NOTA means "None of the Above." Good luck!

1. If $a$ is increasing at a constant rate of 2 units per second, find the rate at which the total area of the regions bounded by $y = x^3 - ax$ and the x-axis is increasing when $a = 3$.
   A. 3 B. 6 C. 27 D. 54 E. NOTA

2. The volume of a cube is numerically equal to its surface area. Find the side length of the cube.
   A. 3 B. $3\sqrt[3]{3}$ C. $3\sqrt{3}$ D. 6 E. NOTA

3. A cone has radius $r$ and height $h$. If $r + h = 18$, find the maximum possible volume of the cone.
   A. $144\pi$ B. $216\pi$ C. $240\pi$ D. $288\pi$ E. NOTA

4. Let 2 vertices of a rectangle lie on the horizontal asymptote of $y = f(x)$ and the other 2 vertices lie on the graph of $y = f(x)$ where $f(x) = \frac{x^2-1}{x^2+1}$. Find the largest possible area of this rectangle.
   A. 1 B. 2 C. 3 D. 4 E. NOTA

5. A particular right square pyramid has edges that are all of equal length. If the surface area of the pyramid is numerically equal to the volume of the pyramid, the side length of the pyramid can be written in the form $\sqrt{m} + \sqrt{n}$ for positive integers $m$ and $n$. Find $m + n$.
   A. 72 B. 124 C. 234 D. 288 E. NOTA

6. Circle $w_1$ has radius 3 and is externally tangent to circle $w_2$, which has radius 4. $\overline{AB}$ is a common external tangent segment with $A$ on circle $w_1$ and $B$ on circle $w_2$. If the center of circle $w_1$ is $O_1$ and the center of circle $w_2$ is $O_2$, find the area of quadrilateral $ABO_2O_1$.
   A. $8\sqrt{3}$ B. $10\sqrt{3}$ C. $12\sqrt{3}$ D. $14\sqrt{3}$ E. NOTA

7. If the radius of the sphere inscribed in a cone of radius 7 and height 24 can be written in the form $\frac{m}{n}$ for relatively prime positive integers $m$ and $n$, find $m+n$.

A. 21 B. 25 C. 29 D. 33 E. NOTA

8. An ellipse's latus rectums intersect the ellipse at 4 points, with those 4 points forming the vertices of a square. Find the eccentricity of the ellipse.

A. $\frac{\sqrt{5}-1}{4}$ B. $\frac{1}{2}$ C. $\frac{\sqrt{5}-1}{2}$ D. $\frac{\sqrt{5}+1}{4}$ E. NOTA

For questions 9-11, let $f(x)=x^2+1$, let line $L_1$ be the line tangent to $y=f(x)$ at point $(a, f(a))$ for $a>0$, and let $R$ be the region bounded by the y-axis, $L_1$, and $y=f(x)$ .

9. Find the value of $a$ that minimizes the sum of the area of $R$ and the y-coordinate of the y-intercept of $L_1$.

A. 1 B. $\sqrt{2}$ C. 2 D. $2\sqrt{2}$ E. NOTA

10. Find the volume of the solid formed when $R$ is revolved about the $y$-axis if $a=2$.

A. $\frac{4\pi}{3}$ B. $2\pi$ C. $\frac{8\pi}{3}$ D. $3\pi$ E. NOTA

11. For some value of $a$, the x-axis cuts $R$ in half. Find the integer closest to $a^2$.

A. 5 B. 6 C. 7 D. 8 E. NOTA

12. A cylinder's height and radius are both integers. If the total surface area of the cylinder equals the volume of the cylinder, find the sum of all possible distinct values for the height of the cylinder.

A. 4 B. 9 C. 10 D. 13 E. NOTA

13. Isosceles trapezoid $ABCD$ has $AB||CD$ and $AB < CD$. Trapezoid $ABCD$ also has $AB = 2, \angle ADC = 60°$, and perimeter 28. Find the area of trapezoid $ABCD$.

A. $12\sqrt{3}$ B. $24\sqrt{3}$ C. $36\sqrt{3}$ D. $48\sqrt{3}$ E. NOTA

14. Consider the graph of $y = |x-1| + |x-2|$ on the interval $0 \le x \le 5$. Find the area bounded between this graph and the x-axis on the interval.

A. 13 B. 14 C. 15 D. 16 E. NOTA

15. A rectangle's area is numerically equal to its perimeter. If the side lengths of the rectangle are $\log_2 x$ and $\log_3 x$, find $x$.

A. $\sqrt{6}$ B. 6 C. $6\sqrt{6}$ D. 36 E. NOTA

16. Triangle $KHE$ has side lengths $KH = 9, KE = 10$, and $HE = 11$. Find the area of the incircle of triangle $KHE$.

A. $6\pi$ B. $8\pi$ C. $9\pi$ D. $10\pi$ E. NOTA

17. Let $f(x) = p\cos^2(x) + q\cos(x) + r$ where $p, q, r > 0$. If the graph of $y = f(x)$ on the interval $[0, \pi]$ is tangent to the x-axis at $x = \frac{2\pi}{3}$ and touches the y-axis at the point $(0, 6)$, find $-f'\left(\frac{\pi}{4}\right)$.

A. $\frac{4+4\sqrt{2}}{3}$ B. $\frac{4+8\sqrt{2}}{3}$ C. $\frac{8+4\sqrt{2}}{3}$ D. $\frac{8+8\sqrt{2}}{3}$ E. NOTA

18. Find the area that the curve $r^2 = 2024\cos(\theta)$ encloses.

A. 506 B. 1012 C. 2024 D. 4048 E. NOTA

19. Find the side length of the regular tetrahedron whose surface area is numerically equal to its volume.

A. $3\sqrt{6}$ B. $6\sqrt{6}$ C. $9\sqrt{6}$ D. $12\sqrt{6}$ E. NOTA

20. Find the volume of the solid formed when the circle defined by the equation $x^2 - 12x + y^2 - 18y + 108 = 0$ is rotated about the line $y = -5x$.

A. $26\pi^2\sqrt{26}$ B. $27\pi^2\sqrt{26}$ C. $39\pi^2\sqrt{26}$ D. $54\pi^2\sqrt{26}$ E. NOTA

21. One diagonal of a rhombus has length 18. Given that the rhombus's other diagonal and side length all have integer lengths, find the sum of all possible areas of the rhombus.

A. 216 B. 936 C. 10530 D. 11340 E. NOTA

22. Circle X is inscribed in a quarter circle of radius 1. The ratio of the area of X to the area of the quarter circle can be written in the form $m - \sqrt{n}$ for positive integers $m$ and $n$. Compute $m + n$.

A. 38 B. 44 C. 140 D. 152 E. NOTA

23. Find the area of the region bounded by the graphs of $y = 2x^2 + 3x + 5$ and $y = 7x + 11$.

A. $\frac{64}{3}$ B. $\frac{70}{3}$ C. $\frac{71}{3}$ D. $\frac{73}{3}$ E. NOTA

24. An ellipse has minor axis of length 6 and major axis of length 8. The latus rectums of the ellipse intersect the ellipse at 4 points. Those 4 points form the vertices of a rectangle. Find the area of this rectangle.

A. $\frac{9\sqrt{7}}{2}$ B. $9\sqrt{7}$ C. 45 D. $18\sqrt{7}$ E. NOTA

25. A cone of maximal volume is inscribed in a sphere. Find the ratio of the height of the cone to the radius of the cone.

A. $\sqrt{2}$ B. $\sqrt{3}$ C. 2 D. 3 E. NOTA

26. A parabola has equation $4Py = x^2$ for some $P > 0$. The latus rectum of the parabola intersects the parabola at point A in the 1st quadrant. Line $L_1$ passes through the origin and point A. If the area of the region bounded by $L_1$ and the parabola is 24, find $P$.

A. $3\sqrt{2}$ B. $6$ C. $6\sqrt{2}$ D. $12$ E. NOTA

27. Given $f(x) = a^2 - x^2$ for some positive real number $a$. Let $k$ be the average value of $f(x)$ on the x-interval $[0, a]$. Let point $P$ be the intersection of $y = f(x)$ and $y = k$ in the 1st quadrant. Find the value of $a^2$ if point $P$ lie on the line $y = 2x$.

A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. $2$ D. $3$ E. NOTA

28. Triangle $KHE$ has side lengths $KH = 7$, $KE = 9$, and $HE = 8$. The area of the circumcircle of triangle $KHE$ can be written in the form $\frac{m}{n}\pi$ for relatively prime positive integers $m$ and $n$. Find $m + n$.

A. $431$ B. $446$ C. $461$ D. $476$ E. NOTA

29. Let $L_1$ be the line tangent to $y = x^3$ at $x = 2$. Find the area of the region in the fourth quadrant bounded below by $L_1$.

A. $\frac{16}{3}$ B. $6$ C. $\frac{32}{3}$ D. $12$ E. NOTA

30. For some $a > 1$, let $R$ be the region bounded by $x = 1, x = -1$, the x-axis, and the graph of $y = a^x$. If the y-axis splits $R$ into two regions such that the ratio of the areas of the two regions is $2:1$, find $a$.

A. $\ln(3)$ B. $\ln(4)$ C. $\ln(5)$ D. $\ln(6)$ E. NOTA