All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means "None of the Above."

~~~~~~~~ *Good luck, and have fun!* ~~~~~~~~

1. Let R be the region in the first quadrant bounded by $y \leq 5 - x$ and $y \leq 11 - 3x$. If the area of R is equal to $\frac{A}{B}$, find $A + B$.

    A. 27   B. 29   C. 70   D. 73   E. NOTA

2. The vectors $\vec{A} = \langle x, 3, -1 \rangle$, $\vec{B} = \langle 4, y, 0 \rangle$, and $\vec{C} = \langle -2, 4, z \rangle$ are all pairwise perpendicular to each other. Find xyz.

    A. -60 B. -45 C. -30 D. -27 E. NOTA

3. Positive integers a and b are independently uniformly randomly selected on the interval $[1,6]$. Find the probability that the angles $\frac{2022\pi}{a}$ and $\frac{2022\pi}{b}$ are coterminal.

    A. $\frac{1}{9}$ B. $\frac{1}{6}$ C. $\frac{5}{18}$ D. $\frac{1}{3}$ E. NOTA

4. Simplify: $\frac{2-7i}{3-4i} - \frac{5-6i}{4+3i}$.

    A. $\frac{-52-36i}{25}$ B. $\frac{-60-20i}{25}$ C. $\frac{38+16i}{25}$ D. $\frac{32+26i}{25}$ E. NOTA

5. A circle of radius 2 is inscribed in a square, which is then inscribed in another circle. Find the area of the larger circle.

    A. 6π B. 8π C. 9π D. 12π E. NOTA

6. Find the number of asymptotes to the graph of the equation $y = \frac{x^4 - 6x^3 + 9x^2 + 4x - 12}{x^3 + 6x^2 + 11x + 6}$.

    A. 2   B. 3   C. 4   D. 5   E. NOTA

7. Which of the following statements regarding the vector cross product is always true?
    $\vec{A}, \vec{B}, \vec{C}$ are arbitrary 3D vectors and θ is the angle between $\vec{A}$ and $\vec{B}$.

    A. $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ B. $|\vec{A} \times \vec{A}| = 1$

    C. $|\vec{A} \times \vec{B}|^2 = |\vec{A}||\vec{B}|\sin\theta$ D. $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{C} + \vec{A} \times \vec{B}$ E. NOTA

8. Find the sum of the solutions to $4\sin^4\theta + \sin^3\theta - 11\sin^2\theta + 6\sin\theta = 0$ for $\theta \in [0, 2\pi)$.

    A. $\frac{5\pi}{2}$ B. 3π C. $\frac{7\pi}{2}$ D. 5π E. NOTA

9. The cosine of which of the following complex numbers equals 3?

    A. $i\ln(1 + \sqrt{2})$ B. $i\ln(2 + \sqrt{2})$ C. $2i\ln(1 + \sqrt{2})$ D. $2i\ln(2 + \sqrt{2})$ E. NOTA

10. Find the area of the ellipse $x^2 - xy + y^2 - (x + y)\sqrt{2} = 10$.

    A. $7\pi\sqrt{3}$ B. $8\pi\sqrt{3}$ C. $10\pi\sqrt{3}$ D. $\frac{20\pi\sqrt{3}}{3}$ E. NOTA

11. Given the following system of equations, find the minimum value of $a^2 + 2b^2 + 3c^2$.

    $$a + b + c = -1$$
    $$ab + bc + ca = -32$$
    $$abc = 60$$

    A. 84   B. 98   C. 130   D. 175   E. NOTA

12. Find the sum of the entries of $\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 1 \\ 3 & -2 & 1 \end{bmatrix}^{-1}$.

    A. 2   B. 3   C. 4   D. 5   E. NOTA

13. Which of the following functions are periodic? (The correct answer choice must include all periodic functions and no nonperiodic functions.)

I: $f(x) = \tan\left(2x + \frac{\pi}{4}\right) + 2\tan\left(x - \frac{\pi}{3}\right)$

II: $g(x) = \sin \pi x + \pi \sin x$

III: $h(x) = \cos\sqrt{|x|}$

A. I   B. I, II   C. III   D. I, II, III   E. NOTA

14. If $f(A, B) = \frac{A+Bi}{A-Bi}$, then $f(4046,2023) = \frac{P+Qi}{R}$. Find $P + Q + R$.

A. 8   B. 12   C. 18   D. 22   E. NOTA

15. Identify the following: $3x^2 - 8y^2 + 10xy - 8x + 10y - 3 = 1$.

A. Cardioid   B. Ellipse   C. Hyperbola   D. Parabola   E. NOTA

16. Find the sum of all real values of x such that $(x^2 - 2x - 4)^{x^2-5x+6} = 1$.

A. 5   B. 6   C. 8   D. 9   E. NOTA

17. Let $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$. $S = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ is a matrix such that $D = S^{-1}AS$ is a diagonal matrix $D = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}$. Using this information, find the sum of the elements of A^{2023}.

A. $5^{2023} - 1$ B. 5^{2023} C. $2 \cdot 5^{2023} - 1$ D. $2 \cdot 5^{2023}$ E. NOTA

18. Find the number of values of x such that $\sin(2023x) = 0$ for $x \in [0,2023\pi)$.

A. 2023^2 B. $2 \cdot 2023^2$ C. $4 \cdot 2023^2$ D. $8 \cdot 2023^2$ E. NOTA

19. If $\sum_{n=0}^{\infty}\left[\left(-\frac{3}{4}\right)^n e^{2\pi in/3}\right] = \frac{A - Bi\sqrt{3}}{C}$, find $A + B + C$.

    A. 25   B. 27   C. 29   D. 31   E. NOTA

20. Determine the value of t for which the parabola parametrically defined by
    $x(t) = 1 \pm \sqrt{t+2}$ and $y(t) = \frac{t}{2} - 3$ intersects its latus rectum.

    A. -1 B. 4   C. 14   D. $\frac{73}{4}$ E. NOTA

21. If the sum of the reciprocals of the positive integer factors of 2023 is equal to $\frac{R}{S}$, find $R + S$.

    A. 4047   B. 4183   C. 4343   D. 4479   E. NOTA

22. Find the sum of the entries in the matrix equal to $\begin{bmatrix} 8 & 1 & 4 & 5 & 0 \\ 7 & 7 & 3 & 2 & 4 \\ 4 & 1 & 4 & 0 & 3 \\ 1 & 3 & 7 & 7 & 6 \\ 0 & 8 & 2 & 6 & 7 \end{bmatrix}\begin{bmatrix} 5 & 0 & 7 & 3 & 5 \\ 1 & 6 & 4 & 7 & 2 \\ 0 & 1 & 8 & 8 & 3 \\ 3 & 6 & 3 & 4 & 4 \\ 2 & 8 & 7 & 3 & 0 \end{bmatrix}$.

    A. 1951   B. 1990   C. 2000   D. 2022   E. NOTA

23. Which of these is equal to $\tan\left(\arcsin\frac{\sqrt{x^2-4}}{x}\right)$, where defined?

    A. $\frac{x^2}{\sqrt{x^2-4}}$ B. $\frac{\sqrt{x^2-4}}{2}$ C. $\frac{\sqrt{x^2-4}}{x}$ D. $\frac{x\sqrt{x^2-4}}{2}$ E. NOTA

24. Find the smallest value of $|k|$ such that $|z - 1 + 2i| = k + |z + 2 - 2i|$ is not a hyperbola.

    A. 4   B. $\sqrt{20}$ C. 5   D. $\sqrt{32}$ E. NOTA

25.
If the area of the ellipse given by the equation $r = \dfrac{5}{4-\sin\theta}$ is equal to $\pi\sqrt{\dfrac{A}{B}}$, find $(A + B) \bmod 7$.

A. 1   B. 2   C. 5   D. 6   E. NOTA

26. There is a unique triplet of positive integers $\{A, H, S\}$ where $A, H, S < 100$ such that $A^5 + H^2 = S^3$. Find $A + H + S$.

A. 20   B. 23   C. 26   D. 29   E. NOTA

27. If $AB = BA$ and $B = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$, which of the following could be A?

A. $\begin{bmatrix} 2022 & 2022 \\ 2022 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 2022 & 2023 \\ 2023 & 2024 \end{bmatrix}$

C. $\begin{bmatrix} 2023 & 2022 \\ 2022 & 1 \end{bmatrix}$ D. $\begin{bmatrix} 2023 & 2024 \\ 2025 & 2026 \end{bmatrix}$ E. NOTA

28. Determine the range (in interval notation) of the function $y = \sin x + \sin(\pi x)$.

A. $(-2, 2)$ B. $[-2, 2]$

C. $\left(-\sqrt{\pi^2 + 1}, \sqrt{\pi^2 + 1}\right)$ D. $\left[-\sqrt{\pi^2 + 1}, \sqrt{\pi^2 + 1}\right]$ E. NOTA

29. If z is a complex number satisfying $z^2 + z + 1 = 0$, find $\sum_{n=1}^{2022}(z^{4n} + z^{-n})^2$.

A. 674   B. 1011   C. 2022   D. 4044   E. NOTA

30. Which of the following describes the graph of $x^2 + y^2 = 2023$?

A. Cardioid   B. Hyperbola   C. Ellipse   D. Parabola   E. NOTA