The acronym NOTA denotes that "None of the Above" answers are correct. All answers are rationalized, simplified, and exact unless otherwise stated. Good luck and have fun!

1. Evaluate:

$$\sum_{n=1}^{10}(4n-5)$$

A. 175 B. 130 C. 215 D. 170 E. NOTA

2. The sum of the positive integers between 119 and N (inclusive) is 2024. Find N.

A. 134 B. 130 C. 102 D. 89 E. NOTA

3. What is a_{131} in a sequence recursively defined as $a_{n-1} = \frac{2a_n - 5}{2}$, where $a_2 = 100$?

A. $\frac{445}{2}$ B. 420 C. $\frac{845}{2}$ D. 425 E. NOTA

4. Sequences $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$ are composed of strictly positive terms. If $\lim_{n\to\infty} a_n = \frac{1}{4}$ and $b_n > a_n$ for all n, which of the following must be true?

I. b_n does not converge. II. $\sum_{n=1}^{\infty} a_n$ converges III. $\prod_{n=1}^{\infty} b_n$ converges

A. II only B. III only C. I and II D. II and III E. NOTA

5. Find the sum of the entries in *m*:

$$m = \prod_{n=1}^{8}\begin{bmatrix} n & 0 \\ 0 & 1 \end{bmatrix}$$

A. 40321 B. 39521 C. 5041 D. 37 E. NOTA

6. How many integers in the interval $[1000,10000)$ are divisible by 5 and 9?

A. 201 B. 200 C. 199 D. 198 E. NOTA

7. Evaluate:

$$\sqrt{2}\prod_{\theta=1^\circ}^{90^\circ}\operatorname{cis}\theta$$

A. $-1-i$ B. $1-i$ C. $i-1$ D. $1+i$ E. NOTA

8. A finite arithmetic sequence has thirty-four terms and a common difference of six. If the sum of the sequence is 238, what is the value of the twenty-second term in the sequence?

A. 31 B. 34 C. 37 D. 40 E. NOTA

9. Express the repeating decimal $0.\overline{48}$ as a fraction.

A. $\frac{16}{33}$ B. $\frac{8}{17}$ C. $\frac{97}{198}$ D. $\frac{148}{297}$ E. NOTA

10. Evaluate:

$$\sum_{n=1}^{35}\frac{n^2}{5}$$

A. 3024 B. 2982 C. 1554 D. 1512 E. NOTA

11. Find the units digit of m.

$$m=\sum_{n=0}^{101}n!$$

A. 2 B. 3 C. 4 D. 6 E. NOTA

12. For an infinite sequence $\{a_n\}_{n\geq 1}$ of rationals, the harmonic mean of a_n, a_{n+1}, and a_{n+2} is $n+1$. If $a_1=5$ and $a_2=6$, then $a_4=-\frac{A}{B}$ for relatively prime positive integers A and B. Find $A+B$.

A. 11 B. 13 C. 17 D. 21 E. NOTA

13. Find the number of positive integers in the interval $[100,10000)$ whose digits in order form an arithmetic sequence. *Note:* 0 *cannot be the leading digit of a positive integer.*

A. 57 B. 72 C. 75 D. 78 E. NOTA

14. Consider $\{a_n\}_{n\in\mathbb{N}} = \left\{\frac{7}{2}, \frac{12}{5}, \frac{17}{8}, \frac{22}{11}, \ldots\right\}$. Compute $\lim_{n\to\infty} a_n$.

A. 1 B. $\frac{3}{2}$ C. $\frac{5}{3}$ D. $\frac{7}{2}$ E. NOTA

15. Which of the following statements is true about the sequence shown below?

$$\{a_n\}_{n\in\mathbb{N}} = \frac{(-1)^n}{n^2+1}$$

I. The sequence is monotonic.
II. The sequence is convergent.
III. The sequence is unbounded.

A. I and II B. II and III C. I and III D. II only E. NOTA

16. Find the sum of the 400 smallest odd positive integers.

A. 160000 B. 159201 C. 40000 D. 39801 E. NOTA

17. Evaluate:

$$\sum_{n=0}^{100} \cos\left(\frac{n\pi}{3} + \frac{\pi}{2}\right)$$

A. $-\sqrt{3}$ B. $-\frac{\sqrt{3}}{2}$ C. $\frac{\sqrt{3}}{2}$ D. $\sqrt{3}$ E. NOTA

18. Find the 23rd term in an arithmetic sequence $\{a_n\}$ where $a_{12} = 60$ and $a_{48} = 78$.

A. 64 B. 64.5 C. 65 D. 65.5 E. NOTA

19. For the sequence $\{a_n\}_{n\geq 2}$, $\sqrt{a_n+\sqrt{a_n+\sqrt{a_n+\cdots}}}=n$. Find $\sum_{n=2}^{10} a_n$.

A. 276 B. 330 C. 360 D. 384 E. NOTA

20. A harmonic sequence is generally written in the form $a_n=\frac{1}{k+(n-1)d}$, where k is a real number, d is the common difference of the denominator and a_n represents the nth term in the sequence. If, for a given harmonic sequence, $\frac{a_2}{a_3}=\frac{4}{3}$, find $\frac{k}{d}$.

A. -6 B. -5 C. 1 D. 2 E. NOTA

21. Let $\{h_n\}_{n\geq 0}$ be a sequence of integers. For all $n\geq 1$, let $h_n=3h_{n-1}+1$ if h_{n-1} is odd and $h_n=\frac{h_{n-1}}{2}$ if h_{n-1} is even. If $h_0=11$, find the smallest value of n such that $h_n=1$.

A. 12 B. 14 C. 15 D. 17 E. NOTA

22. Find the sum of the series $\frac{1}{5}-\frac{1}{25}+\frac{1}{125}-\frac{1}{625}+\cdots$.

A. $\frac{104}{625}$ B. $\frac{156}{625}$ C. $\frac{1}{6}$ D. $\frac{1}{4}$ E. NOTA

23. Find the coefficient of the x^4 term in the expansion of n.

$$n=\left(8+2x+\frac{1}{2}x^2+\frac{1}{8}x^3+\cdots\right)(16+24x+36x^2+54x^3+\cdots)$$

A. $\frac{3109}{4}$ B. $\frac{1555}{2}$ C. $\frac{1024}{3}$ D. 341 E. NOTA

24. Evaluate:

$$\sum_{n=1}^{10}\left(\frac{1-i\sqrt{3}}{2}\right)^n$$

A. $\frac{-3-i\sqrt{3}}{2}$ B. $\frac{-1-i\sqrt{3}}{2}$ C. 1 D. $-i\sqrt{3}$ E. NOTA

25. Which of the following represents, in interval notation, all values of x for which the following series converges?

$$\sum_{n=1}^{\infty} n^{x^3+4x^2-2x-9}$$

A. $(-4, -\sqrt{2})$ B. $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, 4)$
C. $(-\infty, -4) \cup (\sqrt{2}, \infty)$ D. $(-\infty, -4) \cup (-\sqrt{2}, \sqrt{2})$ E. NOTA

26. Evaluate: $\tan\left(\frac{\pi}{6} + \frac{3\pi}{6} + \frac{5\pi}{6} + \cdots + \frac{47\pi}{6}\right)$.

A. $-\frac{\sqrt{3}}{3}$ B. 0 C. $\frac{\sqrt{3}}{3}$ D. $\sqrt{3}$ E. NOTA

27. The sum of an infinite geometric series with strictly positive terms is 9. If the sum of the first two terms is 6, find the common ratio between consecutive terms in the series.

A. $\frac{\sqrt{3}}{3}$ B. $\frac{2\sqrt{3}}{5}$ C. $\frac{1}{2}$ D. $\frac{\sqrt{2}}{3}$ E. NOTA

28. $\{a_n\}_{1 \le n \le 177}$ is an arithmetic sequence with $a_1 = 3$ and $a_{177} = 2024$. Evaluate:

$$\sum_{i=1}^{176} \frac{1}{a_i a_{i+1}}$$

A. $\frac{1}{69}$ B. $\frac{3}{138}$ C. $\frac{1}{23}$ D. $\frac{2}{23}$ E. NOTA

29. Evaluate:

$$\sum_{n=2}^{\infty} \frac{1}{n^2 + 3n}$$

A. $\frac{13}{36}$ B. $\frac{13}{12}$ C. $\frac{77}{180}$ D. $\frac{11}{18}$ E. NOTA

30. The roots of the cubic $x^3 - 6x^2 + px + q$ with real coefficients form an arithmetic sequence. Find the maximum possible value of pq

A. 30 B. 32 C. 36 D. 48 E. NOTA