Mental Math
Test #401

Name: _____

ID Number: _____

School: _____

Division (circle one):

Mu Alpha Theta Sponsor

_____ 1. Evaluate: $\sum_{i=0}^{10} i$

_____ 2. Evaluate: $1 - 2 + 3 - 4 \ldots + 99$

_____ 3. Evaluate: $7!$

_____ 4. Evaluate: $\frac{2}{\frac{1}{40}+\frac{1}{60}}$

_____ 5. Round to the nearest integer: $\sum_{i=0}^{5} \sqrt{2^i}$

_____ 6. Find the sum of the reciprocals of the roots of the following equation:

$$f(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$$

_____ 7. Evaluate: $4^{\log_2 3} + 27^{\log_3 2}$

_____ 8. Evaluate: $\begin{vmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{vmatrix}$

_____ 9. Buffy has a favorite quadratic polynomial $f(x)$. He tells you that $f(-1) = -5,\ f(0) = 0$ and $f(1) = -5$ What is $f(4)$?

_____ 10. Evaluate: $\sum_{i=1}^{\infty} i \cdot \left(\frac{1}{2}\right)^i$

_____ 11. For integers x, y and z, find a valid solution to

$$x^2 + y^2 = 3z^2.$$

_____ 12. Deez's favorite number is 4 mod 7 and 3 mod 11. Given that his number is a positive integer less than 77, what is his favorite number?

_____ 13. Find the greatest prime factor of $7! + 8! + 9!$

_____ 14. Find the units digit of $\sum_{i=1}^{5} i^i$.

_____ 15. Let x, y, z be positive integers. Find the number of solutions to $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

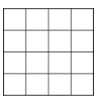
_____ 16. Find the radius of the circle whose area is equal to its circumference.

_____ 17. Find the side length of the equilateral triangle whose area is equal to its perimeter.

_____ 18. Find the side length of the regular tetrahedron whose volume is equal to its surface area.

_____ 19. A right triangle has legs of length 20 and 21. Compute the hypotenuse of this right triangle.

_____ 20. How many rectangles are in the following figure:



_____ 21. How many 5-digit palindromes are there?

_____ 22. How many distinguishable ways are there to rearrange the letters in $JEFFREY$?

_____ 23. Find the constant term of $\left(x^2 + 2 + \frac{1}{x}\right)^3$?

_____ 24. How many 3-digit numbers have the property that the ten's digit is equal to the sum of the hundred's and unit's digit?

_____ 25. How many ways are there for Konwoo to distribute 8 apples among him, Jeffrey, Andy, and Buffy given that he wants at least one?

_____ 26. Help Andy find the number of ways to draw a red and then a black card when drawing two cards from a shuffled standard 52-card deck.

_____ 27. Mr. Lu flips four coins. Given that at least one of them flips heads, what is the probability he flipped all heads?

_____ 28. Mr. Lu rolls three 6-sided dice. He gets \$1 if any of them are a 1, otherwise he loses \$1. What are his expected profits from one game?

_____ 29. Mr. Lu is now playing a game where him and I each roll a 6-sided die. Whoever rolls the higher value gets \$1 (the loser loses \$1), and if there is a tie, both players get \$1. What are Mr. Lu's expected profits from one game?

_____ 30. Mr. Lu just rolled 2022 dice. Find the probability that the sum of his rolls is even?

_____ 31. A perfect number is a number equal to the sum of its proper divisors. What is the smallest perfect number?

_____ 32. Evaluate: $\sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}$

_____ 33. Evaluate: $\sqrt{2\sqrt{2\sqrt{2 \ldots}}}$

A **24 puzzle** involves using the operations $+$, $-$, $\cdot$, $/$ among four numbers (each number used exactly once) to compute 24.

Example: $1, 2, 3, 4 \rightarrow 1 \cdot 2 \cdot 3 \cdot 4 = 24$.

_____ 34. Find a solution for the following 24: $3, 3, 3, 3$

_____ 35. Find a solution for the following 24: $5, 5, 5, 5$

For the last five questions, find the next two terms in the **sequence** indicated by the _

_____ 36. $M, T, W, T, \_\_, \_\_$

_____ 37. $S, F, W, S, S, \_\_, \_\_$

_____ 38. $O, T, T, F, F, \_\_, \_\_$

_____ 39. $M, V, E, M, J, \_\_, \_\_$

_____ 40. $D, H, Q, D, \_\_, \_\_$