DACEA BCBAB CBCCD ABDBD BCBAD BAEAB

1.	**D**	$\sum_{n=1}^{10}(4n-5) = 4(1+2+3+4+5+6+7+8+9+10) - 5 \cdot 10 = 170$
2.	**A**	The sum of the integers between 119 and $119+k$ is $119(k+1)+\frac{k(k+1)}{2} = \frac{1}{2}(k^2+239k+238)$. Setting this equal to 2024 gives $k^2+239k-3810=0$, for $k=15$ (or $k=-254$). $N=134$.
3.	**C**	$2a_{n-1} = 2a_n - 5$ $a_n = a_{n-1} + \frac{5}{2}$ Each consecutive term increase by $\frac{5}{2}$ $a_n = a_1 + (n-1)\frac{5}{2}$ Using the given information: $a_1 = a_2 - \frac{5}{2} = \frac{195}{2}$ $a_{131} = \frac{195}{2} + (131-1)\frac{5}{2}$ $a_{131} = \frac{195}{2} + 325 = \frac{845}{2}$
4.	**E**	I. This is not necessarily true, sequence b_n could converge where $\lim_{n\to\infty} b_n > \frac{1}{4}$ II. This is not true as the sum will continue to increase by ¼ for each term as n→∞ III. This is not necessarily true, only true if $\lim_{n\to\infty} b_n < 1$
5.	**A**	$\prod_{n=1}^{8}\begin{bmatrix} n & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1\times 2\times 3\times \ldots \times 8 & 0 \\ 0 & 1 \end{bmatrix}$ $\prod_{n=1}^{8}\begin{bmatrix} n & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 40320 & 0 \\ 0 & 1 \end{bmatrix}$ The sum of the entries is $40320 + 1 = 40321$
6.	**B**	The Least common multiple of 5 and 9 is 45. Any number divisible by 5 and 9, has a factor of 45. $23(45) = 1035$ and $222(45) = 9990$ There are $222 - 23 + 1 = 200$ valid numbers on the interval.

7.	**C**	$$\prod_{\theta=1}^{90^\circ}\left(cis\left(\frac{\theta}{2}\right)\right)=cis(1+2+3+\cdots+90)^\circ$$ $$\prod_{\theta=1}^{90^\circ}\left(cis\left(\frac{\theta}{2}\right)\right)=cis\left(90\times\frac{(1+90)}{2}\right)^\circ$$ $$\prod_{\theta=1}^{90^\circ}\left(cis\left(\frac{\theta}{2}\right)\right)=cis(45\times 91)^\circ$$ $$\sqrt{2}\prod_{\theta=1}^{90^\circ}\left(cis\left(\frac{\theta}{2}\right)\right)=\sqrt{2}cis(45\times 3)^\circ=i-1$$
8.	**B**	The median of the sequence is $\frac{238}{34}=7$. Because there are an even number of terms, this is the average of a_{17} and a_{18}, so $a_{18}=10$. $a_{22}=10+(22-18)\cdot 6=34$.
9.	**A**	$$0.\overline{48}=0.484848484848\ldots$$ $$48.\overline{48}=100\times 0.484848484848\ldots$$ $$48.\overline{48}-0.\overline{48}=48$$ $$48=99\times 0.\overline{48}$$ $$0.\overline{48}=\frac{48}{99}=\frac{16}{33}$$
10.	**B**	$$\sum_{n=1}^{75}\left(\frac{n^2}{6}\right)=\frac{1}{6}\sum_{n=1}^{75}(n^2)$$ Common Summations → $\sum_{k=1}^{n}(k^2)=\frac{n(n+1)(2n+1)}{6}$ $$\sum_{n=1}^{35}\left(\frac{n^2}{5}\right)=\frac{1}{5}\left[\frac{35\big(35+1)(2(35)+1\big)}{6}\right]$$ $$\sum_{n=1}^{35}\left(\frac{n^2}{5}\right)=\frac{7\big(35+1)(2(35)+1\big)}{6}$$ $$\sum_{n=1}^{35}\left(\frac{n^2}{5}\right)=7\big(6)(71\big)$$ $$\sum_{n=1}^{35}\left(\frac{n^2}{5}\right)=42\times 71=2982$$
11.	**C**	$$m=\sum_{n=0}^{101}n!=1+1+2+6+24+120+720+\cdots$$ $n!$ will be a multiple of 10 for $n>4$ $1+1+2+6+4=14$ → The units digit is 4
12.	**B**	$\frac{3}{\frac{1}{5}+\frac{1}{6}+\frac{1}{a_3}}=2$ gives $a_3=\frac{15}{17}$. $\frac{3}{\frac{1}{6}+\frac{17}{15}+\frac{1}{a_4}}=3$ gives $a_4=-\frac{10}{3}$. $10+3=13$.

13.	**C**	There are 18 three- and four-digit numbers with all repeating digits. The rest can be found by listing integers. There are 16 ascending three-digit numbers, 20 descending three-digit numbers, 9 ascending four-digit numbers, and 12 descending four-digit numbers for a total of 75 arithmetic sequence integers.
14.	**C**	$$a_n = \frac{7+(n-1)5}{2+(n-1)3}$$ $$a_n = \frac{5n+2}{3n-1}$$ $$\lim_{n\to\infty}(a_n) = \frac{5}{3}$$
15.	**D**	I. The sequence is not monotonic II. The sequence converges to 0 III. The sequence is bounded because it decreases in magnitude over time. The sequence never exceeds an absolute value of $\frac{1}{2}$
16.	**A**	$$S = 1+3+5+7+\cdots$$ $$a_n = 1+2(n-1)$$ $$S = \frac{a_1+a_n}{2}\times n$$ $$S = \frac{1+\left(1+2(399)\right)}{2}\times 400$$ $$S = \frac{800}{2}\times 400 = 160{,}000$$
17.	**B**	$$\sum_{n=0}^{100}\left[\cos\left(\frac{n\pi}{3}+\frac{\pi}{2}\right)\right] = 0-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}+0+\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}+\cdots$$ Every full rotation (every six terms), the sequence cancels itself out. $$\sum_{n=0}^{100}\left[\cos\left(\frac{n\pi}{3}+\frac{\pi}{2}\right)\right] = 16(0)+\sum_{n=96}^{100}\left[\cos\left(\frac{n\pi}{3}+\frac{\pi}{2}\right)\right]$$ $$= 0-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}+0+\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$$ Alternatively, $$\sum_{n=0}^{100}\left[\cos\left(\frac{n\pi}{3}+\frac{\pi}{2}\right)\right] = \sum_{n=0}^{101}\left[\cos\left(\frac{n\pi}{3}+\frac{\pi}{2}\right)\right]-\cos\left(\frac{101\pi}{3}+\frac{\pi}{2}\right)$$ $$= 0-\cos\left(-\frac{\pi}{3}+\frac{\pi}{2}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$
18.	**D**	$$a_{48}-a_{12} = 78-60$$ $$18 = [a_1+(48-1)d]-[a_1+(12-1)d]$$ $$18 = 36d$$ $$a_{23} = a_{12}+(11)d$$ $$a_{23} = 60+11\left(\frac{1}{2}\right) = 65.5$$

19.	**B**	We have $\sqrt{a_n + n} = n$, so $a_n + n = n^2$ and $a_n = n^2 - n$. $\sum_{n=2}^{10}(n^2 - n) = \sum_{n=1}^{10}(n^2 - n) = \frac{10\cdot 11\cdot 21}{6} - \frac{10\cdot 11}{2} = 385 - 55 = 330$.
20.	**D**	From the given: $a_2 = \frac{1}{k+d(2-1)}$ $a_3 = \frac{1}{k+d(3-1)}$ $$\frac{a_2}{a_3} = \frac{4}{3}$$ $$3a_2 = 4a_3$$ $$\frac{3}{k+d(2-1)} = \frac{4}{k+d(3-1)}$$ $$\frac{3}{k+d} = \frac{4}{k+2d}$$ $$3k + 6d = 4k + 4d$$ $$k = 2d$$ $$\frac{k}{d} = 2$$
21.	**B**	The sequence is $\{11,34,17,52,26,13,40,20,10,5,16,8,4,2,1\}$, where $h_{14} = 1$.
22.	**C**	$$s = \frac{\left(\frac{1}{5}\right)}{1 - \left(-\frac{1}{5}\right)} = \frac{1}{6}$$
23.	**B**	Let C represent the coefficient. The contributing terms are $$C = \left(\frac{x^4}{32}\right)(16) + \left(\frac{x^3}{8}\right)(24x) + \left(\frac{x^2}{2}\right)(36x^2) + (2x)(54x^3) + (81x^4)(8)$$ $$C = \frac{1}{2} + 3 + 18 + 108 + 648 = \frac{1555}{2}$$
24.	**A**	$$\sum_{n=1}^{10}\left(\frac{1 - i\sqrt{3}}{2}\right)^n = \sum_{n=1}^{10} \text{cis}\left(-\frac{\pi n}{3}\right)$$ $$= \sum_{n=0}^{11} \text{cis}\left(-\frac{\pi n}{3}\right) - \text{cis}(0) - \text{cis}\left(-\frac{11\pi}{3}\right)$$ $$= 0 - 1 - \text{cis}\left(\frac{\pi}{3}\right) = -\frac{3 + i\sqrt{3}}{2}$$
25.	**D**	The series converges if $x^3 + 4x^2 - 2x - 9 < -1$ $$x^3 + 4x^2 - 2x - 8 < 0$$ $$x^2(x+4) - 2(x+4) < 0$$ $$(x^2 - 2)(x+4) < 0$$ $$(x - \sqrt{2})(x + \sqrt{2})(x + 4) < 0$$ $$x \in (-\infty, -4) \cup (-\sqrt{2}, \sqrt{2})$$

26.	**B**	$\frac{\pi}{6}+\left(\frac{\pi}{6}+\frac{\pi}{3}\right)+\left(\left(\frac{\pi}{6}+\frac{\pi}{3}\right)+\frac{\pi}{3}\right)+\cdots+\left(\frac{\pi}{6}+23\left(\frac{\pi}{3}\right)\right)$ This is an arithmetic sequence $s=24\times\frac{\left(\frac{\pi}{6}+\frac{47\pi}{6}\right)}{2}=12\times\left(\frac{48\pi}{6}\right)=96\pi$ $\tan\left(\frac{\pi}{6}+\frac{\pi}{2}+\frac{5\pi}{6}+\cdots+\frac{47\pi}{6}\right)=\tan(96\pi)$ $\tan\left(\frac{\pi}{6}+\frac{\pi}{2}+\frac{5\pi}{6}+\cdots+\frac{47\pi}{6}\right)=\tan(0\pi)=0$
27.	**A**	$a_1+a_2+\cdots+a_n=9 \rightarrow 9=\frac{a_1}{1-r} \rightarrow a_1=9(1-r)$ $a_1+a_2=6$ $a_1+(a_1\times r)=6$ $a_1(1+r)=6$ $a_1=\frac{6}{(1+r)}$ $\frac{6}{(1+r)}=9(1-r)$ $9(1-r^2)=6$ $r^2=\frac{1}{3}$ If $a_n>0$ for all n, then $r=\frac{\sqrt{3}}{3}$.
28.	**E**	Let $a_{i+1}-a_i=d$. Then $\frac{1}{a_ia_{i+1}}=\frac{a_{i+1}-a_i}{da_ia_{i+1}}=\frac{1}{d}\left(\frac{1}{a_i}-\frac{1}{a_{i+1}}\right)$. The sum therefore telescopes and is equal to $\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_{177}}\right)=\frac{1}{d}\left(\frac{a_{177}-a_1}{a_1a_{177}}\right)=\frac{1}{d}\left(\frac{176d}{a_1a_{177}}\right)=\frac{176}{3\cdot 2024}=\frac{2}{69}$.
29.	**A**	$\sum_{n=2}^{\infty}\frac{1}{n(n+3)}=\sum_{n=2}^{\infty}\frac{1}{3}\left[\frac{1}{n}-\frac{1}{n+3}\right]$ $\sum_{n=2}^{\infty}\frac{1}{n(n+3)}=\frac{1}{3}\sum_{n=2}^{\infty}\left[\frac{1}{n}-\frac{1}{n+3}\right]$ $\sum_{n=2}^{\infty}\frac{1}{n(n+3)}=\frac{1}{3}\left[\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{3}-\frac{1}{6}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\left(\frac{1}{5}-\frac{1}{8}\right)+\cdots\right]$ $\sum_{n=2}^{\infty}\frac{1}{n(n+3)}=\frac{1}{3}\left[\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right]$ $\sum_{n=2}^{\infty}\frac{1}{n(n+3)}=\frac{1}{3}\left[\frac{13}{12}\right]=\frac{13}{36}$
30.	**B**	By Vieta's, the roots of the polynomial are $2-a$, 2, and $2+a$. The sum of these taken two at a time is $12-a^2$, so $p=12-a^2$. The product of these is $8-2a^2$, so $q=2a^2-8$. Let $y=a^2$. The maximum of $2(12-y)(y-4)$ occurs when $y=8$ and is equal to 32.