Precalculus Hustle Solutions

$\frac{15}{2}$	$\frac{13}{183}$	$[-\sqrt{2},\sqrt{2}]$	$2-\sqrt{3}$	Obtuse
720	2041	337	262144	15
$-\sqrt{3}$	32	$\sqrt{457}$	$\frac{37}{30}$	$\frac{\sqrt{43}}{2}$
10	$\frac{3}{8}$	2	30	$\frac{\sqrt{10}}{10}$
152π	$\frac{-3-\sqrt{3}}{2}$	DNE	15	107

1. $\begin{vmatrix} 5 & x \\ 2 & 3 \end{vmatrix} = 0, 2x = 15, x = \frac{15}{2}$.

2. $-25x + 208x = 13 \;\; 183x = 13 \therefore x = \frac{13}{183}$

3. $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$, max and min of sin are 1 and –1, so range is $[-\sqrt{2}, \sqrt{2}]$.

4. $\left|e^{\frac{\pi}{3}i} - e^{\frac{\pi}{2}i}\right|^2 = \left|\text{cis}\frac{\pi}{3} - \text{cis}\frac{\pi}{2}\right|^2 = \left|\frac{1}{2} + \frac{\sqrt{3}}{2}i - i\right|^2 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} - 1\right)^2}^2 = 2 - \sqrt{3}$.

5. $49:63:84 = 7:9:12$, and since$7^2 + 9^2 < 12^2$, the triangle is Obtuse.

6. There is one 5 digit palindrome for every 3 digit number. Therefore those that do not have a 3 as their first or 2nd digit, will have less than one 3 in its corresponding palindrome.

Number of total palindromes – 1st digit 3 – 2nd digit 3 +10 = 900 – 100 – 90 + 10 = 720.

7. $6 \bmod 5 \equiv 1 \bmod 5$, $7 \bmod 6 \equiv 1 \bmod 6$, $5 \bmod 4 \equiv 1 \bmod 4$, LCM(4,5,6) = 60 so x is 1 mod 60. 2040 is divisible by 60, so $x = 2041$.

8. $(x+y)^2 - 2xy = x^2 + y^2$ $\qquad 7^2 - 2 \cdot 12 = 25$

$(x^2+y^2)^2 - 2(xy)^2 = x^4 + y^4$ $\qquad 25^2 - 2 \cdot (12^2) = 337$

9. Let $x = y = 1$. Then the sum is $8^6 = 2^{18} = 262144$.

10. $1(S_1) - 3(1) = 0, S_1 = 3 \quad 1(S_2) - 3(S_1) - 3(2) = 1(S_2) - 3(3) - 3(2) = 0 \; S_2 = 15$.

11. $\tan 120° = -\sqrt{3}$

12. $(4)(2) + (6)(4) = 32$

13. $c^2 = 7^2 + 24^2 - 2(7)(24)\cos 60° = 457 \to c = \sqrt{457}$

14. $\frac{6}{(n+1)(n+4)} = \frac{A}{n+1} + \frac{B}{n+4} = \frac{An+4A+Bn+B}{(n+1)(n+4)}$. $A + B = 0$ and $4A + B = 6$ A= 2 B = -2

$\sum_{n=3}^{\infty}\left(\frac{2}{n+1} - \frac{2}{n+4}\right) = \left(\frac{2}{4} - \frac{2}{7}\right) + \left(\frac{2}{5} - \frac{2}{8}\right) + \left(\frac{2}{6} - \frac{2}{9}\right) \ldots \ldots = \frac{2}{4} + \frac{2}{5} + \frac{2}{6} = \frac{37}{30}$

15. Shift one point down to (0, 0, 0), so (0, 0, 0), (2, 3, -3), and (-1, -3, 4)

Area $=\frac{|(2,3,-3)X(-1,-3,4)|}{2}=\frac{\sqrt{43}}{2}$.

16. $\frac{x-3}{7}=sec\,t$ and $\frac{6-y}{5}=tan\,t$, therefore $\left(\frac{x-3}{7}\right)^2-\left(\frac{6-y}{5}\right)^2=1$, Conjugate axis = $2\cdot 5=10$.

17. Solve inequality: $0\leq x\leq 2$, $0\leq y\leq 2$, $y\geq -x+2$, and $y\leq -x+3$, area enclosed by inequalities is $\frac{3}{8}$.

18. $\frac{1}{1+cos\,x}+\frac{1}{1+sec\,x}=\frac{2+sec\,x+cos\,x}{1+1+sec\,x+cos\,x}=1$, same with $\frac{1}{1+sin\,x}+\frac{1}{1+csc\,x}$, $\therefore 1+1=2$

19. Keychain permutation: $\frac{(n-1)!}{2}$, Divide by 2 again since there are two R's in DARRYL, $\frac{\frac{(6-1)!}{2}}{2}=30$.

20. $\frac{(2)(4)+(2)(2)}{(2\sqrt{2})(2\sqrt{5})}=\frac{3}{\sqrt{10}}=cos\,\theta$ $\sqrt{1-\left(\frac{3}{\sqrt{10}}\right)^2}=\sin\theta=\frac{\sqrt{10}}{10}$

21. $2(3^2\pi-1^2\pi)+(2)(3)(17)\pi+(2)(1)(17)\pi=152\pi$

22. $\left(sin\frac{7\pi}{12}\right)\left(cos\frac{5\pi}{4}\right)\left(tan\frac{\pi}{3}\right)\left(csc\frac{\pi}{6}\right)=\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)\left(-\frac{\sqrt{2}}{2}\right)\left(\sqrt{3}\right)(2)=\frac{-3-\sqrt{3}}{2}$.

23. $\lim_{x\to 0}\frac{sin\,x+1}{sin\,x}$, Top goes to 1, while bottom goes to 0, therefore, the limit does not exist.

24. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 so there are 15 prime numbers.

25. $(8)100-(97+98+101+77+99+110+111)=107$.