1. D The lines $y = 5 - x$ and $y = 11 - 3x$ intersect at the point $(3,2)$. The line $x = 3$ splits the region into a right triangle with legs of 2 and $\frac{2}{3}$ and a trapezoid with bases 5 and 2 and height 3. The total area is $\frac{1}{2} \cdot 2 \cdot \frac{2}{3} + \frac{5+2}{2} \cdot 3 = \frac{2}{3} + \frac{21}{2} = \frac{67}{6}$. $A + B = 73$.

2. B The dot product of any two of these vectors equals 0. This gives the following system of equations.

$$4x + 3y = 0$$
$$-8 + 4y = 0$$
$$-2x - z = -12$$

The solution to this system is $x = -\frac{3}{2}$, $y = 2$, and $z = 15$. $xyz = -45$.

3. C $\frac{2022}{1} = 2022$, $\frac{2022}{2} = 1011$, $\frac{2022}{3} = 674$, $\frac{2022}{4} = 505.5$, $\frac{2022}{5} = 404.4$, and $\frac{2022}{6} = 337$. $n = 1$ and $n = 3$ correspond to the angle $\theta = 0$, and $n = 2$ and $n = 6$ correspond to the angle $\theta = \pi$. $n = 4$ and $n = 5$ correspond to angles of unique direction. There is a $\frac{2}{3}$ chance to pick an angle that is part of a pair, and a $\frac{1}{3}$ chance to pick a unique angle, so the probability the two angles are coterminal is $\frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{6} = \frac{5}{18}$.

4. D Multiply the numerator and denominator by the conjugate of the denominator and subtract. $\frac{2-7i}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{6-28i^2-21i+8i}{3^2+4^2} = \frac{34-13i}{25}$ and $\frac{5-6i}{4+3i} \cdot \frac{4-3i}{4-3i} = \frac{20+18i^2-24i-15i}{4^2+3^2} = \frac{2-39i}{25}$, so $\frac{34-13i}{25} - \frac{2-39i}{25} = \frac{32+26i}{25}$.

5. B The radius of the inner circle is half the side length of the square, so the square has side length 4. The diagonal of the square is twice the radius of the outer circle, so the outer circle has radius $2\sqrt{2}$ and area 8π.

6. B $\frac{x^4-6x^3+9x^2+4x-12}{x^3+6x^2+11x+6} = \frac{(x+1)(x-2)^2(x-3)}{(x+1)(x+2)(x+3)}$. There is a hole at $x = -1$ and vertical asymptotes at $x = -2$ and $x = -3$. The degree of the numerator is exactly one greater than the degree of the denominator, so there is also a slant asymptote (its equation is $y = x - 12$) for a total of 3.

7. D $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$, $|\vec{A} \times \vec{A}| = 0$, and $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin \theta_{bet}$. However, the distributive (plus commutative) property for cross products is correctly stated: $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{C} + \vec{A} \times \vec{B}$.

8. A Set $u = \sin \theta$. $4u^4 + u^3 - 11u^2 + 6u = u(u-1)(u+2)(4u-3)$. $u = -2$ will give non-real solutions, so the values that satisfy the equation are the solutions to $\sin \theta = 0$, $\sin \theta = 1$, and $\sin \theta = \frac{3}{4}$. The first two of these equations have solution sets $\theta = \left\{0, \frac{\pi}{2}, \pi\right\}$. $\arcsin \frac{3}{4}$ is not a rational multiple of π, but since sine is symmetric about the x-axis, the solutions to the third equation are φ and $\pi - \varphi$. The sum of all of the solutions is $\frac{5\pi}{2}$.

9. C For complex numbers, $\cos z = \frac{e^{iz}+e^{-iz}}{2}$. Setting this equal to 3 yields $e^{iz} - 6 + e^{-iz} = 0$. Multiplying by e^{iz} and setting $u = e^{iz}$ yields the quadratic $u^2 - 6u + 1 = 0$, or $u = 3 \pm 2\sqrt{2}$. Solving $e^{iz} = 3 \pm 2\sqrt{2}$ yields $z = -i\ln\left(3 \pm 2\sqrt{2}\right) = 2i\ln\left(1 \mp \sqrt{2}\right)$.

10. B Solving $\cot 2\theta = \frac{1-1}{-1} = 0$ yields $\theta = \frac{\pi}{4}$. Then $x = x'\cos\frac{\pi}{4} - y'\sin\frac{\pi}{4} = \frac{x'-y'}{\sqrt{2}}$ and $y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4} = \frac{x'+y'}{\sqrt{2}}$. Substituting these into the given equation yields $\frac{(x'-y')^2}{2} - \frac{x'^2-y'^2}{2} + \frac{(x'+y')^2}{2} - \frac{2x'}{\sqrt{2}\sqrt{2}} = 10$. This simplifies to $\frac{x'^2}{2} + \frac{3y'^2}{2} - 2x = 10$, or $x'^2 - 4x + 3y'^2 = 20$. Completing the square and dividing yields the ellipse $\frac{(x'-2)^2}{24} + \frac{y'^2}{8} = 1$, which has an area of $8\pi\sqrt{3}$.

11. B By Vieta's formula, a, b, and c are the solutions to the equation $x^3 + x^2 - 32x - 60 = 0$. This factorizes to $(x+2)(x+5)(x-6) = 0$, so $(a,b,c) = (-2,-5,6)$ in some order. The ordering that minimizes $a^2 + 2b^2 + 3c^2$ is $(6,-5,-2)$, which corresponds to $36 + 50 + 12 = 98$.

12. A The determinant is $(-1+0-8) - (-6-2+0) = -1$, so the matrix is invertible.

$$\begin{bmatrix}1 & 0 & 2\\ 2 & -1 & 1\\ 3 & -2 & 1\end{bmatrix} \lll\ggg \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \lll\ggg R_2 -= 2R_1, R_3 -= 3R_1$$
$$\begin{bmatrix}1 & 0 & 2\\ 0 & -1 & -3\\ 0 & -2 & -5\end{bmatrix} \lll\ggg \begin{bmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ -3 & 0 & 1\end{bmatrix} \lll\ggg R_2 \div= -1$$
$$\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 3\\ 0 & -2 & -5\end{bmatrix} \lll\ggg \begin{bmatrix}1 & 0 & 0\\ 2 & -1 & 0\\ -3 & 0 & 1\end{bmatrix} \lll\ggg R_3 += 2R_2$$
$$\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 3\\ 0 & 0 & 1\end{bmatrix} \lll\ggg \begin{bmatrix}1 & 0 & 0\\ 2 & -1 & 0\\ 1 & -2 & 1\end{bmatrix} \lll\ggg R_2 -= 3R_3, R_1 -= 2R_3$$
$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \lll\ggg \begin{bmatrix}-1 & 4 & -2\\ -1 & 5 & -3\\ 1 & -2 & 1\end{bmatrix}$$

The sum of the entries in the inverse matrix is 2.

13. A The periods of the subfunctions in I are $\frac{\pi}{2}$ and π, so the overall function has period π. The periods of the subfunctions in II are 2 and 2π. The ratio of these periods is irrational, so the overall function is aperiodic. III is an aperiodic function. Only I is periodic.

14. B Consider $f(2n,n)$. $\frac{2n+ni}{2n-ni} = \frac{2+i}{2-i} = \frac{(2+i)^2}{4-i^2} = \frac{(2+i)^2}{5} = \frac{4+i^2+4i}{5} = \frac{3+4i}{5}$. $P + Q + R = 12$.

15. C $\begin{vmatrix}A & B/2 & D/2\\ B/2 & C & E/2\\ D/2 & E/2 & F\end{vmatrix} = \begin{vmatrix}3 & 5 & -4\\ 5 & -8 & 5\\ -4 & 5 & -4\end{vmatrix} = (96 - 100 - 100) - (-128 + 75 - 100) = 49$, so the conic is not degenerate. $B^2 - 4AC = 100 - 4 \cdot 3 \cdot (-8) = 196 > 0$, so the conic is a hyperbola.

16. B There are several cases that would produce values of x satisfying the equation. The first is when the exponent is 0 and the base is not. Solving $x^2 - 5x + 6 = 0$ yields $x = 2$ and $x = 3$, neither of which makes the base equal 0. The second case is when the base equals 1. The sum of the solutions to $x^2 - 2x - 4 = 1$ is 2 by Vieta's formula. The final case is when the base equals -1 and the exponent is an even integer. Solving $x^2 - 2x - 4 = -1$ yields $x = -1$ and $x = 3$ as solutions; the latter of these has already been confirmed as a solution. $x^2 - 5x + 6 = 12$ for $x = -1$, so this is a solution as well. The sum of all of the solutions is 6.

17. D Left-multiply by S and right-multiply by S^{-1} to obtain $A = SDS^{-1}$. $S^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. $A^2 = SDS^{-1}SDS^{-1} = SD^2S^{-1}$, and it can easily be shown through induction that $A^n = SD^nS^{-1}$. Thus, $A^{2023} = SD^{2023}S^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & n \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -1 & n \\ 1 & 2n \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} n-2 & n+1 \\ 2n+2 & 2n-1 \end{bmatrix}$ where $n = 5^{2023}$. The sum of the elements of this matrix is $2n = 2 \cdot 5^{2023}$.

18. A The number of solutions to $\sin nx = 0$ over $[0, \pi)$ where n is an integer is n. There are 2023 solutions to $\sin 2023x = 0$ over $[0, \pi)$, and the range $[0,2023)$ contains 2023 half-periods of $\sin x$, so there are $2023 \cdot 2023 = 2023^2$ solutions to $\sin 2023x = 0$ over $[0,2023\pi)$.

19. C The sum can be rewritten as $\sum_{n=0}^{\infty}\left[\left(-\frac{3}{4}\right)^n e^{2\pi i n/3}\right] = \sum_{n=0}^{\infty}\left(-\frac{3}{4}e^{2\pi i/3}\right)^n$. This is a geometric series with first term 1, so its sum is $\left(1 + \frac{3}{4}e^{2\pi i/3}\right)^{-1} = 8\left(8 + 6e^{2\pi i/3}\right)^{-1} = 8\left(8 - 3 + 3i\sqrt{3}\right)^{-1} = \frac{8}{5+3i\sqrt{3}} = \frac{40-24i\sqrt{3}}{25+27} = \frac{10-6i\sqrt{3}}{13}$. $A + B + C = 29$.

20. A If $y = \frac{t}{2} - 3$, then $t = 2y + 6$. $x = 1 \pm \sqrt{2y+8}$, so $2(y+4) = (x-1)^2$. $4f = 2$, so the focal distance is $\frac{1}{2}$. The latus rectum of the parabola is contained by $y = -4 + \frac{1}{2} = -\frac{7}{2}$. Solving $\frac{t}{2} - 3 = -\frac{7}{2}$ gives the relevant value of t as $t = -1$.

21. D If f is a factor of 2023, then so is $f' = \frac{2023}{f}$, and $\frac{1}{f} + \frac{1}{f'} = \frac{1}{2023}\left(\frac{2023}{f'} + \frac{2023}{f}\right)$. Thus, the sum of the reciprocals of the factors of 2023 is the sum of the factors of 2023 divided by 2023. Factoring, $2023 = 7 \cdot 17^2$, so the sum of the factors of 2023 is $(1+7)(1+17+289) = 8 \cdot 307 = 2456$ and the desired value is $\frac{2456}{2023}$. $2456 + 2023 = 4479$.

22. C The sum of the entries in the product of two matrices is the sum of the products of the sums of the entries in corresponding columns and rows (in that order) of the two matrices. All columns in the first matrix have sum 20. All rows in the second matrix also have sum 20. The sum of the entries in the resultant is $5 \cdot 20 \cdot 20 = 2000$. Just for fun, this resultant matrix is

$$\begin{bmatrix} 56 & 40 & 107 & 83 & 74 \\ 56 & 89 & 135 & 114 & 66 \\ 27 & 34 & 85 & 60 & 34 \\ 41 & 115 & 138 & 126 & 60 \\ 40 & 142 & 115 & 117 & 46 \end{bmatrix}$$

23. E $\frac{\sqrt{x^2-4}}{x}$ is the sine of an angle in a right triangle whose opposite side is $\sqrt{x^2-4}$ and whose hypotenuse is x, making the adjacent leg 2. The tangent of this angle is $\frac{\sqrt{x^2-4}}{2}$. However, if x is negative, $\frac{\sqrt{x^2-4}}{x}$ is negative, thus the angle is in Q4, making tangent negative. The correct answer is $\frac{\sqrt{x^2-4}}{2} \cdot \frac{|x|}{x}$

24. C Rearrange to say $|z-1+2i| - |z+2-2i| = k$. The difference in distance of z from the points $(1, -2i)$ and $(-2, 2i)$ in the Argand plane is k. This distance is at most 5, when z lies on a ray beginning at one of the two points and extending directly away from the other point. WLOG, when $k = 5$, this represents a ray from $-2+2i$ pointing in the direction opposite of $1-2i$.

25. B If $r = \frac{5}{4-\sin\theta}$, then $4r - r\sin\theta = 5$ and $4r = 5 + y$. Squaring, $16r^2 = y^2 + 10y + 25$, or $16x^2 + 15y^2 - 10y = 25$. Combining, $16x^2 + 15\left(y - \frac{1}{3}\right)^2 = 25 + \frac{5}{3} = \frac{80}{3}$. Dividing, we have the ellipse $\frac{x^2}{5/3} + \frac{\left(y-\frac{1}{3}\right)^2}{16/9} = 1$, which has an area of $\pi\sqrt{\frac{80}{27}}$. $A + B = 107$ and $107 \bmod 7 = 2$.

26. A By inspection, $3^5 + 10^2 = 7^3$ (from $243 + 100 = 343$). $3 + 10 + 7 = 20$.

27. C Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Multiplying, we have $\begin{bmatrix} 2a-3b & -3a+5b \\ 2c-3d & -3c+5d \end{bmatrix} = \begin{bmatrix} 2a-3c & 2b-3d \\ -3a+5c & -3b+5d \end{bmatrix}$. Comparing the elements on the main diagonals, we have $b = c$. From the other long main diagonal, $c + d = a$. The only choice that satisfies these two conditions is $\begin{bmatrix} 2023 & 2022 \\ 2022 & 1 \end{bmatrix}$.

28. A For $\sin x + \sin(\pi x)$ to equal 2, $\sin x = \sin(\pi x) = 1$. This is impossible, since x and πx cannot be of the form $\frac{\pi}{2} + 2\pi n$ at the same time. However, since π is irrational, the function is aperiodic and there exists values such that the function becomes arbitrarily close to ± 2. Thus, the range of $\sin x + \sin(\pi x)$ is $(-2, 2)$.

29. D The roots of $z^2 + z + 1 = 0$ are ω and ω^2, where ω is a non-real third root of unity. The powers of ω are cyclic with period 3. Thus, the sum is equal to $674\sum_{n=1}^{3}(\omega^n + \omega^{2n})^2$. Consider the summand: $(\omega + \omega^2)^2 + (\omega^2 + \omega^4)^2 + (\omega^3 + \omega^6)^2 = 2(\omega + \omega^2)^2 + 4 = 2(-1)^2 + 4 = 6$. The total sum is $674 \cdot 6 = 4044$.

30. C A circle is an ellipse.