1. D $\cos x = \frac{4}{5}$ and $\cos 3x = 4\cos^3 x - 3\cos x \rightarrow 4\left(\frac{4}{5}\right)^3 - 3\left(\frac{4}{5}\right) = -\frac{44}{125}$

2. B $\cot\theta\cos\theta = \frac{\cos^2\theta}{sin\theta}$. The only way this is negative is if $\sin\theta < 0$. The only angle that satisfies this requirement is $\frac{27\pi}{5}$ .

3. D All terms cancel except for $\cos 180°$ which is $-1$

4. C $5\theta = \frac{\pi}{2} + 2\pi k$ and $5\theta = \frac{3\pi}{2} + 2\pi k$ which yields $\theta = \frac{\pi}{10} + \frac{2\pi}{5}k$ and $\theta = \frac{3\pi}{10} + \frac{2\pi}{5}k$. The two smallest solutions are $\frac{\pi}{10}$ and $\frac{3\pi}{10}$ and their product is $\frac{3\pi^2}{100}$.

5. A $\sin^2 x\cos^2 x = \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) = \frac{1}{4} - \frac{1}{4}\cos^2 2x = \frac{1}{4} - \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2}\cos 4x\right) = \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos 4x = \frac{1}{8} - \frac{1}{8}\cos 4x$.

6. A Sketching the picture we find that the central angle of the smaller arc is $130°$ therefore making the central angle of the larger arc $230°$. Subtract the 2 angles and find the arc length: $S = \left(\frac{100\pi}{180}\right)(10) = \frac{50\pi}{9}$ .

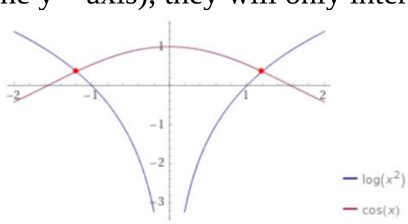
7. B Equation simplifies to $\frac{1+2\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)} = 4 \rightarrow \frac{2+2\sin x}{\cos x(1+\sin x)} = 4 \rightarrow \frac{2}{\cos x} = 4 \rightarrow \cos x = \frac{1}{2}$. The two angles are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

8. D Because of cyclic nature of sine, all of the terms will cancel except for the last five terms. Therefore, $\sin\left(\frac{2017\pi}{6}\right) + \sin\left(\frac{2018\pi}{6}\right) + \sin\left(\frac{2019\pi}{6}\right) + \sin\left(\frac{2020\pi}{6}\right) + \sin\left(\frac{2021\pi}{6}\right) = 2 + \sqrt{3}$

9. B Equation factors to $(\cos^2 x - 1)(2\cos x + 1) = 0$ producing solutions of $0, \frac{2\pi}{3}, \pi$, and $\frac{4\pi}{3}$. Their sum is $3\pi$.

10. A We can rewrite the equation as $x = \tan\left(Arctan\, 1 + Arctan\left(\frac{24}{7}\right)\right)$. Using tangent addition we get $x = \frac{1+\frac{24}{7}}{1-1\cdot\frac{24}{7}} = -\frac{31}{17}$. Therefore $|-31 + 17| = 14$

11. B If you sketch both curves (the other branch of the log graph is simply reflected over the y – axis), they will only intersect twice.




12. D $\sin 141° = \sin 39° = \sin 3(13°) \rightarrow x = 13°$. Since $\sin 3x = 3\sin x - 4\sin^3 x$ , we have $3\sin 13° - 4\sin^3(13°)$.

13. C
$$\frac{8!}{3!\,2!} = 3360$$

14. B We need all set of x's for which both cosine and sine will be positive. Answer choice B is the only one that satisfies this requirement.

15. C This is an infinite geometric series with $r = 3\tan^2 x$. Therefore $3\tan^2 x \geq 1 \rightarrow \tan x \geq \frac{\sqrt{3}}{3}$. So $x \geq \frac{\pi}{6}$. The other bound here is where $\tan x$ is undefined which is at $\frac{\pi}{2}$.

16. B Let $m_1 = \frac{17}{7}$ and $m_2 = 1$. Therefore the tangent of the angle between them is $\tan(m_1 - m_2) = \left(\frac{\left(\frac{17}{7}-1\right)}{1+\frac{17}{7}\cdot 1}\right) = \frac{5}{12}$. This means that the sine of the angle is $\frac{5}{13}$ and the cosine of this angle is $\frac{12}{13}$. Using tangent half angle $\tan\left(\frac{\theta}{2}\right) = \frac{1-\frac{12}{13}}{\frac{5}{13}} = \frac{1}{5}$. Add this tangent back to tan 1 to get the slope of the bisector: $\tan(A+B) = \frac{1+\frac{1}{5}}{1-\frac{1}{5}\cdot 1} = \frac{3}{2}$.

17. E $j(x) = \frac{e^{2x}(\cos^2 x - \sin^2 x)}{e^{2x}(\cos^2 x + \sin^2 x)} = \cos^2 x - \sin^2 x = \cos 2x$. Therefore $\cos 2x = 0$ which produces the zeros $\frac{\pi}{4}$, $\frac{3\pi}{4}$, and $\frac{5\pi}{4}$. Exclude $\frac{7\pi}{4}$ due to the given domain restriction.

18. A Spherical coordinates are in the form of $(\rho\cos\theta\sin\phi, \rho\sin\theta\sin\phi, \rho\cos\phi)$. This makes the two points $\left(\frac{15}{4}, \frac{5\sqrt{3}}{4}, \frac{5}{2}\right)$ and $(2, 0, 0)$. Therefore $d = \sqrt{\left(\frac{7}{4}\right)^2 + \left(\frac{5\sqrt{3}}{4}\right)^2 + \left(\frac{5}{2}\right)^2} = \sqrt{14}$

19. D The determinant simplifies to $-(\sin x + 1)(1 + \sin x\cos x)$. Plugging in $\pi$ you get -1.

20. C Only III, IV, and V have an eccentricity of 1 therefore these are the parabolas.

21. D
$$\left(\frac{4500\ rot}{\text{min}}\right)\left(\frac{4\pi\ inch}{1\ rot}\right)\left(\frac{1\ ft}{12\ in}\right)\left(\frac{1\text{min}}{60\ secs}\right) = 25\pi\frac{ft}{sec}$$

22. B
$$r\left(\cos\theta\sin\frac{\pi}{6} + \sin\theta\cos\frac{\pi}{6}\right) = 2 \rightarrow \frac{1}{2}x + \frac{\sqrt{3}}{2}y = 2 \rightarrow x + \sqrt{3}y = 4$$

23. D Answer choice should be $z_1 z_2 = r^2 cis(\alpha + \beta)$

24. C $r^2 + 8r^2\cos^2\theta = 72 \rightarrow 9x^2 + y^2 = 72 \rightarrow \frac{x^2}{8} + \frac{y^2}{72} = 1$. Area of the ellipse is $\pi ab = \pi\left(\sqrt{8\cdot 24}\right) = 24\pi$

25. B $|u| = \sqrt{34}$, $|v| = \sqrt{10}$, and $|u + v| = 2\sqrt{17}$. Using the Law of Cosines…
$$10 = 34 + 68 - 2\sqrt{34\cdot 68}\cos\theta$$
$$-92 = -68\sqrt{2}\cos\theta$$
$$\cos\theta = \frac{92}{68\sqrt{2}} \rightarrow \sec\theta = \frac{17\sqrt{2}}{23}$$

26. C is false because it is missing the $\pm$

27. D This simplifies to $(\cos^2 x + \sin^2 x)^2 + x^2$ which is $1 + x^2$. Therefore the minimum value is 1.

28. D $r - 4r\sin\theta = 2 \rightarrow r^2 = (4y+2)^2 \rightarrow x^2 - 15y^2 - 16y - 4 = 0$. Therefore, $\frac{(1(-15)+2[(-4)^2-(1)(-16)])}{(-16)(-4)} = \frac{49}{64}$. Note that all terms in the fraction are quadratic, so even with the arbitrary scaling of the equation, the value of the fraction remains the same.

29. A Sketch a picture here and label the distance to the shore as x. Working from Lighthouse B and using Law of Cosines we get:

$$30^2 = 25^2 + 50^2 - 2(25)(50)\cos B$$
$$\cos B = \frac{89}{100}$$

This means that $\sin C = \frac{\sqrt{2079}}{100}$. But, since $\sin B = \frac{x}{25}$, we can equate the two expressions and solve for x:

$$\frac{x}{25} = \frac{\sqrt{2079}}{100} \rightarrow x = \frac{3\sqrt{231}}{4}$$

30. A This question will involve some knowledge of limits and where they exist. If you direct substitute in 0, you'll produce a 0 in the denominator so we'd need a 0 in the numerator in order to make the limit undefined which allows us to move forward with the problem.

$$a + \cos bx = 0 \rightarrow a + \cos b(0) = 0 \rightarrow a + 1 = 0 \rightarrow a = -1$$

Now that we know the value of $a$, we can rewrite the limit as $\lim_{x \to 0} \frac{-1+\cos bx}{x^2} = -4$ which is equivalent to $\lim_{x \to 0} \frac{-b\sin bx}{2x} = -4$. Using the special trig limit $\lim_{x \to 0} \frac{\sin Ax}{B} = \frac{A}{B}$, we can solve:

$$-\frac{b^2}{2} = 4 \rightarrow b^2 = 8$$

Therefore, $a + b^2 = -1 + 8 = 7$.