

1.	C	I. False II. True III. True
2.	A	$f_{avg} = \frac{1}{4-1} \int_1^4 (x \ln(x)) dx$ <p>Using integration by parts: $u = \ln(x)$, $du = \frac{dx}{x}$, $dv = x dx$, $v = \frac{1}{2} x^2$</p> $\int (x \ln(x)) dx = \frac{1}{2} x^2 \ln(x) - \int \frac{1}{2} x^2 \left(\frac{dx}{x}\right)$ $\int (x \ln(x)) dx = \frac{1}{2} x^2 \ln(x) - \frac{1}{4} x^2 + C$ $\int_1^4 (x \ln(x)) dx = \left[\frac{1}{2} (4^2) \ln(4) - \frac{1}{4} (4^2) \right] - \left[\frac{1}{2} \ln(1) - \frac{1}{4} \right]$ $\int_1^4 (x \ln(x)) dx = 8 \ln(4) - 4 + \frac{1}{4} = 8 \ln(4) - \frac{15}{4}$ $f_{avg} = \frac{1}{4-1} \int_1^4 (x \ln(x)) dx = \frac{8}{3} \ln(4) - \frac{5}{4}$
3.	B	$\lim_{x \rightarrow 0} \left(\frac{4 \sin^2(x)}{3 \sin(x) \cos^2(x) - 2 \sin^2(x)} \right) = \lim_{x \rightarrow 0} \left(\frac{4 \sin(x)}{3 \cos^2(x) - 2 \sin(x)} \right)$ $\lim_{x \rightarrow 0} \left(\frac{4 \sin^2(x)}{3 \sin(x) \cos^2(x) - 2 \sin^2(x)} \right) = \frac{4 \sin(0)}{3 \cos^2(0) - 2 \sin(0)}$ $\lim_{x \rightarrow 0} \left(\frac{4 \sin^2(x)}{3 \sin(x) \cos^2(x) - 2 \sin^2(x)} \right) = \frac{0}{3-0} = 0$
4.	A	$V = \pi \int_{-1}^1 \left[(3 - (-2))^2 - (x^2 - 1 - (-2))^2 \right] dx$ $V = \pi \int_{-1}^1 [25 - (x^2 + 1)^2] dx$ $V = \pi \int_{-1}^1 [25 - x^4 - 2x^2 - 1] dx$ $V = \pi \left[25x - \frac{1}{5} x^5 - \frac{2}{3} x^3 - x \right]_{-1}^1$ $V = \pi \left[\left(25 - \frac{1}{5} - \frac{2}{3} - 1 \right) - \left(-25 + \frac{1}{5} + \frac{2}{3} + 1 \right) \right] = \pi \left(50 - \frac{2}{5} - \frac{4}{3} - 2 \right)$ $V = \frac{694}{15} \pi$
5.	D	$\int_2^3 \left(\frac{6x}{x^2+1} \right) dx = 3 \ln x^2+1 \Big _2^3$ $\int_2^3 \left(\frac{6x}{x^2+1} \right) dx = 3 \ln(9+1) - 3 \ln(4+1)$ $\int_2^3 \left(\frac{6x}{x^2+1} \right) dx = 3[\ln 10 - \ln 5] = \ln 8$
6.	A	$s = \langle \csc^2(t), \tan^2(t) \rangle$ $\frac{ds}{dt} = \langle 2 \csc(t)(-\csc(t) \cot(t)), 2 \tan(t) (\sec^2(t)) \rangle$ $\frac{ds}{dt} \Big _{\frac{3\pi}{4}} = \left\langle 2 \csc\left(\frac{3\pi}{4}\right) \left(-\csc\left(\frac{3\pi}{4}\right) \cot\left(\frac{3\pi}{4}\right)\right), 2 \tan\left(\frac{3\pi}{4}\right) \left(\sec^2\left(\frac{3\pi}{4}\right)\right) \right\rangle$ $\frac{ds}{dt} \Big _{\frac{3\pi}{4}} = \langle 2(\sqrt{2})(-\sqrt{2})(-1), 2(-1)(2) \rangle$ $\frac{ds}{dt} \Big _{\frac{3\pi}{4}} = \langle 4, -4 \rangle$ $speed = \left \frac{ds}{dt} \Big _{\frac{3\pi}{4}} \right = 4\sqrt{2}$

7.	B	<p>I. Divergent by comparison test with $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}}\right)$</p> <p>II. Converges by the ratio test</p> <p>III. Divergent by comparison test with $\sum_{n=2}^{\infty} \left(\frac{n}{\ln(n)}\right)$</p>
8.	C	$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ $M(x) = 2 + 4\left(x - \frac{1}{2}\right) + 8\left(x - \frac{1}{2}\right)^2$ $M\left(\frac{3}{4}\right) = 2 + 4\left(\frac{1}{4}\right) + 8\left(\frac{1}{4}\right)^2 = \frac{7}{2}$ $\varepsilon\left(\frac{3}{4}\right) = \frac{1}{1-(3/4)} - \frac{7}{2} = \frac{1}{2}$
9.	A	$r = -4 \sin(\theta) \qquad x = r \cos(\theta) \qquad y = r \sin(\theta)$ $r' = -4 \cos(\theta) \qquad x' = r' \cos(\theta) - r \sin(\theta) \qquad y' = r' \sin(\theta) + r \cos(\theta)$ $r\left(\frac{5\pi}{3}\right) = -4 \sin\left(\frac{5\pi}{3}\right) = 2\sqrt{3}$ $r'\left(\frac{5\pi}{3}\right) = -4 \cos\left(\frac{5\pi}{3}\right) = -2$ $x'\left(\frac{5\pi}{3}\right) = (-2) \cos\left(\frac{5\pi}{3}\right) - (2\sqrt{3}) \sin\left(\frac{5\pi}{3}\right) = 2$ $y'\left(\frac{5\pi}{3}\right) = (-2) \sin\left(\frac{5\pi}{3}\right) + (2\sqrt{3}) \cos\left(\frac{5\pi}{3}\right) = 2\sqrt{3}$ $\frac{dy}{dx} = \sqrt{3}$ $\text{slope of normal line} = -\left(\frac{1}{\sqrt{3}}\right) = -\frac{\sqrt{3}}{3}$
10.	B	$y = y_0 + f(t_0, y_0)(t - t_0)$ $y(3.2) = 0 + (3 - 0 + 1)(0.2) = 0.8$ $y(3.4) = 0.8 + (3.2 - 0.8 + 1)(0.2) = 1.48$
11.	D	$\int_0^{\frac{\pi}{4}} \sin(4x) \sin(x) dx = \int_0^{\frac{\pi}{4}} 2 \sin(2x) \cos(2x) \sin(x) dx$ $\int_0^{\frac{\pi}{4}} \sin(4x) \sin(x) dx = \int_0^{\frac{\pi}{4}} 4 \sin(x) \cos(x) \cos(2x) \sin(x) dx$ $\int_0^{\frac{\pi}{4}} \sin(4x) \sin(x) dx = \int_0^{\frac{\pi}{4}} 4 \sin^2(x) \cos(x) \cos(2x) dx$ $\int_0^{\frac{\pi}{4}} \sin(4x) \sin(x) dx = \int_0^{\frac{\pi}{4}} 4 \sin^2(x) \cos(x) [1 - 2 \sin^2(x)] dx$ $\int_0^{\frac{\pi}{4}} \sin(4x) \sin(x) dx = \int_0^{\frac{\pi}{4}} 4 \sin^2(x) \cos(x) dx - \int_0^{\frac{\pi}{4}} 8 \sin^4(x) \cos(x) dx$ $\int_0^{\frac{\pi}{4}} \sin(4x) \sin(x) dx = \frac{4}{3} \sin^3(x) \Big _0^{\frac{\pi}{4}} - \frac{8}{5} \sin^5(x) \Big _0^{\frac{\pi}{4}}$ $\int_0^{\frac{\pi}{4}} \sin(4x) \sin(x) dx = \frac{4}{3} \left(\frac{\sqrt{2}}{2}\right)^3 - \frac{8}{5} \left(\frac{\sqrt{2}}{2}\right)^5$ $\int_0^{\frac{\pi}{4}} \sin(4x) \sin(x) dx = \frac{4}{3} \left(\frac{2\sqrt{2}}{8}\right) - \frac{8}{5} \left(\frac{4\sqrt{2}}{32}\right)$ $\int_0^{\frac{\pi}{4}} \sin(4x) \sin(x) dx = \frac{\sqrt{2}}{3} - \frac{\sqrt{2}}{5} = \frac{2\sqrt{2}}{15}$

12.	A	$\frac{1}{(x-2)(x+3)} = \frac{2}{x-2} - \frac{2}{x+3}$ $\int_3^7 \frac{10dx}{(x-2)(x+3)} = \int_3^{10} \left(\frac{2}{x-2} - \frac{2}{x+3} \right) dx$ $\int \frac{10dx}{(x-2)(x+3)} = 2 \ln x-2 - 2 \ln x+3 + c$ $\int \frac{10dx}{(x-2)(x+3)} = 2 \ln \left \frac{x-2}{x+3} \right + c$ $\int_3^7 \frac{10dx}{(x-2)(x+3)} = 2 \ln \left(\frac{7-2}{7+3} \right) - 2 \ln \left(\frac{3-2}{3+3} \right)$ $\int_3^7 \frac{10dx}{(x-2)(x+3)} = 2 \ln \left(\frac{1}{2} \right) - 2 \ln \left(\frac{1}{6} \right) = \ln 9$
13.	E	$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\tan(2x)) dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\frac{\sin(2x)}{\cos(2x)} \right) dx$ $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\tan(2x)) dx = -\frac{1}{2} \ln \cos(2x) \Big _{\frac{\pi}{6}}^{\frac{\pi}{3}}$ <p>$\ln \cos(2x)$ is not continuous on the interval $\left[\frac{\pi}{6}, \frac{\pi}{3} \right]$ because $\ln \left \cos \left(2 \left(\frac{\pi}{4} \right) \right) \right$ does not exist</p>
14.	B	$\lim_{x \rightarrow 0} \left[\frac{(1-e^0) \cos(0)}{e^1 (\sin(0))} \right]$ <p>$\lim_{x \rightarrow 0} \left[\frac{0}{0} \right]$ is undefined, use L'Hopital's rule</p> $\lim_{x \rightarrow 0} \left[\frac{(1-e^x) \cos(x)}{e^x (\sin(x))} \right] = \lim_{x \rightarrow 0} \left[\frac{\frac{d}{dx} [(1-e^x) \cos(x)]}{\frac{d}{dx} [e^x (\sin(x))]} \right]$ $\lim_{x \rightarrow 0} \left[\frac{(1-e^x) \cos(x)}{e^x (\sin(x))} \right] = \lim_{x \rightarrow 0} \left[\frac{-\sin(x) - e^x \cos(x) + e^x \sin(x)}{e^x \sin(x) + e^x \cos(x)} \right]$ $\lim_{x \rightarrow 0} \left[\frac{(1-e^x) \cos(x)}{e^x (\sin(x))} \right] = \lim_{x \rightarrow 0} \left[\frac{-\sin(0) - e^0 \cos(0) + e^0 \sin(0)}{e^0 \sin(0) + e^0 \cos(0)} \right]$ $\lim_{x \rightarrow 0} \left[\frac{(1-e^x) \cos(x)}{e^x (\sin(x))} \right] = \lim_{x \rightarrow 0} \left[\frac{0 - (1)(1) + 0}{0 + (1)(1)} \right] = -1$
15.	A	$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x)^n}{n}$ $\ln(x) = \ln(1+(x-1)) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-1)^n}{n}$
16.	B	<p>The shape of the graph is a conical Frustum</p> $SA = 2\pi \int y \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx$ $SA = 2\pi \int_2^{14} \left(\frac{1}{2} x \right) \sqrt{1 + \frac{1}{4}} dx = \frac{\pi\sqrt{5}}{2} \int_2^{14} x dx$ $SA = \frac{\pi\sqrt{5}}{2} \left(\frac{1}{2} x^2 \right) \Big _2^{14}$ $SA = \frac{\pi\sqrt{5}}{2} (98) - \frac{\pi\sqrt{5}}{2} (2) = 48\sqrt{5}\pi$

17.	A	$\lim_{x \rightarrow 4} \left(\frac{x^3 - 2x^2 - 2x - 24}{x^2 - 16} \right) = \frac{0}{0} \text{ is undefined, use L'Hopital's rule}$ $\lim_{x \rightarrow 4} \left(\frac{x^3 - 2x^2 - 2x - 24}{x^2 - 16} \right) = \lim_{x \rightarrow 4} \left(\frac{\frac{d}{dx}(x^3 - 2x^2 - 2x - 24)}{\frac{d}{dx}(x^2 - 16)} \right)$ $\lim_{x \rightarrow 4} \left(\frac{x^3 - 2x^2 - 2x - 24}{x^2 - 16} \right) = \lim_{x \rightarrow 4} \left(\frac{3x^2 - 4x - 2}{2x} \right)$ $\lim_{x \rightarrow 4} \left(\frac{x^3 - 2x^2 - 2x - 24}{x^2 - 16} \right) = \frac{3(4)^2 - 4(4) - 2}{2(4)}$ $\lim_{x \rightarrow 4} \left(\frac{x^3 - 2x^2 - 2x - 24}{x^2 - 16} \right) = \frac{48 - 16 - 2}{8} = \frac{15}{4}$
18.	C	$L = \int_a^b \left(\sqrt{1 + \left(\frac{dy}{dx} \right)^2} \right) dx$ $\frac{dy}{dx} = 3\sqrt{x}$ $L = \int_7^{11} (\sqrt{1 + 9x}) dx$ $L = \frac{2(1+9x)^{\frac{3}{2}}}{\frac{3}{2}} \Big _7^{11} = \frac{2(1+9(11))^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2(1+9(7))^{\frac{3}{2}}}{\frac{3}{2}}$ $L = \frac{2(1+9x)^{\frac{3}{2}}}{\frac{3}{2}} \Big _7^{11} = \frac{2(100)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2(64)^{\frac{3}{2}}}{\frac{3}{2}}$ $L = \frac{2(1+9x)^{\frac{3}{2}}}{\frac{3}{2}} \Big _7^{11} = \frac{2(1000) - 2(512)}{\frac{3}{2}} = \frac{976}{\frac{3}{2}}$
19.	D	$\sin(8\theta) \text{ has a period of } \frac{2\pi}{8} \text{ or } \frac{\pi}{4}$ $A = \frac{1}{2} \int_a^b r^2 d\theta$ $A = \frac{1}{2} \int_0^{\frac{\pi}{8}} (\sin(8\theta))^2 d\theta$ $\text{Recall } \cos(2n\theta) = 1 - 2\sin^2(n\theta)$ $A = \frac{1}{2} \int_0^{\frac{\pi}{8}} -\frac{1}{2} [\cos(16\theta) - 1] d\theta$ $A = -\frac{1}{4} \int_0^{\frac{\pi}{8}} [\cos(16\theta) - 1] d\theta$ $A = -\frac{1}{4} \times \left(\frac{1}{16} \sin(16\theta) - \theta \Big _0^{\frac{\pi}{8}} \right)$ $A = -\frac{1}{4} \times \left[\frac{1}{16} (0 - 0) - \left(\frac{\pi}{8} - (0) \right) \right] = \frac{\pi}{32}$

20.	C	$A = \int_a^b (f(x) - g(x)) dx \text{ where } f(x) = 8 - 4x \text{ and } g(x) = x^2 - 3x + 2$ <p>From Algebra: $f(x)$ and $g(x)$ intersect at $x = -3$ and $x = 2$</p> $A = \int_a^b (f(x) - g(x)) dx$ $A = \int_{-3}^2 (6 - x - x^2) dx = \left(6x - \frac{1}{2}x^2 - \frac{1}{3}x^3 \right) \Big _{-3}^2$ $A = \left(12 - 2 - \frac{8}{3} \right) - \left(-18 - \frac{9}{2} + 9 \right) = \frac{125}{6}$ $\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} (f^2(x) - g^2(x)) dx$ $\bar{y} = \frac{1}{2A} \int_{-3}^2 [(64 - 64x + 16x^2) - (x^4 - 6x^3 + 13x^2 - 12x + 4)] dx$ $\bar{y} = \frac{1}{2A} \int_{-3}^2 (60 - 52x + 3x^2 + 6x^3 - x^4) dx$ $\bar{y} = \frac{1}{2A} \left[\left(60x - 26x^2 + x^3 + \frac{3}{2}x^4 - \frac{1}{5}x^5 \right) \Big _{-3}^2 \right]$ $\bar{y} = \frac{1}{2A} \left[\left(120 - 104 + 8 + 24 - \frac{32}{5} \right) - \left(-180 - 234 - 27 + \frac{243}{2} + \frac{243}{5} \right) \right]$ $\bar{y} = \frac{1}{2A} \left[\left(48 - \frac{32}{5} \right) - \left(-441 + \frac{243}{2} + \frac{243}{5} \right) \right]$ $\bar{y} = \frac{1}{2A} \left(489 - \frac{32-243}{5} - \frac{243}{2} \right) = \frac{1}{2A} \left(489 - 55 - \frac{243}{2} \right)$ $\bar{y} = \frac{1}{2A} \left(434 - \frac{243}{2} \right) = \frac{1}{2A} \left(\frac{625}{2} \right)$ $\bar{y} = \frac{6}{2 \times 125} \left(\frac{625}{2} \right) = \frac{15}{2}$
21.	B	$\frac{dy_h}{dt} + [\sin(t)]y_h = 0$ $\int \frac{dy_h}{y_h} \left(\frac{1}{y_h} \right) = \int -\sin(t)$ $\ln y_h = \cos(t) + C_1$ $y_h = C_2 e^{\cos(t)}$ $4 = C_2 e^{\cos(\frac{\pi}{3})} \rightarrow C_2 = 4e^{-\frac{1}{2}}$ $y_h = \left(4e^{-\frac{1}{2}} \right) e^{\cos(t)}$ $y(\pi) = \left(4e^{-\frac{1}{2}} \right) e^{\cos(\pi)} = 4e^{-\frac{3}{2}}$
22.	D	$\frac{d}{dx} \left(\int_a^u [f(t)] dt \right) = \frac{d}{du} \left(\int_a^u [f(t)] dt \right) \times \frac{du}{dx} \text{ where } a \text{ is a constant}$ $\frac{d}{dx} \left(\int_a^u [f(t)] dt \right) = \frac{d}{du} (F(u) - F(a)) \times \frac{du}{dx}$ $\frac{d}{dx} \left(\int_a^u [f(t)] dt \right) = f(u) \times \frac{du}{dx}$ $h(x) = \frac{d}{dx} \left(\int_1^{x^2} [e^{4t}] dt \right) = 2xe^{4x^2}$ $h(2) = 2(2)e^{4(2)^2} = 4e^{16}$

23.	B	$x = t^3 \quad y = 3t^5 - t^7$ $t = \sqrt[3]{x} \rightarrow y = 3x^{\frac{5}{3}} - x^{\frac{7}{3}}$ $\frac{dy}{dx} = 5x^{\frac{2}{3}} - \frac{7}{3}x^{\frac{4}{3}}$ $\frac{d^2y}{dx^2} = \frac{10}{3}x^{-\frac{1}{3}} - \frac{28}{9}x^{\frac{1}{3}}$ $\left. \frac{d^2y}{dx^2} \right _{x=9} = \frac{10}{3}(8)^{-\frac{1}{3}} - \frac{28}{9}(8)^{\frac{1}{3}}$ $\left. \frac{d^2y}{dx^2} \right _{x=9} = \frac{10}{6} - \frac{56}{9} = -\frac{41}{9}$
24.	C	<p>Conditionally convergent if $\sum(a_n)$ converges but $\sum a_n$ diverges</p> <p>I. Converges Absolutely (comparison test $e^x = \sum_{n=1}^{\infty} \left(\frac{x^n}{n!}\right)$)</p> <p>II. Converges Absolutely (geometric series with $r < 1$)</p> <p>III. Conditionally convergent (comparison test $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n^2}}\right)$ diverges)</p>
25.	A	<p>Let $u = \ln(\sqrt{x})$ and $dv = dx$</p> $du = \frac{1}{\sqrt{x}} \left(\frac{1}{2}x^{-\frac{1}{2}}\right) dx \quad v = x$ $du = \frac{1}{2x} dx$ $\int \ln(\sqrt{x}) dx = x \ln(\sqrt{x}) - \int x \left(\frac{1}{2x}\right) dx$ $\int \ln(\sqrt{x}) dx = x \ln(\sqrt{x}) - \int \frac{1}{2} dx$ $\int \ln(\sqrt{x}) dx = x \ln(\sqrt{x}) - \frac{1}{2}x + C$ $\int_4^{16} \ln(\sqrt{x}) dx = \left[16 \ln(\sqrt{16}) - \frac{1}{2}(16)\right] - \left[4 \ln(\sqrt{4}) - \frac{1}{2}(4)\right]$ $\int_4^{16} \ln(\sqrt{x}) dx = [16 \ln(4) - 8] - [4 \ln(2) - 2]$ $\int_4^{16} \ln(\sqrt{x}) dx = [32 \ln(2) - 8] - [4 \ln(2) - 2] = 28 \ln(2) - 6$ $A + B + C = 28 + 2 + 6 = 36$
26.	A	$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$ $L = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{(\csc(\theta))^2 + (-\csc(\theta) \cot(\theta))^2} d\theta$ $L = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{\csc^2(\theta) + \csc^2(\theta) \cot^2(\theta)} d\theta$ $L = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} [\csc(\theta)] \sqrt{1 + \cot^2(\theta)} d\theta$ $L = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} [\csc^2(\theta)] d\theta$ $L = -\cot(\theta) \Big _{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $L = \left(-\frac{\sqrt{3}}{3}\right) - (-\sqrt{3}) = \frac{2\sqrt{3}}{3}$

27.	C	$xy \frac{dy}{dx} + 4x^2 + y^2 = 0$ $\left(\frac{y}{x}\right) \frac{dy}{dx} = -4 - \left(\frac{y}{x}\right)^2$ <p>Let $u = \frac{y}{x}$. Then $\frac{dy}{dx} = u + x \frac{du}{dx}$.</p> $u \left(u + x \frac{du}{dx}\right) = -4 - u^2$ $\frac{u}{4+2u^2} du = -\frac{1}{x} dx$ $\frac{1}{4} \ln(4 + 2u^2) = -\ln x + C$ $\sqrt[4]{4 + 2u^2} = \frac{C}{x}$ $u^2 = \frac{1}{2} \left(\frac{K}{x^4} - 4\right)$ $y^2 = \frac{K - 4x^4}{2x^2}$ <p>With the initial condition, $K = 576$ and $y = \sqrt{\frac{288 - 2x^4}{x^2}}$. The largest value of x in the domain of this solution is the one that causes the numerator to equal 0. Solving $288 - 2x^4 = 0$ gives $x = \sqrt{12}$.</p>
28.	D	<p>First, simplify using long division</p> $\int_{-1}^1 \left(\frac{1 - 2x^2 + x^4}{x^2 + 1}\right) dx = \int_{-1}^1 \left(x^2 - 3 + \frac{4}{x^2 + 1}\right) dx$ $\int_{-1}^1 \left(\frac{1 - 2x^2 + x^4}{x^2 + 1}\right) dx = \left[\frac{1}{3}x^3 - 3x\right]_{-1}^1 + 4 \int_{-1}^1 \left(\frac{1}{x^2 + 1}\right) dx$ $\int_{-1}^1 \left(\frac{1 - 2x^2 + x^4}{x^2 + 1}\right) dx = \left[\left(\frac{1}{3} - 3\right) - \left(-\frac{1}{3} + 3\right)\right] + 4[\arctan(x)]_{-1}^1$ $\int_{-1}^1 \left(\frac{1 - 2x^2 + x^4}{x^2 + 1}\right) dx = \left[-\frac{16}{3}\right] + 4[\arctan(1) - \arctan(-1)]$ $\int_{-1}^1 \left(\frac{1 - 2x^2 + x^4}{x^2 + 1}\right) dx = \left[-\frac{16}{3}\right] + 4\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right]$ $\int_{-1}^1 \left(\frac{1 - 2x^2 + x^4}{x^2 + 1}\right) dx = 2\pi - \frac{16}{3} = \frac{6\pi - 16}{3}$

29.	D	<p>Consider $\int_0^{2\pi} \frac{dx}{a+b \cos x+c \sin x}$ for $a^2 > b^2 + c^2$. $b \cos x + c \sin x = \sqrt{b^2 + c^2} \sin(x + \theta)$ for some θ. Periodicity can eliminate the $+\theta$ with a substitution. Let $k = \sqrt{b^2 + c^2}$.</p> $\int_0^{2\pi} \frac{dx}{a+\sqrt{b^2+c^2} \sin x} = \int_0^{2\pi} \frac{dx}{a+k \sin x} = \int_0^{\pi} \frac{dx}{a+k \sin x} + \int_0^{\pi} \frac{dx}{a-k \sin x} = \int_0^{\pi} \frac{2a}{a^2-k^2 \sin^2 x} dx = \int_0^{\pi/2} \frac{4a}{a^2(\sin^2 x+\cos^2 x)-k^2 \sin^2 x} dx = \int_0^{\pi/2} \frac{4a \sec^2 x}{a^2+(a^2-k^2) \tan^2 x} dx.$ <p>Let $u = \tan x$. Then $\frac{du}{dx} = \sec^2 x$.</p> $\int_0^{\pi/2} \frac{4a \sec^2 x}{a^2+(a^2-k^2) \tan^2 x} dx = \int_0^{\infty} \frac{4a}{a^2+(a^2-k^2)u^2} du = \frac{1}{a^2-k^2} \int_0^{\infty} \frac{4a}{u^2+\left(\frac{a}{\sqrt{a^2-k^2}}\right)^2} du.$ <p>Evaluating, this equals $\frac{4a \arctan \frac{x}{\frac{a}{\sqrt{a^2-k^2}}}}{(a^2-k^2) \cdot \frac{a}{\sqrt{a^2-k^2}}}$ $\Big _0^{\infty} = \frac{2\pi}{\sqrt{a^2-k^2}}$. With $a = 7$ and $k^2 = 6^2 + 3^2 = 45$, this evaluates to π.</p>
30.	B	<p>$\frac{d}{dx} f(x) = \int f(x) dx$. $f(x) = e^x$ and $f(x) = e^{-x}$ are well-known solution to this equation. Differentiating gives $\frac{d^2}{dx^2} f(x) = f(x)$, which also provides the solutions $f(x) = \sinh x = \frac{e^x - e^{-x}}{2}$ and $f(x) = \cosh x = \frac{e^x + e^{-x}}{2}$ for a total of 4.</p>