The answer choice "NOTA" stands for "None of the Above". Good luck, and have fun!

1. Gemini is Aditi's favorite competition at Nationals, and she studies for it every other day religiously. She prefers to sit on stairs while she studies, and the number of steps in the staircase she chooses to sit in affects her productivity. If the number of steps in the staircase she chooses to sit in today has stair equal to $\sqrt[3]{9-4\sqrt{5}}+\sqrt[3]{9+4\sqrt{5}}$, then how many steps are there?

   A. $2\sqrt[3]{3}$ B. 3 C. $2\sqrt{5}$ D. 4 E. NOTA

2. Navya and Anjana like to hide donuts to surprise people, and decide to hide donuts in Albert's hotel room. Since Albert is busy watching SpongeBob, he is not paying attention and the girls are able to hide the donuts in x minutes if $x_7 = 12_6$. What is x?

   A. 8 B. 9 C. 10 D. 11 E. NOTA

3. If $z = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$, then what is the value of $(1-z)(1-z^2)(1-z^3)(1-z^4)$?

   A. 25 B. 10 C. 5 D. 0 E. NOTA

4. Tony and Jeremy like to run, and they decide to have a 100 unit race. If Tony's speed is governed by $T = \sqrt{20-\sqrt{20-\cdots}}$, and Jeremy's speed is governed by $J = \sqrt{20+\sqrt{20+\cdots}}$, who won the race by how much time?

   A. Tony, 5 B. Jeremy, 5 C. Tony, 10 D. Jeremy, 10 E. NOTA

5. The graph of $(x^2+y^2-2x+4y+5)(x^2+y^2+4x+2y+5)=0$ consists of two points. Find the distance between the two points.

   A. $3\sqrt{2}$ B. $2\sqrt{5}$ C. $\sqrt{10}$ D. 5 E. NOTA

6. Find the sum of the reciprocals of the positive integral factors of 28.

   A. 1 B. 2 C. 3 D. 4 E. NOTA

7. Even though Anjana likes to surprise people with donuts, she also likes to steal people's lunchboxes (she is a very balanced person). One day she steals Jeffrey's lunchbox and hides it at the coordinate $(4,7)$. If Jeffrey is at $(4,-1)$, and there is a long line of kids in front of the microwave at $x = 1$ where he must go to ask Kevin where his lunchbox is, what is the shortest possible distance that he travels to get his lunchbox?

A. 10 B. 11 C. $8\sqrt{2}$ D. $3+\sqrt{73}$ E. NOTA

8. Shivi and Jubili love going on walks and bike rides, and they go twice a day, every day. However, one day they could not decide whether to walk or to bike and decided that Shivi will walk and Jubili will bike. If their speeds are $Shivi$: $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{vmatrix}$ and $Jubili$: $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$, how are they moving relative to each other?

A. Jubili is faster B. Shivi is faster
C. They are moving together D. Neiter of them are moving E. NOTA

9. Inspired by Shivi and Jubili, Angela, Anjana, and Navya decide to go on a walk, but can never get their times to match. They decide to take a walk at night (defined by 10 PM to 6AM). Anjana and Angela have the same availability, and plan to walk together for four continuous hours, and Navya plans to walk for six continuous hours. What is the probability that the three of them will be walking together for at least three hours?

A. $\frac{13}{16}$ B. $\frac{5}{8}$ C. $\frac{15}{16}$ D. $\frac{7}{8}$ E. NOTA

10. Seeing how everyone is being active and going on walks, Tony and Ani decide to have a race for fun. Instead of running in the same direction, they decide to run towards each other (starting with a distance of 50 miles). A dragonfly, gets excited and decides to fly between them as they run at a speed of 100 mph (it starts in front of Ani and flies to Tony and turns immediately to fly back until they meet). If Ani runs at speed 30 mph and Tony runs at speed 20 mph how much distance does the dragonfly travel?

A. 100 mi B. 150 mi C. 200 mi D. 50 mi E. NOTA

11. A square is formed by connecting (in consecutive order) the midpoints of alternating sides of a regular octagon. Find the ratio of the area enclosed by the square to the area enclosed by the regular octagon.

A. $\frac{1+\sqrt{2}}{4}$ B. $\frac{2+2\sqrt{2}}{8}$ C. $\frac{5}{8}$ D. $\frac{3+4\sqrt{2}}{8}$ E. NOTA

12. Find the sum of all positive integers n that make $\frac{(1-\sqrt{3}i)^{24}}{(\sqrt{2}+\sqrt{2}i)^{n}}$ an integer.

A. 84 B. 60 C. 156 D. 48 E. NOTA

13. Aditi is going to study for Gemini again, and is climbing a 17-step staircase to sit on. How many ways ("way" being sequence of stair numbers she steps on) can Aditi get to the 17th step if she only goes up two or three stairs at a time?

A. 4 B. 25 C. 49 D. 64 E. NOTA

14. Navya and Angela have come back from their walk, so they decide to have a karaoke session. While both are incredibly talented singers, Shivi still gets a headache from their singing, and they agree to stop singing if Shivi can solve the following problem: If a, b, c are positive real numbers such that $a^3 + b^3 + c^3 = 3abc$, what is the value of $\frac{(a+b)(b+c)(c+a)}{abc}$? Assuming Shivi gets the correct answer, what does she get?

A. 6 B. 2 C. 4 D. 8 E. NOTA

15. If the cross product of the two vectors $a = \langle 4, -11, 10\rangle$ and $b = \langle x, -2x, -3x\rangle$ satisfies $a \times b = \langle y, 154,\ z\rangle$, what is $a \cdot b$?

A. 0 B. 217 C. -28 D. -63 E. NOTA

16. Evaluate the following limit: $\lim_{x\to 0^+} \frac{1-\frac{1}{5x}}{2-\frac{2}{3x}}$

A. $\frac{10}{3}$ B. $\frac{1}{2}$ C. $\frac{15}{2}$ D. DNE E. NOTA

17. Michael likes to speak Spanish but has not taken a Spanish class in years. This results in his Spanish abilities deteriorating at a rate of 4 units of knowledge per night while he sleeps. Not able to fathom this, he frantically begins to watch Spanish YouTube videos during the day at a rate of 5 videos per day (assume one video is equivalent to one unit of knowledge) to recover his level of fluency. If his knowledge has already decreased by 100 units, how many days will it take him for him to first recover his abilities?

A. 95 B. 96 C. 85 D. 86 E. NOTA

18. What is the axis of symmetry of the parabola defined by the parametric equations $x = \frac{t+4}{3} \quad y = \frac{t^2}{2} - t$?

A. $x = -1$ B. $x = \frac{5}{3}$ C. $x = \frac{5}{6}$ D. $x = \frac{4}{3}$ E. NOTA

19. Isa, Eddie, Bach, and Saber love to play video games, but they are relatively novice and only play until 10pm. They decide to take advice from some older math teamers on how to play video games later in the night without falling asleep and getting caught in Mr. Frazer's class (Mr. Frazer is very experienced in catching videogame-induced-lack-of-sleep). They are advised to follow this ratio for hours of videogame and hours of sleep to not get caught: $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. What is the ratio?

A. 1 B. e^n C. n^e D. e E. NOTA

20. Simplify (where defined): $\frac{\sec(x)\csc(x)-\tan(x)\csc(x)}{(\sec(x)-\tan(x))^2+1}$

A. $\frac{\cos(x)}{2}$ B. $\frac{\csc(x)}{2}$ C. $\frac{\tan(x)}{2}$ D. $\frac{\cot(x)}{2}$ E. NOTA

21. Aditi and Kevin are very social people. They often compare the number of friends they have with each other to see who has more. Aditi currently has 350 friends, and Kevin currently has 700, and the rates that they make friends are 7 friends per day and 3.5 friends per day, respectively. How many days will it take Aditi to have the same number of friends as Kevin?

A. 250 B. 200 C. 100 D. 50 E. NOTA

22. What is $33_9 \times 44_9 =$?

A. 5500_9 B. 1573_9 C. 1563_9 D. 1543_9 E. NOTA

23. An open-top rectangular box has height 6, and a square base. The volume of the box is 72. What is the total area of all exposed surfaces (assume that the thickness of the walls is 0)?

A. $144\sqrt{3} + 24$ B. $96\sqrt{3} + 24$ C. $48\sqrt{3} + 72$ D. $72\sqrt{3} + 48$ E. NOTA

24. Mr. Lu is incredibly knowledgeable, and Amy and Navya are not nearly as knowledgeable as Mr. Lu is. In fact, to increase their knowledge, the two of them like to ask Mr. Lu discrete mathematics questions. One of the questions they asked him was this: If Sylvia flips a fair two-sided coin repeatedly, and gets 18 heads and 2 tails, what is the chance that she gets tails on the next flip? Mr. Lu taught them how to do it, arriving at the final answer of

A. 0.1 B. 0.2 C. 0.5 D. 0.9 E. NOTA

25. The sum of Chloe's two favorite numbers is 54 and their product is 704. What is the positive difference between these two numbers?

A. 40 B. 20 C. 30 D. 10 E. NOTA

26. Abhiram paints the outside of a cube with side length 8 blue. Since Eric does not like blue, he cuts it into 512 congruent cubes with side length 1. This upsets Abhiram, but to make him feel better Robert tells him to find the number of smaller cubes that have exactly two blue faces. Assuming Abhiram gets the answer correct, what does he answer?

A. 188 B. 72 C. 216 D. 64 E. NOTA

27. Given that $\ln 5 = n$. What is the value of $\ln(-5)$ assuming the principal argument? (Assume logarithms have complex domain and range)

A. $ni\pi$ B. $n + i\pi$ C. $-ni\pi$ D. $i\pi - n$ E. NOTA

28. Devika and Erick have a very similar sense of humor, and often complete each other's jokes. Timmy is skeptical of this, and says that if they really do think similarly, then they should be able to solve this problem accurately:
When $\sqrt{15 \cdot 17 \cdot 19 \cdot 21 + 16}$ is simplified, it is a three-digit integer. What is the sum of the digits?
They remember doing this problem in 2019, and correctly tell Timmy the answer. What did they tell Timmy?

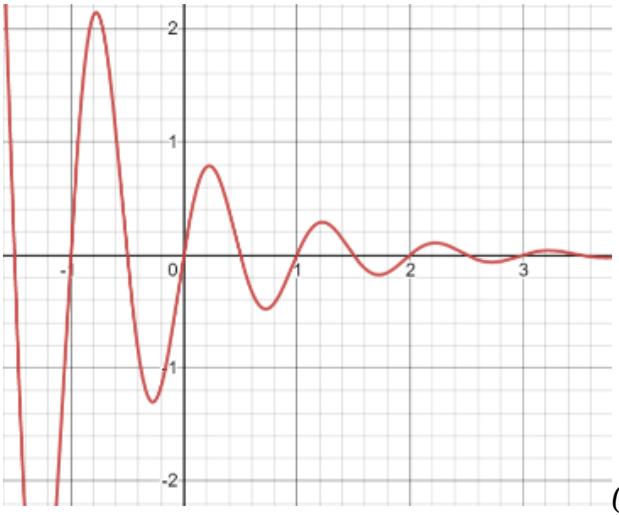
A. 7 B. 11 C. 13 D. 15 E. NOTA

29. Find the value for $\theta \in [0, \pi)$ that satisfies the given equation:

$$\tan\theta = \frac{\tan\frac{5\pi}{36} - 1}{\tan\frac{\pi}{9} - 1}$$

A. $\frac{2\pi}{9}$ B. $\frac{\pi}{36}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{4}$ E. NOTA

30. Good job on making it this far! Here is a graph of one of my favorite equations. Given that the equation for the graph below is listed as an answer choice, which one is it?



(graph from desmos.com)

A. $f(x) = e^{-x}\sin(2\pi x)$ B. $f(x) = e^{-x}\cos(4\pi x)$
C. $f(x) = e^{-x}\cos(2\pi x)$ D. $f(x) = e^{-x}\sin(4\pi x)$
E. $f(x) = e^{x}\sin(2\pi x)$