1. E
2. C
3. D
4. B
5. C
6. D
7. A
8. D
9. C
10. C
11. E
12. A
13. C
14. C
15. E
16. E
17. D
18. D
19. D
20. D
21. E
22. E
23. D
24. D
25. C
26. C
27. D
28. B
29. A
30. E

1. E The polynomial $F(x)$ factors into $(x-1)(x+2)(4x-3)(2x+7)$. This yield four solutions, the smallest one being $\frac{-7}{2}$.
2. C 84 can be written as $10^2 - 4^2, and\ 2 \times 4 \times 10 = 80$. So the expression simplifies to $\sqrt{(10-4i)^2} = 10 - 4i$.
3. D $\sin n\theta$, if n is even, then the number of petals is $2n$, so the number of petals is 8.
4. B Adding the three equations together results in $a^2 + b^2 + c^2 + 2ab + 2bc + 2ac = 144, (a+b+c)^2 = 144,\ a+b+c = 12$.
5. C $1 - (\sin x)^2 + 0.5 \sin x + 0.5 = 0, (\sin x)^2 - 0.5 \sin x - 0.5 = 0, (\sin x + 0.5)(\sin x - 1) = 0, \sin x = 0.5, -1.\ x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$. Total sum is $\frac{7\pi}{2}$.
6. D $$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{ab+bc+ac}{abc} = \frac{\frac{13}{4}}{\frac{29}{4}} = \frac{13}{29}.$$
7. A $7x - y = 4.\ 6x + 3y = 5.$ Solving the system of equations, we get $x = \frac{17}{27}$.
8. D Removing all the 10s, we get $3 * 4 * 6 * 7 * 8 * 9 * 11 * 6 * 13 * 14 * 3 * 16 * 17 * 18 * 19 * 2 * 21 * 22$. Just looking at the units digits as we multiply, the final answer is 8.
9. C We can have the following square pairs: $(0,5), (5,0), (-5,0), (0,-5), (3,4), (4,3), (3,-4), (-4,3), (-3,4), (4,-3), (-3,-4), (-4,-3)$. That's 12 pairs.
10. C $x^3 + y^3 = (x+y)(x^2 - xy + y^2) = (6)(36-18) = 108$.
11. E It may seem like $x^2 + y^2 = (x+y)^2 - 2xy = (x+y)^2 - 2(4(x+y) - 10) = (x+y-4)^2 + 4$ has a minimum of 4. This occurs when $x + y = 4, xy = 6$. A further look reveals that this has no real solutions. Since $(x+y-4)^2 + 4$ must be greater than 4 otherwise, the answer is E.
12. A Orthogonal means their dot products are zero. So $6 - 6b + 4a = 0$, and $-6 + a - 12b = 0$. Solving the system of equations, we get $b = \frac{-5}{7}$.
13. C $A(x+1) + B(x-6) = 3x - 6.\ A + B = 3.\ A - 6B = -6.\ A = \frac{12}{7}.\ B = \frac{9}{7}.\ A - B = \frac{3}{7}$
14. C Chicken McNugget Theorem: $mn - m - n = 11 * 14 - 11 - 14 = 154 - 25 = 129$.
15. E $0.AB = \frac{AB}{100}.\ 0.\overline{AB} = \frac{AB}{99}. \frac{AB}{99} = \frac{AB}{100} + \frac{1}{450}. \frac{AB}{9900} = \frac{1}{450}.\ AB = 22$. Note that $\lfloor 10x \rfloor + \lfloor 100x \rfloor$ is equivalent to $A + B$.
16. E $2\sin 2x - 2\sqrt{3}\sin x + 2\cos x - \sqrt{3} = 0. So\ 4\sin x \cos x - 2\sqrt{3}\sin x + 2\cos x - \sqrt{3} = 0$. Factoring it, you get $(2\cos x - \sqrt{3})(2\sin x + 1) = 0$. $\cos x = \frac{\sqrt{3}}{2},\ x = \frac{\pi}{6}.\ \sin x = \frac{-1}{2}, x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$. Sum of the solutions is $\frac{19\pi}{6}$
17. D Since you're dividing by a quadratic, the remainder will be in form $ax + b$. Plugging in 2, the polynomial becomes 6, so $2a + b = 6$. Plugging in 1, the remainder becomes 5, so $a + b = 5$. Therefore, $a = 1, and\ b = 4$. Remainder is $x + 4$.

18. D Let's call a the normal speed of Mr. Lu's rowing, and c the speed of the current. $(a + c)(2) = 4(a - c) = 1200.$ Therefore, c = 150, and a = 450.

19. D Simplifying each term, we get $\frac{11+8i}{5} + \frac{4-7i}{5} = \frac{15+i}{5}$.

20. D Notice that we want $f(1)$. Using finite differences, we can quickly verify that $f(1) = 15$.

21. E Case one: $2x + x - 2 = 6. x = \frac{8}{3}. 2x - x + 2 = 6. x = 4. -2x - x + 2 = 6. x = \frac{-4}{3}. -2x + x - 2 = 6. x = -8.$ Smallest solution is $x = -8$.

22. E $4(\cos x)^2 + 1 = (\cos x)^2 + 4. (\cos x)^2 = 1. \cos x = -1, 1. \; x = 0, \pi$, but $\cot(x)$ is undefined, so there are no solutions.

23. D This is the Pythagorean triplet 20, 99, 101. Therefore, the answer is $\frac{99}{101}$.

24. D The characteristic polynomial is the determinant of $\begin{bmatrix} -1-\lambda & 2 & 1 \\ 0 & -2-\lambda & 5 \\ 6 & 4 & 3-\lambda \end{bmatrix}$. Expanding gives $-\lambda^3 + 33\lambda + 98$.

25. C Factoring the equation gives $x^6(x^6 + 1) = 2(2 + 1) \rightarrow x^6 = 2, -3$. The radius of the roots will be alternating between $2^{\frac{1}{6}}, 3^{\frac{1}{6}}$ with angle $\frac{\pi}{6}$ in between them. The total area is $12 \cdot a \cdot b \cdot \frac{\sin(\theta)}{2} = 12 \cdot 2^{\frac{1}{6}} \cdot 3^{\frac{1}{6}} \cdot \frac{1}{4} = 2^{\frac{1}{6}} \cdot 3^{\frac{7}{6}}$.

26. E Let $\log(x) = y \rightarrow x = 2^y$. The equation $y^2 - 36y + 3 = 0$ has sum of roots 36. $2^{r_1 + r_2} = 2^{36}. \log_{10}(2^{36}) = 36 \log(2) = 36 * 0.3010 = 10.83 \ldots \rightarrow 11$

27. D The three solution pairs are $(25,4), (14,9), (3,14)$.

28. B Notice the fact that there is only one way to create x^{2024} and it is by doing $x^4(x^5)^4(x^{25})^0(x^{125})^1(x^{625})^3$. These correspond to $2^4 \cdot 4^4 \cdot 16 \cdot 32^3 = 2^{4+8+4+15} = 2^{31}$.

29. A Plugging in the formula into the quadratic equation, we get the derivative (also the velocity) is $2x + 3$. Plugging in 2, we get $x = 7$.

30. E $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$