

1. If  $\sin x = \frac{3}{5}$  and  $0 < x < \frac{\pi}{2}$ , find  $\cos 3x$ .
- A.  $\frac{4}{5}$       B.  $\frac{12}{5}$       C.  $-\frac{36}{125}$       D.  $-\frac{44}{125}$       E. NOTA
2. Which of the following values of  $\theta$  satisfies the inequality  $\cot \theta \cos \theta < 0$ ?
- A.  $450^\circ$       B.  $\frac{27\pi}{5}$       C.  $-\frac{\pi}{2}$       D.  $\frac{4\pi}{7}$       E. NOTA
3. Find the exact value of  $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 178^\circ + \cos 179^\circ + \cos 180^\circ$ .
- A. 1      B. 0      C. 2      D. -1      E. NOTA
4. Find the product of the 2 smallest positive solutions of  $\cos 5\theta = 0$ .
- A.  $\frac{3\pi^2}{4}$       B.  $\frac{\pi^2}{20}$       C.  $\frac{3\pi^2}{100}$       D.  $\frac{3\pi^2}{20}$       E. NOTA
5. Which of the following is equivalent to  $\sin^2 x \cos^2 x$ ?
- A.  $\frac{1}{8} - \frac{1}{8}\cos 4x$       B. 1  
C.  $\frac{1}{2}\cos 4x - \frac{1}{4}$       D.  $\frac{1}{8} - \frac{1}{2}\cos x$       E. NOTA
6. Two tangents to the same circle form a  $50^\circ$  angle. The radius of the circle is 10. Let  $A$  be the length of the smaller of the intercepted arcs and  $B$  be the length of the larger of the intercepted arcs. Find  $B - A$ .
- A.  $\frac{50\pi}{9}$       B.  $\frac{80\pi}{9}$       C.  $\frac{65\pi}{9}$       D.  $\frac{115\pi}{9}$       E. NOTA
7. Find the sum of the solutions to the equation on  $[0, 2\pi]$ :  $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 4$
- A.  $\pi$       B.  $2\pi$       C.  $3\pi$       D.  $5\pi$       E. NOTA

8. What is the value of  $\sin \frac{\pi}{6} + \sin \frac{2\pi}{6} + \sin \frac{3\pi}{6} + \dots + \sin \frac{2019\pi}{6} + \sin \frac{2020\pi}{6} + \sin \frac{2021\pi}{6}$ ?
- A.  $\frac{1+\sqrt{3}}{2}$       B.  $\frac{3+\sqrt{3}}{2}$       C.  $1 + \sqrt{3}$       D.  $2 + \sqrt{3}$       E. NOTA
9. Find the sum of the solutions of  $2\cos^3 x + \cos^2 x - 2\cos x - 1 = 0$  on the interval  $[0, 2\pi)$ .
- A.  $\frac{5\pi}{3}$       B.  $3\pi$       C.  $5\pi$       D.  $\frac{11\pi}{3}$       E. NOTA
10. If  $x = \tan\left(\frac{\pi}{4} + 2\sin^{-1}\left(\frac{3}{5}\right)\right)$  and  $x = \frac{a}{b}$  in simplest form, find  $|a + b|$ .
- A. 14      B. 17      C. 31      D. 48      E. NOTA
11. How many real values of  $x$  satisfy the equation  $\ln(x^2) = \cos x$ ?
- A. 0      B. 2      C. 4      D.  $\infty$       E. NOTA
12. Which of the following is equal to  $\sin 141^\circ$ ?
- A.  $3\sin^3 13^\circ$       B.  $1 - 8\sin^3 13^\circ$   
C.  $\sin 13^\circ - 2\sin^2 13^\circ$       D.  $3\sin 13^\circ - 4\sin^3 13^\circ$       E. NOTA
13. How many distinct arrangements can be made from the word COSECANT, using each letter exactly once and placing the vowels in alphabetical order? (Note that the vowels do not have to be adjacent to each other.)
- A. 360      B. 840      C. 3360      D. 20160      E. NOTA
14. What is the domain of  $y = (\log(\cos x))^2 - (\log(\sin x))^2$ ?
- A.  $\left\{x \in R, x \neq n\pi, x \neq \frac{n\pi}{2}, n \in Z\right\}$       B.  $\left\{2n\pi < x < \frac{\pi}{2} + 2\pi n, n \in Z\right\}$   
C.  $\left\{0 < x < \frac{\pi}{2}\right\}$       D.  $\left\{n\pi < x < \frac{\pi}{2} + \pi n, n \in Z\right\}$       E. NOTA

15. Which of the following intervals contains all values of  $x$  in the interval  $\left(0, \frac{\pi}{2}\right)$  such that the sum of the infinite geometric series  $1 + 3 \tan^2 x + 9 \tan^4 x + 27 \tan^6 x + \dots$  diverges?
- A.  $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$       B.  $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$       C.  $\left[\frac{\pi}{6}, \frac{\pi}{2}\right)$       D.  $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$       E. NOTA
16. Two intersecting lines have slopes of 1 and  $\frac{17}{7}$ . What is the slope of the line that bisects the acute angle formed by these lines?
- A.  $\frac{12}{7}$       B.  $\frac{3}{2}$       C.  $\frac{17}{10}$       D. 2      E. NOTA
17. If  $f(x) = e^x \cos x$  and  $g(x) = e^x \sin x$ , find the zeros of the function  $j(x) = \frac{f^2(x) - g^2(x)}{f^2(x) + g^2(x)}$  on the interval  $\left[0, \frac{3\pi}{2}\right]$ .
- A.  $\frac{\pi}{4}, \frac{5\pi}{4}$       B.  $\frac{\pi}{2}, \frac{3\pi}{2}$       C.  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$       D.  $\emptyset$       E. NOTA
18. What is the distance between the spherical coordinates  $(5, 30^\circ, 60^\circ)$  and  $(2, 0^\circ, 90^\circ)$ ?
- A.  $\sqrt{14}$       B.  $\sqrt{15}$       C. 4      D.  $\sqrt{17}$       E. NOTA
19. Evaluate  $\begin{vmatrix} \sin x & \sin x & -1 \\ 1 & \cos x & 1 \\ -\sin x & 1 & -\sin x \end{vmatrix}$  when  $x = \pi$ .
- A. 2      B. 1      C. 0      D. -1      E. NOTA
20. Which of the following represents an expression for a parabola in polar coordinates?
- I.  $r = \frac{2}{3+\cos\theta}$       II.  $r = \frac{2}{3-\sin\theta}$       III.  $r = \frac{2}{1+\cos\theta}$   
 IV.  $r = \frac{2}{1-\sin\theta}$       V.  $r = \frac{2}{\sin\theta-1}$
- A. III only      B. I, II only      C. III, IV, V only      D. IV, V only      E. NOTA

21. A pottery wheel spins with an angular velocity of 4500 revolutions per minute. What is the linear velocity, measured in feet per second, of a point 2 inches from the center of the pottery wheel?
- A.  $300\pi$       B.  $50\pi$       C.  $75\pi$       D.  $25\pi$       E. NOTA
22. What is the equivalent rectangular equation of the polar equation  $r \sin(\theta + \frac{\pi}{6}) = 2$  ?
- A.  $\sqrt{3}x + y = 2$       B.  $x + \sqrt{3}y = 4$   
C.  $x + 2y = 2\sqrt{3}$       D.  $\sqrt{3}x + y = 4$       E. NOTA
23. Which of the following statements is false?
- A. DeMoivre's Theorem is true for all complex base and integer exponent.  
B.  $e^{i\pi} = -1$   
C. The polar graph of  $\ln r = \theta$  is a spiral  
D. If  $z_1 = r \ cis \alpha$  and  $z_2 = r \ cis \beta$ , then  $z_1 z_2 = r^2 \ cis (\alpha\beta)$   
E. NOTA
24. Find the area bounded by  $r^2 = \frac{72}{1+8 \cos^2 \theta}$ .
- A.  $72\pi$       B.  $36\pi$       C.  $24\pi$       D.  $6\pi$       E. NOTA
25. If  $\vec{u} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$  and  $\vec{v} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ , find the secant of the angle between  $\vec{u}$  and  $\overrightarrow{u+v}$ .
- A.  $\frac{\sqrt{85}}{6}$       B.  $\frac{17\sqrt{2}}{23}$       C.  $\frac{69\sqrt{2}}{34}$       D.  $\frac{10\sqrt{85}}{69}$       E. NOTA
26. Which one of the following is not true for all real values of  $x$  ?
- A.  $\sin x = \cos \left( \frac{\pi}{2} - x \right)$       B.  $\sin x \cos x = \frac{1}{2} \sin 2x$   
C.  $\sin \left( \frac{x}{2} \right) = \sqrt{\frac{1}{2}(1 - \cos x)}$       D.  $2 \cos^2 x = 1 + \cos 2x$       E. NOTA

27. What is the minimum value of  $\sin^4 x + x^2 + 0.5(\sin 2x)^2 + \cos^4 x$  ?  
A. 0      B.  $\frac{1}{2}$       C.  $\frac{\sqrt{2}}{2}$       D. 1      E. NOTA
28. The polar equation  $r(1 - 4 \sin \theta) = 2$  can be rewritten in Cartesian form as  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ , where  $A > 0$ . Find the value of  $\frac{AC+2(F^2-AE)}{EF}$ .  
A.  $\frac{81}{32}$       B.  $\frac{31}{64}$       C.  $\frac{21}{32}$       D.  $\frac{49}{64}$       E. NOTA
29. Two lighthouses are 50 miles apart on a straight shoreline. A cruise ship is located in the water 30 miles from Lighthouse A and 25 miles from Lighthouse B. How far is the ship from the shoreline?  
A.  $\frac{3\sqrt{231}}{4}$       B.  $\frac{25\sqrt{3}}{2}$       C.  $\frac{\sqrt{2079}}{100}$       D.  $12\sqrt{231}$       E. NOTA
30. If  $\lim_{x \rightarrow 0} \left( \frac{a+\cos bx}{x^2} \right) = -4$ , find the value of  $a + b^2$ .  
A. 7      B. 8      C. 9      D. 5      E. NOTA