Good Luck! :)

1. Robb's running schedule requires he never run more than 2 days in a row, nor does he go more than 2 days in a row without running. In 2 weeks, what is the positive difference between the fewest number of days and the greatest number of days that he could run?

   A. 3 B. 4 C. 5 D. 6 E. NOTA

2. Rickon randomly chooses 2 distinct numbers from the set $\{1,2,4,8,16,32\}$. The probability the sum is greater than 18 is $\frac{B}{30}$. What is $B$?

   A. 7 B. 12 C. 14 D. 16 E. NOTA

3. A regular right hexagonal pyramid has height $\sqrt{13}$ and slant height 5. What is the volume of the pyramid?

   A. $6\sqrt{39}$ B. $8\sqrt{39}$ C. $16\sqrt{13}$ D. $39\sqrt{3}$ E. NOTA

4. Evaluate: $\lim_{x\to 0} \frac{|3x-2|-|3x+2|}{x}$.

   A. $-6$ B. $-4$ C. 4 D. DNE E. NOTA

5. The line through the origin with angle of inclination 30 degrees intersects with the graph $y = -3x^2 + 6x$ at the origin and $(a, b)$. What is the value of $b$?

   A. $\frac{6\sqrt{3}-1}{9}$ B. $\frac{6\sqrt{3}-1}{27}$ C. $\frac{\sqrt{3}}{162}$ D. $\frac{2\sqrt{22}}{27}$ E. NOTA

6. Evaluate: $\sum_{n=1}^{\infty} \frac{n}{3^n}$

   A. $\frac{7}{16}$ B. $\frac{9}{16}$ C. $\frac{11}{16}$ D. $\frac{3}{4}$ E. NOTA

7. Find the number of distinct real values for $x$ such that

$$\log(25x) + \log(4x) - 4\log(x-1) = 2$$

A. 0 B. 1 C. 2 D. 4 E. NOTA

8. What is/are the horizontal asymptotes of $g(x) = \frac{e^{2x}-4x^2}{5e^{2x}+2x^2}$?

A. $y = -2$ B. $y = .2$ C. $y = -2$ and $y = 0$ D. $y = .2$ and $y = 0$ E. NOTA

9. Compute $\langle -2,1,3\rangle \times \langle 4,-1,0\rangle$.

A. $\langle 3,12,2\rangle$ B. $\langle 8,1,0\rangle$ C. $\langle 3,12,-2\rangle$ D. $\langle -2,0,-3\rangle$ E. NOTA

10. Let $A = \{1,2,3 \ldots 89,90\}$ and $B \subset A$ with $|B| = 80$. If the sum of the elements of $B$ is $S$, how many possible values are there for $S$?

A. 80 B. 100 C. 800 D. 801 E. NOTA

11. If $y = \cos^2 t + 9\sin^3 t + x^2$, and $x = 3\sin t$, then which of the following values of $x$ is a solution to $y = 0$ when $y$ is a function of only the variable $x$?

A. 2 B. 3 C. 9 D. 27 E. NOTA

12. What is the coefficient of the 4th term in the binomial expansion of $(4a + b)^{\frac{1}{2}}$?

A. $\frac{1}{16}$ B. $\frac{1}{128}$ C. $\frac{1}{256}$ D. $\frac{1}{512}$ E. NOTA

13. In triangle $ABC$, $AB = 5$, $BC = 7$, $CA = 9$, and D is on $\overline{CA}$ with $BD = 5$. If $\frac{AD}{DC} = \frac{x}{y}$ where $x, y$ are relatively prime positive integers, what is the value of $x + y$?

A. 9 B. 12 C. 27 D. 81 E. NOTA

14. The quadratic sequence $\{a_1, a_2, a_3, a_4, a_5, \ldots\} = \{-3, -2, 3, 12, 25, \ldots\}$ can be represented by the rule $a_n = An^2 + Bn + C$. What is the value of $A - B - C$?

A. $-6$ B. 2 C. 3 D. 7 E. NOTA

15. Evaluate: $\begin{vmatrix} -1 & 2 & 3 & 1 \\ 0 & 3 & 4 & 5 \\ 1 & 0 & 0 & -2 \\ 4 & 1 & -3 & 2 \end{vmatrix}$

A. 12 B. 13 C. 48 D. 54 E. NOTA

16. In how many ways can 3 lattice points be picked such that both coordinates of each point are positive integers less than 5 and the 3 points form a triangle?

A. 488 B. 516 C. 520 D. 528 E. NOTA

17. If the zeroes of $J(x) = x^4 - 13x^3 + 47x^2 - 23x - 84$ are $S, N, O, W$ with $S > N > O > W$, find the value of $S + N + W$.

A. 9 B. 10 C. 11 D. 13 E. NOTA

18. Find all asymptotes of $\tanh x$.

A. $x = 1$ B. $y = -1$ C. $x = \pm 1$ D. $y = 1$ E. NOTA

19. How many positive perfect cubes divide $3! \cdot 5! \cdot 7!$?

A. 4 B. 5 C. 6 D. 14 E. NOTA

20. Given the equation, $\sec^2(x) - 2\tan(x) = 4$, how many real solutions exist for $x$ over the domain $(-\pi, \pi]$?

A. 0 B. 1 C. 2 D. 3 E. NOTA

21. Trapezoid $BRAN$ is circumscribed around a circle. Points $S$ and $T$ are midpoints of legs $\overline{BN}$ and $\overline{RA}$ respectively. If $ST = 10$, what is the perimeter of $BRAN$?

A. 30 B. 38 C. 40 D. 48 E. NOTA

22. A circle centered at the origin has radius 25. How many lattice points lie on the circle?

A. 8 B. 12 C. 16 D. 20 E. NOTA

23. Find $\left(\sec\frac{\pi}{9}\right)\left(\sec\frac{2\pi}{9}\right)\left(\sec\frac{4\pi}{9}\right)$.

A. 4 B. 6 C. 8 D. 12 E. NOTA

24. For $k \in \mathbb{R}$, find the product of the solutions to: $\log_4 k + \log_{k^2}\frac{1}{8} = 1$.

A. $\frac{-1}{4}$ B. $\frac{1}{4}$ C. $\frac{3}{4}$ D. 4 E. NOTA

25. Evaluate the following expression: $\cos\left(2\arcsin\frac{5}{6}\right)$.

A. $-\frac{25}{18}$ B. $-\frac{7}{18}$ C. $\frac{7}{18}$ D. $\frac{25}{18}$ E. NOTA

26. Find the radius of the circle that passes through the points $(1,-2)$, $(10,5)$, and $(5,4)$.

A. 8 B. $\sqrt{65}$ C. $4\sqrt{17}$ D. $\sqrt{70}$ E. NOTA

27. What is the sum of the period and phase shift of: $y = 5\sin\left(5x - \frac{\pi}{2}\right)$? (Magnitude of the phase shift should be minimized.)

A. $\frac{3\pi}{10}$ B. $\frac{\pi}{2}$ C. $\frac{7\pi}{10}$ D. $\frac{9\pi}{10}$ E. NOTA

28. From point A outside of circle R, two secants, $\overline{AS}$ intersecting the circle at T and $\overline{AK}$ going through the center (point R). The degree measure of angle SRT is 74 and the degree measure of angle SAK is 28. What is the degree measure of minor arc SK?

A. 56 B. 68 C. 74 D. 81 E. NOTA

29. Evaluate: $\lim_{x\to 0} \frac{1+2^{\frac{1}{x}}}{3+2^{\frac{1}{x}}}$

A. DNE B. 0 C. $\frac{1}{3}$ D. 1 E. NOTA

30. If $\cot x = A$ and $\cot y = B$, find $\cot(x-y)$ in terms of $A$ and $B$, given that $AB \neq 0$ and $|A| \neq |B|$.

A. $\frac{AB-1}{A+B}$ B. $\frac{AB-1}{A-B}$ C. $\frac{AB+1}{A-B}$ D. $\frac{AB+1}{B-A}$ E. NOTA