cis(-45°)$ which gives us $(4+2i)\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)=3\sqrt{2}-\sqrt{2}i$.
2. C We will use the fact that $\text{sic}(\theta)=\text{cis}(\frac{\pi}{2}-\theta)$. Thus, we have $\text{cis}\left(\frac{\pi}{6}\right)*\text{cis}\left(\frac{3\pi}{4}\right)\div\text{cis}\left(\frac{4\pi}{3}\right)=\text{cis}\left(\frac{\pi}{6}+\frac{3\pi}{4}-\frac{4\pi}{3}\right)=\text{cis}\left(\frac{-5\pi}{12}\right)=\text{sic}\left(\frac{11\pi}{12}\right)$.
3. A Let vertex A be located at $(0,a)$ and B be located at $(b,1)$. If we rotate point B 120 degrees counterclockwise about L, we will get to point A (rotating A 120 degrees counterclockwise to get to A would result in a hexagon with points in the fourth quadrant). We can do this by shifting L to the origin. After this shift, A' is $(-2,a-4)$ and B' is $(b-2,-3)$. We will perform the rotation by treating them as complex numbers. We have $(b-2-3i)*cis(120°)=-2+(a-4)i\rightarrow(b-2-3i)*(-1+\sqrt{3}i)=-2+(a-4)i$. Expanding and matching up the real and imaginary parts gives us $2+3\sqrt{3}-b=-4\rightarrow b=6+3\sqrt{3}$ and $\sqrt{3}(b-2)+3=2a-8\rightarrow\sqrt{3}(4+3\sqrt{3})+3=2a-8\rightarrow a=10+2\sqrt{3}$.
4. B n must have a remainder of 3 when divided by 4. The lowest possible value is $4(2)+3=11$ and the highest is $4(24)+3=99$, which encompasses $24-2+1=23$ possible values.
5. E Expanding we get $(cosh(1)^2-sinh(1)^2+2sinh(1)cosh(1)i.\ cosh^2(x)-sinh^2(x)=1$ and the third term is $\frac{2(e+\frac{1}{e})(e-\frac{1}{e})}{4}i=\frac{e^2-\frac{1}{e^2}}{2}i$. Thus, our final answer is $1+\frac{e^2-\frac{1}{e^2}}{2}i$.
6. A This is the same as i times a polynomial with all real coefficients, so the answer is the same as it would be if the coefficients were all real to begin with. The minimum number of real roots for an odd degree polynomial is 1, so that is our answer.
7. B This is equal to $-i\sqrt{3}*i\sqrt{12}=6$.
8. C x^5-1 can be factored as $(x-1)(x-w)(x-w^2)(x-w^3)(x-w^4)$, where w is any fifth root of unity ($cis\left(\frac{-4\pi}{5}\right)$ is a fifth root of unity). If $x=2$, we have $2^5-1=(2-1)(2-w)(2-w^2)(2-w^3)(2-w^4)=31$, which is our desired expression.
9. D The twentieth roots of unity form a regular icosagon with radius 1. The area can be found by splitting the figure into twenty triangles at the center. Each triangle is isosceles with two sides of length 1 and vertex angle of measure 18°. Thus the area of each triangle is $\frac{1}{2}*1*1*sin(18°)=\frac{-1+\sqrt{5}}{8}$. Multiplying this by 20 gives our desired area of $\frac{-5+5\sqrt{5}}{2}$.
10. B Let $(a+bi)^2=15+112i\rightarrow a^2-b^2+2abi=15+112i$. Matching the real and imaginary parts, we get $a^2-b^2=15$ and $ab=56$. By inspection (or by substituting $b=\frac{56}{a}$ into the first equation) we find that the solutions are $a=8,b=7$ and $a=-8,b=-7$. Either way, the absolute difference between the real and imaginary parts is 1.

11. A $f(a+bi) = b+ai. z = 2-i$, so $f(z) = -1+2i$ and $z+f(z) = 2-i-1+2i = 1+i$. Thus, we have $(1+i)^{12} = (\sqrt{2})^{12}cis\left(\frac{\pi}{4}\right)^{12} = -2^6 = -64.$

12. B We have $\frac{b+ai}{a+bi} = \frac{(a+bi)i+2b}{a+bi} = i + \frac{2b}{a+bi}$. As b approaches infinity, the limit of $\frac{2b}{a+bi}$ will be the ratio of the coefficients of b in the numerator and denominator, which is $\frac{2}{i} = -2i$. Thus, the limit is $i - 2i = -i$. Alternatively, you could have multiplied the numerator and denominator by $a-bi$ and gotten the answer.

13. A Converting to polar, we have $\frac{2^{1000}cis(\frac{\pi}{3})^{1000}}{(\sqrt{2})^{2000}cis(\frac{-\pi}{4})^{2000}} = cis\left(\frac{1000\pi}{3} + 500\pi\right) = \frac{-1}{2} - \frac{\sqrt{3}}{2}i.$

14. A For a given m, there are 11 possible values for n. $\frac{m}{n}$ will be pure imaginary when m and n form a right angle on the complex plane. There will always be 2 such values of n. $\frac{m}{n}$ will be real when they are $180°$ apart on the complex plan. There will always be 1 such n. Thus, our desired probability is $\frac{2+1}{11} = \frac{3}{11}$.

15. D The complex number $(3+2i)^3$ will have an argument of 3θ. This equals $-9+46i$. The tangent of the argument of this number is $\frac{-46}{9}$, so the cotangent is $\frac{-9}{46}$.

16. A i^{3k+17} cycles every 4 numbers $(1, -i, -1, i)$, and each cycle adds up to 0, so the sum is 0.

17. C
   I. $cosh(x) = \frac{e^x+e^{-x}}{2}$ and $cos(ix) = \frac{e^{i(ix)}+e^{-i(ix)}}{2} = \frac{e^{-x}+e^x}{2}$, so they are equal. TRUE
   II. $-isinh(x) = \frac{e^x-e^{-x}}{2i}$ and $sin(ix) = \frac{e^{-x}-e^x}{2i}$. Thus, these are negations of each other and not equal. FALSE
   III. Using similar techniques to I and II, we get that $cosh(ix) = cos(x)$ and $-isinh(ix) = sin(x) \rightarrow sinh^4(x) = sin^4(x)$. Thus the two expressions are equal. TRUE.

18. A The left side represents the distance between z and the complex number $2-i$ while the right side represents the distance from z to the imaginary axis. Thus, this equation describes a set of points equidistant from a particular point and a particular line, so it is a parabola.

19. B This is an ellipse with foci at $4i$ and -2. The distance between them is $2c$ which equals $2\sqrt{5}$, so $c = \sqrt{5}$. The right side of the equation is $2a$ (the length of the major axis), so $a = 4$. Since $c^2 = a^2 - b^2$ for an ellipse, we have $5 = 16 - b^2 \rightarrow b = \sqrt{11}$. So the area of the ellipse is $4\sqrt{11}\pi$.

20. B We will consider this in the Cartesian Plane. $Re(z) = x$ and $Im(z) = y$, so our graph is a line. We want the shortest distance between the point $(6,3)$ and the line $y = 3x - 4 \rightarrow 3x - y = 4$. Using the formula for distance between a point and a line, we get $\frac{6(3)+3(-1)-4}{\sqrt{3^2+1^2}} = \frac{10}{\sqrt{10}} = \sqrt{10}$.

21. A We will rewrite x as $e^{ln(x)}$, so our expression becomes $e^{2in(x)i} = cos(2lnx) + isin(2lnx) = cos(ln(x^2)) + isin(ln(x^2))$. Thus $k = ln(x^2)$.

22. D
$$|4+3i+2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) - \sqrt{3}| = |4+4i| = 4\sqrt{2}.$$

23. D $\sqrt{2}cis(\frac{-\pi}{4}) = 1 - i$, so $e^{\sqrt{2}cis(\frac{-\pi}{4})} = e^{1-i} = e * e^{-i} = ecos(1) - eisin(1)$

24. E Squaring $sin(z) - cos(z) = 4$ gives us $sin^2(z) + cos^2(z) - 2sin(z)cos(z) = 16 \rightarrow 1 - sin(2z) = 16 \rightarrow sin(2z) = -15$. Rewriting $sin(2z)$ in exponential form, we get $\frac{e^{2iz} - e^{-2iz}}{2i} = -15 \rightarrow e^{2iz} - \frac{1}{e^{2iz}} = -30i$. Letting $e^{2iz} = k$, we get a quadratic in k: $k^2 + 30ik - 1 = 0 \rightarrow k = \frac{-30i \pm \sqrt{-896}}{2} = (-15 \pm 4\sqrt{14})i$. The possible magnitudes are $15 \pm 4\sqrt{14}$.

25. C $\frac{z + \bar{z}}{2}$ is the real part of z. Our expression can be written as $(cis(37°) + cis(53°))^2$. Since $cis(37°)$ and $cis(53°)$ have the same magnitude, their sum will have an argument that is halfway between their arguments. Thus the argument of the sum is $45°$. When the sum is squared, its argument becomes $90°$, meaning its real part is 0.

26. B Dividing both sides by $\sqrt{2}$, we get $\sqrt{\frac{1+cos(\theta)}{2}} + i\sqrt{\frac{1-cos(\theta)}{2}} = e^{\frac{\pi}{5}i}$. The left side contains the half angle formulas for cosine and sin while the right side is $cis(\frac{\pi}{5})$. Thus, our equation becomes $cis(\frac{\theta}{2}) = cis(\frac{\pi}{5})$. Thus, $\frac{2\pi}{5}$ is a possible value of θ.

27. B We know that $tan(\theta) = \frac{4}{3}$, so we can figure out that $sin(\theta) = \frac{4}{5}$ and $cos(\theta) = \frac{3}{5}$ by drawing a right triangle (it turns out to be a 3-4-5 right triangle). If we consider the sum, $\sum_{n=1}^{\infty} \frac{cis(n\theta)}{2^{n-1}}$, then our desired sum, $\sum_{n=1}^{\infty} \frac{cos(n\theta)}{2^{n-1}}$ is just the real part. Because $cis(n\theta) = cis(\theta)^n$, we can rewrite $\sum_{n=1}^{\infty} \frac{cis(n\theta)}{2^{n-1}}$ as $\sum_{n=1}^{\infty} \frac{cis(\theta)^n}{2^{n-1}}$, which is a geometric series. The first term is $cis(\theta)$, which equals $\frac{3}{5} + \frac{4}{5}i$, and the common ratio is $\frac{cis(\theta)}{2} = \frac{3}{10} + \frac{2}{5}i$ so our sum equals $\frac{\frac{3}{5}+\frac{4}{5}i}{1-\frac{3}{10}-\frac{2}{5}i}$. Rationalizing the denominator, we get $\frac{2}{13} + \frac{16}{13}i$, so our answer is the real part of this, which is $\frac{2}{13}$

28. D Substituting $\frac{x}{i}$ for x will essentially cancel out the is and give us the desired polynomial. We have $\left(\frac{x}{i}\right)^3 - 4\left(\frac{x}{i}\right)^2 + 3\left(\frac{x}{i}\right) + 1 = ix^3 + 4x^2 - 3ix + 1$. Now we must divide off the leading coefficient, which gives us our polynomial of $x^3 - 4ix^2 - 3x - i$. Thus $P(1) = -2 - 5i$.

29. C Dividing $-9 + 3i$ by $6 + 3i$, we get $-1 + i$, which has an argument of $\frac{3\pi}{4}$, which is our answer.

30. C We can factor this polynomial into two quadratics. After a bit of work, we find that it factors as $(2x^2 + 8x - 5)(x^2 + 1)$. The first factor has real roots while the second does not, so the polynomial has 2 real roots.