1. B
2. D
3. C
4. B
5. C
6. B
7. A
8. B
9. D
10. A
11. A
12. A
13. C
14. D
15. C
16. E
17. B
18. B
19. D
20. D
21. C
22. B
23. B
24. C
25. D
26. B
27. B
28. C
29. A
30. A

1. B If $A = \sqrt[3]{9-4\sqrt{5}}$ and $B = \sqrt[3]{9+4\sqrt{5}}$ then $A^3 + B^3 = 18$ and $AB = 1$
$(A+B)^3 - 3AB(A+B) = 18 \quad x = A + B = \sqrt[3]{9-4\sqrt{5}} + \sqrt[3]{9+4\sqrt{5}}$
$x^3 - 3x - 18 = 0 \quad (x-3)(x^2+3x+6) = 0 \quad x = 3$
2. D $12_6 = 1(6) + 2 = 8 = 1(7) + 1 = 11_7$
3. C We must identify z, z^2, z^3, z^4 as roots of $x^4 + x^3 + x^2 + x + 1 = 0$.
By the Factor Theorem, $x^4 + x^3 + x^2 + x + 1 = (x-z)(x-z^2)(x-z^3)(x-z^4)$
By substituting $x = 1$: $(1-z)(1-z^2)(1-z^3)(1-z^4) = 1+1+1+1+1 = 5$
4. B By the transitive property:
$T = \sqrt{20-T} \quad J = \sqrt{20+J}$
$T^2 = 20 - T \quad J^2 = 20 + J$
$(T-4)(T+5) = 0 \quad (J-5)(J+4) = 0$
$T = 4$ & $J = 5$
$(Rate)(Time) = Distance$
Since they both traveled the same distance, and Jeremy’s rate of 5 was faster than Tony’s rate of 4, we know that Jeremy won the race.
100/4 – 100/5 = 5
5. C The two factors each correspond to a single point, which can be found by completing the square:
$x^2 + y^2 - 2x + 4y + 5 = (x-1)^2 + (y+2)^2 \qquad (1, -2)$
$x^2 + y^2 + 4x + 2y + 5 = (x+2)^2 + (y+1)^2 \qquad (-2, -1)$
$\sqrt{(1+2)^2 + (-2+1)^2} = \sqrt{10}$
6. B The sum of the reciprocal of the positive integral factors is equal to the sum of the positive integral factors divided by the number. 28 is a perfect number, which means that the sum of its positive integral factors is twice its value, and when we divide that by 28, we get $\frac{56}{28} = 2$.
7. A In order to find the shortest distance, we must reflect the point (4, 7) across the line x=1, to get the point (-2, 7). If we calculate the distance between this point and (4, -1) we will be able to account for the distance that he traveled to Kevin and then from there to his lunchbox.
$\sqrt{(-2-4)^2 + (7-(-1))^2} = 10$
8. B We can find the determinants of the matrices and compare, and we see that Shivi is moving at a rate of 1, while Jubili’s rate is 0.
9. D Imagine a 8x8 grid, where the x-axis represents the time in hours after 10PM when Angela and Anjana begin to walk, and the y-axis represents the time in hours after 10PM when Navya starts to walk. Knowing that Angela and Anjana must begin walking at 2AM at the latest, and Navya at 12AM at the latest, we can reduce our possible area down to a 4x2 rectangle. If Navya starts at 12AM, Angela and Anjana would have to start no earlier than 11PM in order to have 3 hours of overlap (represented by point (1,2)). If Angela and Anjana start at 2AM, Navya would have to start no earlier than 11PM to have 3 hours of overlap (represented by point (4,1)).

From these points, we can draw line segments of slope 1. The favorable area is between the line segments, and the area is $(4)(2)-(2)\left(\frac{1}{2}\right)(1)(1)=7$, and the total area is 8, so the probability is $\frac{7}{8}$.

10. A The time it takes for Ani and Tony to meet is the total distance divided by the sum of their rates: $\frac{50}{20+30}=1\ hour$. The dragonfly flies this entire time, so by multiplying the rate at which it flies, to the time that it flies for would result in the distance it traveled: (1)(100) = 100 miles

11. A If we say x is the length of the side of the octagon, as in the diagram. If we think of an octagon as a square with four isosceles triangles removed, then the enclosed area of the octagon is

$\left(x+2\left(\frac{x}{\sqrt{2}}\right)\right)^2-(4)\left(\frac{1}{2}\right)\left(\frac{x}{\sqrt{2}}\right)^2=x^2(2+2\sqrt{2})$ Then the area of the square is

$$\frac{\left(x+2\left(\frac{x}{\sqrt{2}}\right)\right)^2}{4}=x^2\left(\frac{3}{2}+\sqrt{2}\right)$$

So the ratio is $\frac{\left(\frac{3}{2}+\sqrt{2}\right)}{(2+2\sqrt{2})}=\frac{1+\sqrt{2}}{4}$.

12. A $$\frac{(1-\sqrt{3}i)^{24}}{(\sqrt{2}+\sqrt{2}i)^n}=\frac{(2cis300°)^{24}}{(2cis45°)^n}=\frac{2^{24}cis7200°}{2^n cis(45°n)}=\frac{2^{24-n}}{cis(45°n)}$$

For this to be an integer, n must be greater than 0, but =less than or equal to 24, and must be a multiple of 4 so that $45n$ is coterminal with either $0°$ or $180°$. The possible values are 4, 8, 12, 16, 20, 24, with the sum being 84.

13. C Let us call x_n the number of ways to get to the nth step.

$x_1=0 \quad x_2=x_3=1 \quad x_n=x_{x-2}+x_{n-3}$

The sequence is then $0,1,1,1,2,2,3,4,5,7,9,12,16,21,28,37,49$ with 49 as the 17th term.

14. D $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Since the numbers are positive: $a^2+b^2+c^2-ab-bc-ca=0\rightarrow(a-b)^2+(b-c)^2+(c-a)^2=0$ which implies that $a=b=c$.

$$\frac{(a+b)(b+c)(c+a)}{abc}=\frac{(2a)(2b)(2c)}{abc}=8$$

15. C Using the fact that $au\times v=a(u\times v)$, and $au\cdot v=a(u\cdot v)$, we get $22x=154 \quad x=7,\ -4x=-28$

16. E Multiply the top and bottom by 15x, and then plug in 0 and simplify.

$$\lim_{x\to 0^+}\frac{15x-3}{30x-10}=\frac{3}{10}$$

17. B After the first day, Michael has recovered 5 units of knowledge, and loses 4 while he sleeps, resulting in 6 total units of knowledge recovered after the 2nd day. Continuing in this manner, he recovers all 100 units of knowledge on day 96.

18. B Solve for t and plug in to get $y=\frac{9x^2-24x+16}{2}-3x+4=\frac{9}{2}x^2-15x+12$

Axis of symmetry is $-\frac{b}{2a}=-\frac{(-15)}{2(4.5)}=\frac{15}{9}=\frac{5}{3}$

19. D This is one of the definitions of e.

20. D $\dfrac{\csc(x)\,(\sec(x)-\tan(x))}{(\sec(x)-\tan(x))^2+1} = \dfrac{\csc(x)\,(\sec(x)-\tan(x))}{(\sec(x))^2+(\tan(x))^2-2\sec(x)\tan(x)+1}$
$= \dfrac{\csc\,(x)(\sec(x)-\tan(x))}{2(\sec(x))^2-2\sec(x)\tan\,(x)} = \dfrac{\csc\,(x)(\sec(x)-\tan(x))}{2\sec\,(x)(\sec(x)-\tan(x))} = \dfrac{\cot\,(x)}{2}$

21. C We can turn this situation into an equation:
$350+7x=700+3.5x \quad 3.5x=350 \quad x=100$

22. B The easiest way to solve this would be to convert the numbers to base 10, multiply, and then convert back to base 9.
$33_9=(3)(9)+3=30 \quad 44_9=(4)(9)+4=40 \quad 30\times 40=1200$
$1200=(1)(9^3)+(5)(9^2)+(7)(9^1)+(3)(9^0)=1573_9$

23. B The volume of the box is $6x^2=72$, where x is the side length of the base, and $x=2\sqrt{3}$. The surface area will then be the four sides plus the base multiplied by 2 since the box has exposed area on the inside and outside.
$SA=2\left(6\left(8\sqrt{3}\right)+\left(2\sqrt{3}\right)^2\right)=96\sqrt{3}+24$

24. C Since the coin is a fair, two-sided coin, there will always be a 50% chance of either heads or tails coming up.

25. D We can form a quadratic equation with these values and factor it to find the roots.
$x^2-54x+704=0 \quad (x-22)(x-32)=0 \quad 32-22=10$

26. B The edge of the large cube was 8, and the 2 corner cubes will have 3 faces painted, which leaves 6 cubes per edge, and 12 edges. (6)(12) = 72 cubes.

27. B $\ln(-5)=\ln(-1)+\ln(5)=n+\ln(-1)$
$y=\ln(-1) \quad e^y=-1 \quad y=\pi i$
$\ln(-5)=n+\pi i$

28. C $x=18 \quad \sqrt{15\cdot 17\cdot 19\cdot 21+16}=\sqrt{(x-3)(x-1)(x+1)(x+3)+16}$
$=\sqrt{x^4-10x^2+25}=x^2-5=324-5=319 \quad 3+1+9=13$

29. A
$$1=\tan\left(\frac{\pi}{4}\right)=\tan\left(\frac{\pi}{9}+\frac{5\pi}{36}\right)=\frac{\tan\left(\frac{\pi}{9}\right)+\tan\left(\frac{5\pi}{36}\right)}{1-\tan\left(\frac{\pi}{9}\right)\tan\left(\frac{5\pi}{36}\right)}$$
$$1-\tan\left(\frac{\pi}{9}\right)\tan\left(\frac{5\pi}{36}\right)=\tan\left(\frac{\pi}{9}\right)+\tan\left(\frac{5\pi}{36}\right) \quad \tan\left(\frac{5\pi}{36}\right)=\frac{1-\tan\left(\frac{\pi}{9}\right)}{1+\tan\left(\frac{\pi}{9}\right)}$$
$$\tan(\theta)=\frac{\frac{1-\tan\left(\frac{\pi}{9}\right)}{1+\tan\left(\frac{\pi}{9}\right)}-1}{\tan\left(\frac{\pi}{9}\right)-1}=\frac{1-\tan\left(\frac{\pi}{9}\right)-1-\tan\left(\frac{\pi}{9}\right)}{\left(\tan\left(\frac{\pi}{9}\right)\right)^2-1}=\frac{2\tan\left(\frac{\pi}{9}\right)}{1-\left(\tan\left(\frac{\pi}{9}\right)\right)^2}$$
$$=\tan\left(\frac{2\pi}{9}\right)$$

30. A This graph is a dampening sinusoidal wave with roots every half, which means the period of the sinusoid is 1.