Geometry Hustle Solutions

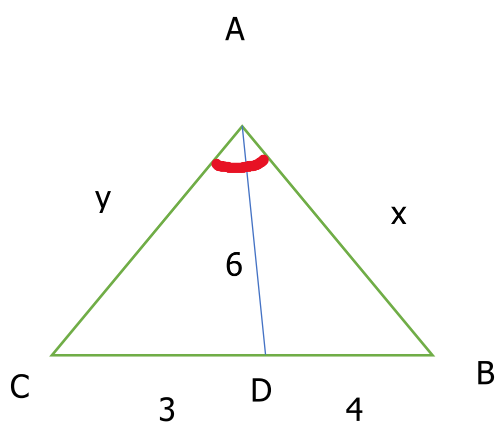
70	32	$\frac{3}{2}$	50	8
58	$\frac{xy}{z}$	105	97.5	220
24	$1+\sqrt{2}+\sqrt{5}$	0	6 and 2	18
90	2184	Inverse	$\frac{30}{11}$	$\sqrt{3}$: 1
$\frac{343\sqrt{2}}{3}$	$4\sqrt{3}$	30	$6\sqrt{3}$	2

1. $x = 25 + (180 - 135) = 70$
2. The side is two times the apothem, or $4\sqrt{2}$. Hence, the area is $\left(4\sqrt{2}\right)^2 = 32$

3. $4x + 2 + 6x + 3 = 180 \rightarrow x = \frac{35}{2}$. Plugging them back in, $m\angle A = 72$ and $m\angle B = 108$. Hence the ratio is $\frac{108}{72} = \frac{3}{2}$.

4. Sketch the graph of the two absolute value functions. This makes a rhombus with diagonals of 10 units each. $Area = \frac{1}{2}d_1 d_2 = \frac{1}{2}(10)(10) = 50$

5.



We can set up a system using the Law of Cosines: $\begin{matrix} 9 = 36 + y^2 - 6y\cos A \\ 16 = 36 + x^2 - 6x\cos A \end{matrix}$. By isolating cos A for both equations, we can set up the proportion $\frac{x^2+20}{6x} = \frac{y^2+27}{6y}$. Since we know that $y = \frac{3}{4}x$ because the triangles are similar, we can substitute into the proportion and then solve:

$\frac{x^2+20}{x} = \frac{\frac{9}{16}x^2+27}{\frac{3}{4}x} \rightarrow \frac{3}{16}x^3 - 12x = 0.$ Solving the equation yields $x = 8$.

6. The ratio between sides is $\frac{28}{14} = 2$ so the perimeter of $\triangle RST$ is twice that of $\triangle ABC$. The perimeter of $\triangle ABC$ is $14 + 11 + 4 = 29$, so the perimeter of $\triangle RST$ is 58.

7. Note that we can count the area in two ways: using the legs, and using the hypotenuse and altitude. Surely, these two must be equal. Let a denote the length of the altitude. Then, $\frac{1}{2}xy = \frac{1}{2}za \rightarrow a = \frac{xy}{z}$.

8. From the Triangle Inequality, we must have $(2x - 7) + (4x + 2) > 5x + 9$ for this NOT to be a triangle. Solving, $x < 15$. Therefore the sum of the values is $\frac{(14)(15)}{2} = 105$

9. We have $30\text{h} - 5.5m = 30(6) - 5.5(15) = 97.5$

10. The midline is half the length of CB. So 2(4x+30)=10x+20 gives x=20. And BC = 220.

11. Using the Pythagorean Theorem we get the missing segments shown to be 5 and 9. So the last base is 5+10+9 = 24.

12. ER = RS = ST = CT = 1 and EC = 2 so CR = $\sqrt{5}$ and CS = $\sqrt{2}$. Perimeter = CS + CR + RS = $\sqrt{2} + \sqrt{5} + 1$

13. The sum of an exterior angle and an interior angle is 180. Therefore, if the exterior angle is an integer, the interior angle must also be an integer. Therefore, there are no regular polygons with exterior angles that are integers and interior angles that are not integers. None (zero)

14. Set up a relationship with the diagonal and the area: $l^2 + w^2 = (2\sqrt{10})^2$ and $lw = 12$. We can then make a substitution to obtain

$\frac{144}{l^2} + l^2 = 40 \rightarrow l^4 - 40l^2 + 144 = 0 \rightarrow (l^2 - 36)(l^2 - 4) = 0 \rightarrow l = 6\ or\ 2$. Therefore the dimensions are 6 and 2.

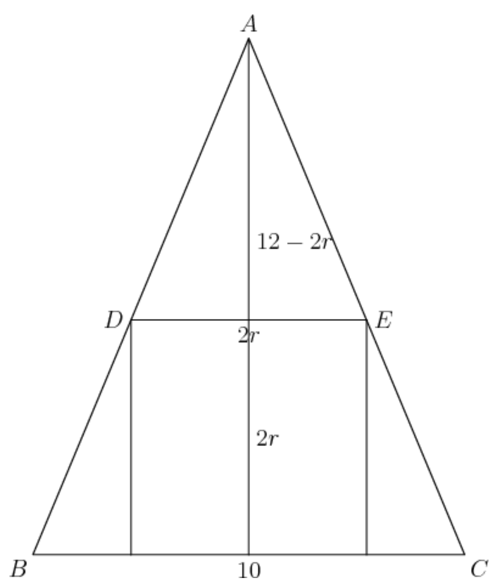
15. The sum of the exterior angles of any polygon is 360. So, $360(8) = 2880$. If $n$is the number of sides, then $\frac{(n-2)180}{n} = 2880 \rightarrow n = 18$.

16. $(180 - x) - (90 - x) = 90$

17. We will have two triangles with base $24$and height 41, and two triangles with base 80 and height 15. Thus, the lateral area is $(24)(41) + (80)(15) = 2184$.

18. The four converses and two contrapositives cancel out, leaving the three inverses. Two of those cancel out to give the inverse.

19.



Let the diameter of the cylinder be $2r$. Examining the cross-section of the cone and cylinder, we find two similar triangles. Hence, $\frac{12-2r}{12} = \frac{2r}{10} \rightarrow r = \frac{30}{11}$

20. The rhombus created forms four 30-60-90 triangles where the longer diagonal will have length $2\sqrt{3}$and the shorter diagonal will have length of 2. The ratio is $\sqrt{3}:1$.

21. Volume: $V = \frac{\sqrt{2}}{3}s^3 = \frac{\sqrt{2}}{3}(7)^3 = \frac{343\sqrt{2}}{3}$

22. Sketching the picture and adding an altitude in the triangle, you'll find that half the base of the triangle is equal to 6. Therefore, $\frac{R\sqrt{3}}{2} = 6$, which means $R = 4\sqrt{3}$.

23. Using properties of chords in a circle we obtain the ratio $15(18) = 3x(10x) \rightarrow 270 = 30x^2 \rightarrow x = 3 \rightarrow XU = 30$.

24. Triangle DXR has area $A = .5(2)(2\sqrt{3}) = 2\sqrt{3}$. Subtract this area from the area of the rhombus which is $A = 4 \bullet 2\sqrt{3} = 8\sqrt{3}$ and you get the area of OXRK, which is $6\sqrt{3}$.

25. Only the last two therefore the answer is 2.