The acronym NOTA denotes that "none of the above answers is correct." All answers are rationalized, simplified, and exact unless otherwise stated. Good luck and have fun!

1. Consider the function $f(x)$ to be integrable and differentiable on the interval $[a, b]$. Which of the following properties must be true?

I. $\left|\int_a^b f(x)dx\right| \geq \int_a^b |f(x)|dx$ II. $\int_a^b |f(x)|dx \geq 0$ III. $\int_a^b |f(x)|dx \geq \int_a^b f(x)dx$

A. I, II, and III B. I and III C. II and III D. II only E. NOTA

2. Find the average value of the function $f(x) = x \ln x$ on the interval $[1,4]$.

A. $\frac{8\ln(4)}{3} - \frac{5}{4}$ B. $\frac{4\ln(4)-4}{3}$ C. $\frac{4\ln(4)}{3} - \frac{5}{4}$ D. $\frac{8\ln(4)-4}{3}$ E. NOTA

3. Evaluate $\lim_{x\to 0}\left(\frac{4\sin^2(x)}{3\sin(x)\cos^2(x)-2\sin^2(x)}\right)$.

A. DNE B. 0 C. -2 D. 4 E. NOTA

4. Find the volume of the solid of revolution formed by rotating the region bounded by $f(x) = x^2 - 1$, $h(x) = 3$, $x = -1$, and $x = 1$ about the line $y = -2$.

A. $\frac{694}{15}\pi$ B. 50π C. $\frac{734}{15}\pi$ D. $\frac{22}{3}\pi$ E. NOTA

5. Compute $\int_2^3 \left(\frac{6x}{x^2+1}\right) dx$.

A. $2\ln 4$ B. $3\ln 3$ C. $\ln 6$ D. $\ln 8$ E. NOTA

6. A particle is moving according to the parameters $x = \csc^2(t)$ and $y = \tan^2(t)$. Find the speed of the particle at $t = \frac{3\pi}{4}$.

A. $4\sqrt{2}$ B. 4 C. 8 D. $8\sqrt{2}$ E. NOTA

7. Which of the following series are divergent?

I. $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{4n+2}}\right)$ II. $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n!}}\right)$ III. $\sum_{n=2}^{\infty}\left(\frac{n}{\ln(n)}\right)$

A. I, II, and III B. I and III C. III only D. II only E. NOTA

8. Let $M(x)$ represent the first three nonzero terms in the Taylor series of $f(x)=\frac{1}{1-x}$ centered about $x=\frac{1}{2}$ and define the error as $\varepsilon(x)=f(x)-M(x)$. What is the value of $\varepsilon\left(\frac{3}{4}\right)$?

A. $\frac{1}{16}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. $\frac{7}{16}$ E. NOTA

9. Find the slope of the normal line to the polar curve $r=-4\sin(\theta)$ at $\theta=\frac{5\pi}{3}$.

A. $-\frac{\sqrt{3}}{3}$ B. $\sqrt{3}$ C. $\frac{\sqrt{3}}{3}$ D. $-\sqrt{3}$ E. NOTA

10. Use Euler's method to obtain an estimate of $y(3.4)$, given the following: $y(3)=0$, the step size is 0.2 and $\frac{dy}{dt}=t-y+1$.

A. 1.28 B. 1.48 C. 1.64 D. 3.88 E. NOTA

11. Which of the following is equivalent to the integral shown below?

$$\int_0^{\frac{\pi}{4}}\sin(4x)\sin(x)\,dx$$

A. $\frac{4\sqrt{2}}{15}$ B. $\frac{\sqrt{2}}{5}$ C. $-\frac{28\sqrt{2}}{15}$ D. $\frac{2\sqrt{2}}{15}$ E. NOTA

12. Evaluate $\int_3^7\frac{10dx}{(x-2)(x+3)}$.

A. $\ln 9$ B. $\ln 3$ C. $\ln 6$ D. $\ln 12$ E. NOTA

13. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan(2x)\,dx$.

A. $\ln(\sqrt{2})$ B. 0 C. $\frac{1}{4}\ln(3)$ D. $\frac{1}{4}\ln\left(\frac{1}{3}\right)$ E. NOTA

14. Evaluate $\lim_{x\to 0} \frac{(1-e^x)\cos x}{e^x \sin x}$.

A. 1 B. -1 C. 0 D. 2 E. NOTA

15. Which of the following represents the Taylor series for the function $f(x) = \ln(x)$ centered at $x = 1$?

A. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-1)^n}{n}$ B. $\sum_{n=1}^{\infty} \frac{(-1)^{n}(x-1)^n}{n}$
C. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n}$ D. $\sum_{n=1}^{\infty} \frac{(-1)^{n}(x+1)^n}{n}$ E. NOTA

16. Compute the area of the surface formed by revolving $y = \frac{1}{2}x$ about the line $y = 0$ on the interval $[2,14]$.

A. $\frac{992\sqrt{2}}{3}\pi$ B. $48\sqrt{5}\pi$ C. $24\sqrt{5}\pi$ D. $\frac{124}{3}\pi$ E. NOTA

17. Evaluate $\lim_{x\to 4} \left(\frac{x^3-2x^2-2x-24}{x^2-16}\right)$.

A. $\frac{15}{4}$ B. $\frac{19}{4}$ C. 0 D. 4 E. NOTA

18. Find the arc length of the function $f(x) = 2x^{\frac{3}{2}}$ on the interval $[7,11]$.

A. $\frac{244}{3}$ B. $\frac{488}{9}$ C. $\frac{976}{27}$ D. 2 E. NOTA

19. Find the area bounded by one petal of the polar curve $r = \sin(8\theta)$.

A. $\frac{\pi}{4}$ B. $\frac{\pi}{8}$ C. $\frac{\pi}{16}$ D. $\frac{\pi}{32}$ E. NOTA

20. Let *R* be the region bound by the graphs $y = x^2 - 3x + 2$ and $4x + y = 8$ on the Cartesian coordinate plane. The centroid of region *R* is $(\bar{x}, \bar{y})$. Find the value of $\bar{y}$.

A. $\frac{125}{12}$ B. $\frac{297}{38}$ C. $\frac{15}{2}$ D. $\frac{95}{12}$ E. NOTA

21. Given $\frac{dy}{dt} + y \sin t = 0$ and $y\left(\frac{\pi}{3}\right) = 4$, which of the following is equivalent to the value of $y(\pi)$?

A. $4e^{\frac{1}{2}}$ B. $4e^{-\frac{3}{2}}$ C. $4e^{-\frac{1}{2}}$ D. $2\sqrt{e}$ E. NOTA

22. Given $h(x) = \frac{d}{dx}\left(\int_1^{x^2} e^{4t} dt\right)$, which of the following is equivalent to $h(2)$?

A. $e^{16} - e^4$ B. e^{16} C. $2e^8$ D. $4e^{16}$ E. NOTA

23. Given $x = t^3$ and $y = 3t^5 - t^7$, compute the value of $\frac{d^2y}{dx^2}$ at $x = 8$.

A. $\frac{4}{9}$ B. $-\frac{41}{9}$ C. $\frac{116}{9}$ D. $-\frac{52}{3}$ E. NOTA

24. Which of the following series are conditionally convergent?

I. $\sum_{n=1}^{\infty}\left(\frac{(-3)^n}{n!}\right)$ II. $\sum_{n=1}^{\infty}\left(\frac{(-2)^{n+1}}{4^n}\right)$ III. $\sum_{n=2}^{\infty}\left(\frac{(-1)^n}{\sqrt{n^2-1}}\right)$

A. I and III B. II and III C. III only D. I and II E. NOTA

25. If $\int_4^{16} \ln(\sqrt{x})\,dx = A\ln B - C$ for positive integers A, B, C with B prime, find $A + B + C$.

A. 36 B. 42 C. 62 D. 68 E. NOTA

26. Find the arc length of the polar curve $r = \csc(\theta)$ on the interval $\frac{\pi}{6} \le \theta \le \frac{\pi}{3}$.

A. $\frac{2\sqrt{3}}{3}$ B. $\frac{\sqrt{3}}{3}$ C. $\frac{4\sqrt{3}}{9}$ D. $\frac{4\sqrt{3}}{3}$ E. NOTA

27. Let $y = f(x)$ be the solution to $xy\frac{dy}{dx} + 4x^2 + y^2 = 0$ that passes through the point $(2,8)$. Find the largest value of x in the domain of $f(x)$.

A. $\sqrt[4]{72}$ B. $\sqrt[4]{96}$ C. $\sqrt{12}$ D. $\sqrt{18}$ E. NOTA

28. Which of the following is equivalent to $\int_{-1}^{1}\left(\frac{1-2x^2+x^4}{x^2+1}\right)dx$?

A. $\frac{6\pi-8}{3}$ B. $\frac{12\pi-16}{3}$ C. $\frac{3\pi-8}{3}$ D. $\frac{6\pi-16}{3}$ E. NOTA

29. Evaluate $\int_0^{2\pi} \frac{dx}{7+6\cos x+3\sin x}$. *Hint*: $\int_0^{2\pi} \frac{dx}{a+k\sin x} = \int_0^{\pi} \frac{dx}{a+k\sin x} + \int_0^{\pi} \frac{dx}{a-k\sin x}$.

A. $\frac{\pi}{2}$ B. $\frac{6\pi}{7}$ C. $\frac{13\pi}{14}$ D. π E. NOTA

30. For how many of the following functions is true that $\frac{d}{dx}f_n(x) = \int f_n(x)dx$? (Note: For the purpose of this question, ignore the constant of integration.)

I. $f_1(x) = e^x$ II. $f_2(x) = e^{-x}$

III. $f_3(x) = \sin x$ IV. $f_4(x) = \cos x$

V. $f_5(x) = \sinh x$ VI. $f_6(x) = \cosh x$

A. 2 B. 4 C. 5 D. 6 E. NOTA