1. Simplify the expression: $\frac{\sin 25° \cos 25° \cos 50° \cos 100° \cos 200°}{\sin 40°}$

A. $\frac{1}{8}$ B. $\frac{1}{16}$ C. $\frac{1}{32}$ D. $\frac{1}{64}$ E. NOTA

2. Find $\sin^4(x) + \cos^4(x)$ if $\cos(2x) = \frac{3}{5}$, and $0 \leq x < \frac{\pi}{2}$.

A. $\frac{7}{25}$ B. $\frac{9}{25}$ C. $\frac{17}{25}$ D. $\frac{21}{25}$ E. NOTA

3. The square of the distance between the polar points $\left(2, \frac{11\pi}{18}\right)$ and $\left(5, -\frac{2\pi}{9}\right)$ can be written in the form of $A + B\sqrt{C}$ in simplest form. Find $A + B + C$.

A. 22 B. 32 C. 36 D. 42 E. NOTA

4. Find the area of the region enclosed by the x-axis, the y-axis, and the polar curve $r = \frac{7}{5\cos\theta + 4\sin\theta}$.

A. $\frac{49}{40}$ B. $\frac{25}{56}$ C. $\frac{49}{20}$ D. $\frac{25}{28}$ E. NOTA

5. A right triangle PQR is inscribed in a circle of diameter 10. Find the maximum possible area of the triangle.

A. 59 B. 24 C. 25 D. 100 E. NOTA

6. What is the value of $\sin\left(2\arccos\left(\frac{1}{5}\right)\right)$

A. $\frac{4\sqrt{6}}{25}$ B. $\frac{2\sqrt{6}}{25}$ C. $\frac{4\sqrt{6}}{5}$ D. $\frac{2\sqrt{6}}{5}$ E. NOTA

7. Find the distance between the foci in the graph of the parametric equations $x = 1 + 3\sec(\theta), y = 3 - \tan(\theta)$ where $\theta \in [0, 2\pi)$

A. $2\sqrt{2}$ B. $4\sqrt{2}$ C. 4 D. 8 E. NOTA

8. Find the cosine of the angle between the vectors $\langle 3,4,12\rangle$ and $\langle 2,5,5\rangle$.

A. $\frac{43\sqrt{3}}{117}$ B. $\frac{43\sqrt{6}}{234}$ C. $\frac{43\sqrt{6}}{117}$ D. $\frac{7\sqrt{6}}{117}$ E. NOTA

9. Calculate $\sin\frac{\pi}{12}+\cos\frac{7\pi}{12}+\tan\frac{\pi}{8}$.

A. $\frac{\sqrt{6}+2\sqrt{2}-2}{2}$ B. $\frac{3\sqrt{2}-2}{2}$ C. $\sqrt{2}+1$ D. $\sqrt{2}-1$ E. NOTA

10. Calculate $(i+\sqrt{3})^3$.

A. $8i$ B. $-8i$ C. 8 D. -8 E. NOTA

11. Find the value of $x \in [0^o, 180^o)$ in degrees that satisfy

$$\tan(x) = (4\cos^2(9^o)-3)(4\cos^2(27^o)-3)$$

A. 12 B. 18 C. 24 D. 27 E. NOTA

12. Find the sum of the solutions to the equation $\sin\frac{\theta}{2}+\cos\frac{\theta}{2}=1, \theta \in [0,8\pi)$.

A. 6π B. 10π C. 14π D. 18π E. NOTA

13. Given the function $g(t) = -7\cos(4\pi t-5)+13$. Let $A =$ the amplitude of the function, let $B =$ the period, and the range be $[C, D]$. Find the sum $A+B+C+D$.

A. $\frac{39}{2}$ B. $\frac{57}{2}$ C. $\frac{67}{2}$ D. 35 E. NOTA

14. Find the eccentricity of the conic with polar equation $r = \frac{7}{4-5\cos\theta}$.

A. $\frac{5}{4}$ B. $\frac{4}{5}$ C. $\frac{4}{7}$ D. $\frac{5}{7}$ E. NOTA

15. Which of the following expressions is equivalent to $\frac{\sec\theta-\cos\theta}{\csc\theta-\sin\theta}$? (where both expressions are defined.)

A. $(\tan\theta)(\sec^2\theta+1)$ B. $(\tan\theta)(\sec^2\theta-1)$
C. $(\tan\theta)(1-\sec^2\theta)$ D. $(\tan\theta)(csc^2\theta-1)$ E. NOTA

16. Given $\sin x+\cos x=\frac{1}{\sqrt{2}}$, and $\sin y+\cos y=0$, find the sum of all possible positive values of $\sin(x+y)$.

A. $\frac{1}{2}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{\sqrt{3}+1}{2}$ D. $\frac{\sqrt{3}+\sqrt{2}+1}{2}$ E. NOTA

17. Find the period of the function $g(x)=8\cos(8\pi x)+11\sin(11\pi x)$.

A. 4 B. $\frac{8}{11}$ C. $\frac{11}{8}$ D. 2 E. NOTA

18. Find the sum of the solutions to $\sin^3(x)\cos(x)-\sin(x)\cos^3(x)=\frac{-\sqrt{2}}{4}\cos(2x)$ where $x\in[0,2\pi)$

A. 3π B. 7π C. $3\pi/2$ D. 14π E. NOTA

19. Find the longest distance between complex roots of the equation $(z+4i-7)^6=16$ when graphed in the complex plane.

A. $2\sqrt[3]{4}$ B. $\sqrt[3]{4}$ C. $2\sqrt{2}$ D. $4\sqrt{2}$ E. NOTA

20. Calculate $\cot 102°\cot 123°-\cot 102°-\cot 123°$.

A. 0 B. -1 C. 1 D. $\frac{1}{2}$ E. NOTA

21. Dave and Margaret start walking from the same point. Dave walks at a heading of 60 degrees at a rate of 5 mph, while Margaret walks at a heading of 315 degrees at a rate of 7 mph. After two hours, the square of the distance between them can be written as $A + B(\sqrt{C} - \sqrt{D})$ where A, B, C, D are positive integers and B is maximized. Find $A + B + C - D$.

A. 230 B. 362 C. 370 D. 374 E. NOTA

22. $\left(\frac{\cos(2x)+1}{2}\right)^2 - \frac{1}{2} = \left(\frac{1-\cos(2x)}{2}\right)^2$. Find x if $x \in \left[0, \frac{\pi}{2}\right)$.

A. $\frac{\pi}{8}$ B. $\frac{\pi}{6}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{3}$ E. NOTA

23. Dave and Margaret are standing 100 ft apart on flat ground and looking up at a kite. The kite is directly above a point that is directly between Dave and Margaret. The angle of elevation from Dave to the kite is 60° and from Margaret it is 45°. How far above the ground is the kite flying?

A. $50\sqrt{3} - 50$ B. $100\sqrt{3} - 100$
C. $150 - 50\sqrt{3}$ D. $50\sqrt{3} + 50$ E. NOTA

24. $\cot\theta = \frac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}}$. Find θ, given $\theta \in \left[0, \frac{\pi}{2}\right)$.

A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{5\pi}{12}$ E. NOTA

25. Simplify $\sin\left(x - \frac{\pi}{2}\right) + \cos\left(x + \frac{\pi}{2}\right) + \tan\left(x + \frac{\pi}{2}\right) + \cot(x + \pi)$.

A. $-(\sin x + \cos x)$ B. $-(\sin x + \cos x + 2\cot x)$
C. $-\sin x + \cos x$ D. $\sin x + \cos x + \cot x$ E. NOTA

26. Find $\lim_{a\to-\infty} \arctan a - \lim_{b\to-\infty} \operatorname{arccot} b$.

A. $-\frac{3\pi}{2}$ B. $-\frac{\pi}{2}$ C. $\frac{\pi}{2}$ D. $\frac{3\pi}{2}$ E. NOTA

27. Find the area of region enclosed by the polar graph $r = 7(\cot^2\theta - \csc^2(\theta))$.

A. 0 B. $\frac{49\pi}{4}$ C. 49π D. 196π E. NOTA

28. The solutions to the equation $x^2 + ax + b = 0$ are $\tan c$ and $\tan d$. Express $\tan(c + d)$ in terms of a and b.

A. $\frac{a}{1-b}$ B. $\frac{a}{b-1}$ C. $\frac{a}{b+1}$ D. $\frac{-a}{b-1}$ E. NOTA

29. Let $w_k = \text{cis}\left(\frac{k\pi}{12}\right)$ for $0 \le k \le 23$. Find the sum of all positive integers $n \le 24$ such that

$$w_0^n + w_1^n + w_2^n + \cdots + w_{23}^n = 0$$

A. 96 B. 144 C. 276 D. 300 E. NOTA

30. Final problem! The expression

$$\frac{\sin 12° + \sin 28° + \sin 66° + \sin 82°}{\cos 12° + \cos 20° + \cos 66° + \cos 74°}$$

can be simplified to $\frac{\cos(x°)}{\cos(y°)}$, where x, y are positive integers no more than 90. Compute the value of $x + y$.

A. 12 B. 48 C. 84 D. 120 E. NOTA