1. B The only real solution is x=1. Since the sum of solutions is 0, this means the sum of all non-real solutions is -1.
2. C Call g the price of a grilled cheese and m the price of a mac and cheese. From the question, we get the equations
$$2g + m = 9$$
$$5g + 3m = 24.25$$
We are looking for $8g + 5m$ which is twice the second equation minus the first equation. This means the price of 8 grilled cheese and 5 mac and cheese is $24.25 * 2 - 9 = 39.50$.
3. E Since the coefficient of the x^2 term is positive, there are infinitely many values of x that make the expression greater than or equal to 0.
4. C $\cos(2\theta) = 1 - 2\sin^2(\theta)$ so our equation turns into $1 - 2\sin^2(\theta) = \sin(\theta)$. Moving everything to the RHS, we get $2\,sin^2(\theta) + \sin(\theta) - 1 = 0$. Factoring this, we have $(2\sin(\theta) - 1)(\sin(\theta) + 1) = 0$. So, our solutions are $\sin(\theta) = -1$ or $2\sin(\theta) = 1$. The solution to the first equation in the range is $\theta = \frac{3\pi}{2}$. The solutions to the second equation are $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$. Summing all this up, we have $\frac{3\pi}{2} + \frac{\pi}{6} + \frac{5\pi}{6} = \frac{5\pi}{2}$.
5. D To get the sum of all the coefficients, that is basically when x=1 and y=1. So, substituting these values into our expressions, we get $(2 * 1 + 1)^5 = (3)^5 = 243$
6. B Call $AC = x$. Then, we will look at two triangles: ACB and ACD.
Then, by triangle inequality on ACB, we have $3 < x < 20$
By triangle inequality on ACD, we have $18 < x < 28$
So, combining both inequalities, we have $18 < x < 20$
Since x must be an integer, the only possible value is 19.
7. A If we look at the function $\cos(x)$ and $\csc(x)$ are decreasing from $0 \le x \le \frac{\pi}{2}$.
This means that the function $[\cos(x)]^{\csc(x)}$is strictly decreasing $0 \le x \le \frac{\pi}{2}$.
This means the closer x is to 0, the larger $f(x)$ is, so we get
$\alpha < \beta < \gamma => f(\alpha) > f(\beta) > f(\gamma)$
8. A Looking at the equation, we see that x=1 is a root. Synthetically dividing this out, we get $x^4 - 8x^3 + 15x^2 + 4x - 20$. Looking at this equation, we see that x=-1 is a root and synthetically dividing it out, we get the equation $x^3 - 9x^2 + 24x - 20$. Repeating this process, we get the 5 roots of the original equation is -1,1,2,2,5. This means the sum of the distinct roots is $-1 + 1 + 2 + 5 = 7$.
9. C Simplifying what we need to find, we need to find $\frac{a^2+b^2}{ab} = \frac{(a+b)^2-2ab}{ab}$. So, the numerator is sum of roots squared minus 2 times product of roots. So the numerator is $(-4)^2 - 4 = 12$. The denominator is just the product of roots which is 2. So, our answer is $\frac{12}{2} = 6$.
10. B Call $\frac{x}{y} = t$ where t is an integer. This means that $x = yt$. Substituting this into our inequality, we get $2 < \frac{ty-3y}{ty+3y} \le 13$. Simplifying our middle expression, we get $2 < \frac{t-3}{t+3} \le 13$. Rewriting our middle expressions, we have $2 < 1 - \frac{6}{t+3} \le 13$. Subtracting 1 from both sides of our inequality, we get $1 < \frac{-6}{t+3} \le 12$. Dividing both

sides by -6, we have $-2 \leq \frac{1}{t+3} < \frac{-1}{6}$. We can basically ignore the left side of the inequality as it is always true because t is an integer, and just focus on $\frac{1}{t+3} < \frac{-1}{6}$. Since t is an integer, the possible values of t are -4,-5,-6,-7,-8. The sum of all these values is -30.

11. C Breaking up our numerator, we make our expression to $\frac{4x^2\cos^2(x)}{2x\cos(x)} + \frac{25}{2x\cos(x)}$.
Simplifying our expression, we have $2x\cos(x) + \frac{25}{2x\cos(x)}$. Applying AM/GM, we have $\frac{2x\cos(x)+\frac{25}{2x\cos(x)}}{2} \geq \sqrt{(2x\cos(x))(\frac{25}{2x\cos(x)})}$ which simplifies to $\frac{2x\cos(x)+\frac{25}{2x\cos(x)}}{2} \geq 5$. So the minimum of our expression is 10.
*Note that this is achievable since the maximum of $x \cdot \cos(x) = \frac{5\pi}{2}$ in the interval, and the minimum is 0 and since it is a continuous distribution, it must hit $\frac{5}{2}$ at some point.

12. C We will use Cramer's rule to just solve for x. From Cramer's rule, we know that

$$x = \frac{\begin{vmatrix} 1 & 8 & 12 \\ 0 & 2 & 6 \\ 8 & 4 & -3 \end{vmatrix}}{\begin{vmatrix} 4 & 8 & 12 \\ 3 & 2 & 6 \\ 5 & 4 & -3 \end{vmatrix}} = \frac{162}{216} = \frac{3}{4}$$

13. D Say $y = \sqrt{3 + 2\sqrt{3 + 2\sqrt{3 + \ldots}}}$. Then, $y = \sqrt{3 + 2y}$.
Squaring both sides, we have $y^2 = 3 + 2y$. Moving everything to the LHS, we have

$$y^2 - 2y - 3 = (y-3)(y+1) = 0$$

Since y must be positive, y=3.

14. D Call f(5)=n. Since this is a cubic polynomial. Then, the differences between the terms of {-4,-2,4,20,n} is {2,6,16,n-20}. The difference of {2,6,16,n-20} is {4,10,n-36}. Finally, take the differences of {4,10,n-36} and you get {6,n-46}. From here, n-46=6 because f(x) is a cubic, so n=52.

15. A Since the tangent line passes through the point (3,2), we know that $3a + b = 2$. We also know that the equation $x^2 - 4x + 5 = ax + b$ has one solution. Moving everything to the LHS, we get $x^2 - (4 + a)x + 5 - b = 0$. Since it only has 1 solution, we know this has a determinant equal to zero, so

$$(4 + a)^2 - 4(5 - b) = a^2 + 8a + 16 - 20 + 4b = 0$$

From our first equation, we know that $b = 2 - 3a$. Substituting that in, we get $a^2 + 8a + 16 - 20 + 8 - 12a = a^2 - 4a + 4 = (a - 2)^2 = 0$ so $a = 2$.
This means that $b = 2 - 3 * 2 = -4$. Therefore, $a + b = 2 - 4 = -2$.

16. D Dividing the bottom equation by the top equation, we get $x^4 = 16$. When solving this, we get $x = \pm 2, \pm 2i$. If we observe either equation, by plugging in any of the values of x, we get a quadratic in terms of y. None of these have a double root, so for each value of x, there are 2 values of y making it so y has 8 distinct values.

17. B Since none of the numbers in the set are negative, we can simplify our expression to $2\log_y(x)$. To check what makes our expression into integers, we'll count how many of values of x make it an integer for each value of y.
For $y = 3^1$, all 15 other values of x will make our expression an integer.
For $y = 3^2$, all 15 other values of x will make our expression an integer.

For $y = 3^3$, $x = 3^{6,}3^{9,}3^{12}, 3^{15}$ make our expression an integer.
For $y = 3^4, x = 3^2, 3^6, 3^8, 3^{10}, 3^{12}, 3^{14}, 3^{16}$ make our expression an integer.
From here, we notice a pattern, for every y that has an odd exponent, the number of values of x that make our expression an integer is $\left\lfloor \frac{16}{exponent} - 1 \right\rfloor$. For every y that has an even exponent, then number of values of x is $\left\lfloor \frac{16}{\frac{exponent}{2}} - 1 \right\rfloor = \left\lfloor \frac{32}{exponent} - 1 \right\rfloor$
So, our probability is $\frac{15+15+4+7+2+4+1+3+2+1+1+1}{16*15} = \frac{7}{30}$

18. C This is just an application of the triangle inequality. We know that

$$|3a| = 3|a| = 3 * 2 = 6, |2b| = 2|b| = 2 * 8 - 16$$

From triangle inequality, we have

$$|3a + 2b + c| \le |3a| + |2b| + |c| = 6 + 16 + 5 = 27$$

19. A Since our divisor is a quadratic, we know that the maximum degree of our remainder is 1, meaning our remainder is in the form of $ax + b$.
We also know that $x^{75} - 25x^{73} + 13 = q(x)(x^2 - 4x - 5) + ax + b$ where $q(x)$ is the quotient when $x^{75} - 25x^{73} + 13$ is divided by $(x^2 - 4x - 5)$ by the definition of division.
Plugging in $x = 5$ and $x = -1$ respectively, we get the two equations

$$5a + b = 13$$
$$-a + b = 37$$

Solving these equations, we get $\mathrm{a} = -4, \mathrm{b} = 33$ so the remainder is $-4x + 33$

20. C First, we will find the “slope” vector. To do this, we will take the cross product of the coefficients of the planes, giving us

$$< 4, -2, -3 > \times < 3,1,4 > = < -5, -25,10 >$$

We can divide a -5 out of each component, giving us $< 1,5, -2 >$
Now, we have to find a point that is on both lines, which on observing the answer choices gives us (1, 2, -1) giving us C.

21. A One way to do this, is to notice that these are the sum of the solutions to $x^9 = 1$ that are in the first and second quadrant. Seeing this, we know that the sum of all the solutions in the first, second, third, and fourth quadrant are -1, since 1 is a solution and the sum of solutions is 0. Since the solutions are symmetric, we just have $\frac{-1}{2}$.
Alternatively,
Multiply our expression by $2\sin(\frac{2\pi}{9})$, we have

$$2\sin(\frac{2\pi}{9})\cos\left(\frac{2\pi}{9}\right) + 2\sin(\frac{2\pi}{9})\cos\left(\frac{4\pi}{9}\right) + 2\sin(\frac{2\pi}{9})\cos\left(\frac{2\pi}{3}\right) + 2\sin(\frac{2\pi}{9})\cos\left(\frac{8\pi}{9}\right).$$

Using our trig identities of double angle and sum and addition of sin, we have

$$\sin(\frac{4\pi}{9}) + \sin(\frac{2\pi}{9} + \frac{4\pi}{9}) + \sin(\frac{2\pi}{9} - \frac{4\pi}{9}) + \sin(\frac{2\pi}{9} + \frac{6\pi}{9}) + \sin(\frac{2\pi}{9} - \frac{6\pi}{9}) + \sin(\frac{2\pi}{9} + \frac{8\pi}{9}) + \sin(\frac{2\pi}{9} - \frac{8\pi}{9})$$
$$= \sin(\frac{4\pi}{9}) + \sin(\frac{6\pi}{9}) + \sin(-\frac{2\pi}{9}) + \sin(\frac{8\pi}{9}) + \sin(-\frac{4\pi}{9}) + \sin(\frac{10\pi}{9}) + \sin(-\frac{6\pi}{9})$$
$$= \sin\left(\frac{4\pi}{9}\right) + \sin\left(\frac{6\pi}{9}\right) - \sin\left(\frac{2\pi}{9}\right) + \sin\left(\frac{8\pi}{9}\right) - \sin\left(\frac{4\pi}{9}\right) - \sin\left(\frac{\pi}{9}\right) - \sin\left(\frac{6\pi}{9}\right)$$
$$= -\sin\left(\frac{2\pi}{9}\right) .$$

So, dividing both sides by $\sin(\frac{2\pi}{9})$, we have

$2(\cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{2\pi}{3}\right) + \cos\left(\frac{8\pi}{9}\right)) = -1$. So, our final answer is $\frac{-1}{2}$.

22. B The roots satisfy

$$r_1^3 = 3r_1^2 - 2r_1 + 1$$
$$r_2^3 = 3r_2^2 - 2r_2 + 1$$
$$r_3^3 = 3r_3^2 - 2r_3 + 1$$

Adding the 3 equations up,

$$r_1^3 + r_2^3 + r_3^3 = 3(r_1^2 + r_2^2 + r_3^2) - 2(r_1 + r_2 + r_3) + 3$$

Substituting the values using Vieta's,

$$r_1^3 + r_2^3 + r_3^3 = 3(3^2 - 2 \cdot 2) - 2(3) + 3 = 12$$

23. A The maximum value of BD is when BCD is a line, so 7.

24. E So, the smallest integer that can be expressed as a linear combination of two numbers is the gcd(greatest common divisor) of the two numbers.

$252 = 2^2 * 3^2 * 7$

$612 = 2^2 * 3^2 * 17$

So, the gcd is $2^2 * 3^2 = 36$.

Furthermore, the only integers that can be expressed as the linear combination of two numbers are multiples of the gcd, so we just count how many of the numbers in the set are multiples of 36 which is 72,108,144 so the answer is 3.

25. B $$\sum_{n=6}^{\infty} \frac{1}{n^2 - 8n + 15} = \sum_{n=6}^{\infty} \frac{1}{(x-3)(x-5)} = \frac{1}{2}\sum_{n=6}^{\infty}\left(\frac{1}{x-5} - \frac{1}{x-3}\right) = \frac{1}{2}\left(1 + \frac{1}{2}\right) = \frac{3}{4}$$

26. D Substitute $z = cis(x)$. Then,

$$z^2 + \frac{1}{z^2} = cis(2x) + cis(-2x) = \cos(2x) + i\sin(2x) + \cos(-2x) + i\sin(-2x)$$

$= \cos(2x) + i\sin(2x) + \cos(2x) - i\sin(2x) = 2\cos(2x) = 1$

Thus, $\cos(2x) = \frac{1}{2}$. So, $2x = \frac{\pi}{3} + 2\pi n => x = \frac{\pi}{6} + \pi n$. Meaning that $z = cis(\frac{\pi}{6})$.

Rewriting what the question asks us for, we have

$$z^{2023} + \frac{1}{z^{2023}} = 2\cos\left(\frac{2023\pi}{6}\right) = \sqrt{3}$$

27. A Let $a_n = a + nd, b_n = br^n$.

$$c_0 = a + b, c_1 = a + d + br, c_2 = a + 2d + br^2, c_3 = a + 3d + br^3$$
$$c_1 - c_0 = d + br - b, c_2 - c_1 = d + br^2 - br, c_3 - c_2 = d + br^3 - br^2$$
$$c_2 - 2c_1 + c_0 = br^2 - 2br + b, c_3 - 2c_2 + c_1 = br^3 - 2br^2 + br$$

Thus, the sequence we get when we subtract consecutive terms twice is geometric.

$$(9-6) - (6-4) = 1, (15-9) - (9-6) = 3, (c_4 - 15) - (15-9) = 9$$
$$\therefore c_4 = 30$$

*Briefly checking if there actually is a solution,

$a = \frac{15}{4}, d = \frac{3}{2}, b = \frac{1}{4}, r = 3$ satisfies the equations.

28. B Rewriting our sum, we see that it is $9^4 + 4 * 4^5$ which is in the form $a^4 + 4b^4$.

Then, we can use the Sophie Germain identity, giving us

$$9^4 + 4 * 4^5 = (9^2 + 2 * 4^2 - 2 * 4 * 9)(9^2 + 2 * 4^2 + 2 * 4 * 9) = (41)(185)$$

41 is prime and dividing 185 by 5, we get 31. So, our sum is $41 * 5 * 37$

This means the sum of the prime factors is $41+5+37=83$

29. B Have $2(a^2+b^2)=c^2+d^2=2x^2$. Then, we can substitute

$$a=x\sin(\alpha), b=x\cos(\alpha)\ c=x\sqrt{2}\sin(\beta), d=x\sqrt{2}\cos(\beta)$$

Then, substituting those values into $\frac{30bc+6bd-30ad+6ac}{c^2+d^2}$, we have

$\frac{30\sqrt{2}x^2\cos(\alpha)\sin(\beta)+6\sqrt{2}x^2\cos(\alpha)\cos(\beta)-30\sqrt{2}x^2\sin(\alpha)\cos(\beta)+6\sqrt{2}x^2\sin(\alpha)\sin(\beta)}{2x^2}$

Simplifying this, we have

$15\sqrt{2}\cos(\alpha)\sin(\beta)+3\sqrt{2}\cos(\alpha)\cos(\beta)-15\sqrt{2}\sin(\alpha)\cos(\beta)+3\sqrt{2}\sin(\alpha)\sin(\beta)$

We can actually simplify this expression into

$$15\sqrt{2}\sin(\beta-\alpha)+3\sqrt{2}\cos(\beta-\alpha)$$

The maximum value of this is

$\sqrt{\left(15\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2}=\sqrt{450+18}=\sqrt{468}$

So the answer is 468.

30. E All of the equations look eerily similar to law of cosine with an angle of $120°$.

If we look at it this way, take ΔABC with $AB=7, AC=8, BC=3$

Take a point P in ΔABC where $\angle APB=120°, \angle APC=120°, \angle BPC=120°$ and $AP=a, BP=b, CP=c$

Now, take the sum of the areas of $\Delta APB, \Delta APC, \Delta BPC$ and set it equal to the area of ΔABC. Use the sine formula for $\Delta APB, \Delta APC, \Delta BPC$ and Heron's formula for ΔABC.

This gives us

$$\frac{1}{2}\sin(120°)(ab+ac+bc)=\frac{\sqrt{3}}{4}(ab+ac+bc)=6\sqrt{3}$$

This gives us

$$ab+ac+bc=24$$

Now, if we add up all of our original equations, we have

$$2a^2+2b^2+2c^2+ab+ac+bc=122$$

Substituting in $ab+ac+bc=24$ and subtracting 24 from both sides, we have

$$2a^2+2b^2+2c^2=98$$

Dividing both sides by 2, we have

$$a^2+b^2+c^2=49$$

Now, to find $a+b+c$,

$$(a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc=49+48=97.$$

Now, we square root both sides the largest value of $a+b+c$ is $\sqrt{97}$.

(There are more solutions, set $c=0$; however they don't produce the largest $a+b+c$)