1. B
2. C
3. C
4. B
5. D
6. B
7. A
8. A
9. B
10. B
11. B
12. D
13. C
14. C
15. D
16. A
17. D
18. D
19. C
20. B
21. C
22. D
23. A
24. C
25. A
26. A
27. D
28. D
29. B
30. D

1. B Assuming the limit exists, $\lim_{x\to 0}\frac{e^{x-1}(e^x-1)}{(x+1)\ln(x+1)} = \lim_{x\to 0}\frac{e^{x-1}}{x+1}\lim_{x\to 0}\frac{e^x-1}{\ln(x+1)} = \frac{1}{e}\lim_{x\to 0}\frac{e^x-1}{\ln(x+1)}$. By l'Hospital's, $\frac{1}{e}\lim_{x\to 0}\frac{e^x-1}{\ln(x+1)} = \frac{1}{e}\lim_{x\to 0}\frac{e^x}{1/(x+1)} = \frac{1}{e}$.

2. C $\lim_{x\to 0}(\sec x)^{\cot^2 x} = \lim_{x\to 0}(1+\tan^2 x)^{\cot^2 x/2} = \lim_{u\to\infty}\left(1+\frac{1}{u}\right)^{u/2} = \sqrt{e}$.

3. C The Maclaurin series of the denominator begins $\frac{x^4}{24}-\frac{x^6}{720}+\cdots$, so the numerator's Maclaurin series must start with an x^4 term. To eliminate the first four terms of the Maclaurin series for e^{2x}, $f(x) = 1+2x+2x^2+\frac{4x^3}{3}$ and $f(3) = 61$.

4. B Let $u = \arctan x$. $\int_0^{\pi/4}\frac{du}{1+u} = \ln(1+u)\big|_0^{\pi/4} = \ln\left(1+\frac{\pi}{4}\right)$. $1+4=5$.

5. D Integrate by parts twice. $\int x^2 e^{-x/3}\ dx = -3x^2e^{-x/3} + \int 6xe^{-x/3}\,dx = -3x^2e^{-x/3} - 18xe^{-x/3} + \int 18e^{-x/3}\,dx = -3x^2e^{-x/3} - 18xe^{-x/3} - 54e^{-x/3} + C = -3e^{-x/3}(x^2+6x+18)+C$. Evaluating at $x=0$ and $x=6$, the value of the integral is $-3e^2\cdot 90 + 3\cdot 18 = 54-\frac{270}{e^2}$. $\frac{270\cdot 2}{54} = 10$.

6. B While a $u=\tan x$ substitution is possible, a Weierstrass substitution $x = \tan\frac{t}{2}$ is better, noting $\sin t = \frac{2x}{1+x^2}$, $\cos t = \frac{1-x^2}{1+x^2}$, and $dt = \frac{2}{1+x^2}dx$. The integral is equal to $\int_0^{\pi/2}\frac{\sin^3 x\cos^2 x}{16}\ dx = \frac{1}{16}\int_0^{\pi/2}\sin x\cos^2 x\,(1-\cos^2 x)\ dx = \frac{1}{16}\int_0^1(u^2-u^4)\ du = \frac{1}{16}\left(\frac{1}{3}-\frac{1}{5}\right) = \frac{1}{120}$. $L = 120$.

7. A From the given information, $\frac{\Delta y}{\Delta x} = -\frac{D_x f(x_0,y_0)}{D_y f(x_0,y_0)}$, so as Δx and Δy approach 0, we have $\frac{dy}{dx} = -\frac{D_x}{D_y}$.

8. A Expanding, $4x^4y+3x^3y^2-x^2y^3+2y-2024=0$. By the Product Rule or the previous question, $\frac{dy}{dx} = -\frac{16x^3y+9x^2y^2-2xy^3}{4x^4+6x^3y-3x^2y^2+2}$. Evaluated at $(3,4)$, this is $-\frac{2640}{542} = -\frac{1320}{271} \in [-5,-4)$.

9. B $D_x = 2x+3y-1$ and $D_y = 2y+3x-4$. Solving the system $2x+3y=1$, $3x+2y=4$ gives $x=2$ and $y=-1$. $f(2,-1)=-1$.

10. B $r + r\sin\theta = 1$, giving $r = 1-y$. Squaring, $x^2+y^2 = y^2-2y+1$. The y^2 terms cancel in simplification, giving the parabola $x^2 = -2y+1$.

11. B $\frac{1}{2}\int_0^{2\pi}(3+2\cos\theta)\ d\theta = \frac{1}{2}\int_0^{2\pi}(9+12\cos\theta+4\cos^2\theta)\ d\theta = \int_0^{2\pi}\left(\frac{9}{2}+(1+\cos 2\theta)\right)\ d\theta = \int_0^{2\pi}\frac{11}{2}\ d\theta = 11\pi$.

12. D $\frac{dr}{d\theta} = -4\sin\theta$. $\int_0^{2\pi}\sqrt{(4+4\cos\theta)^2+16\sin^2\theta}\ d\theta = 4\int_0^{2\pi}\sqrt{1+2\cos\theta+\cos^2\theta+\sin^2\theta}\ d\theta = 4\sqrt{2}\int_0^{2\pi}\sqrt{1+\cos\theta}\ d\theta = 4\sqrt{2}\int_0^{2\pi}\frac{|\sin\theta|}{\sqrt{1-\cos\theta}}\ d\theta = 8\sqrt{2}\int_0^{\pi}\frac{\sin\theta}{\sqrt{1-\cos\theta}}\ d\theta = 8\sqrt{2}\int_0^2\frac{du}{\sqrt{u}} = 32$.

13. C Setting $4x - x^2 = x$ gives $x = 0$ or $x = 3$. $\int_0^3 ((4x - x^2)^2 - x^2)\,dx = \int_1^2 (x^4 - 8x^3 + 15x^2)\,dx = \frac{x^5}{5} - 2x^4 + 5x^3\Big]_0^3 = \frac{243}{5} - 162 + 135 = \frac{108}{5}$. Multiplying by π gives a volume of $\frac{108\pi}{5}$. $108 + 5 = 113$.

14. C With $u = 2x + 1$, $2\pi \int_0^4 x\sqrt{2x+1}\,dx = \frac{\pi}{2}\int_1^9 (u-1)\sqrt{u}\,du = \frac{\pi}{2}\left(\frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3}\right)\Big]_1^9 = \frac{\pi}{2}\left(\frac{486}{5} - 18 - \frac{2}{5} + \frac{2}{3}\right) = \frac{596\pi}{15} \in [39\pi, 40\pi)$. $k = 39$.

15. D The area of the region is $\int_0^1 (\sqrt{x} - x^2)\,dx = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$. Because $y = \sqrt{x}$ and $y = x^2$ are inverses in the first quadrant, the centroid of this region is on the line $y = x$. The distance from this line to $y = x - 4$ is $2\sqrt{2}$, so by Pappus's, the volume of the rotated region is $2\pi \cdot 2\sqrt{2} \cdot \frac{1}{3} = \frac{4\pi\sqrt{2}}{3}$, and $ABC = 24$.

16. A The point corresponds to $t = 2$. $\frac{dy}{dx} = \frac{4t-5}{2t-2} = 2 - \frac{1}{2t-2}$, which equals $\frac{3}{2}$ at $t = 2$. The derivative of $\frac{dy}{dx}$ with respect to t is $\frac{1}{2(t-1)^2}$. This must be divided by $\frac{dx}{dt}$ by the Chain Rule, so $\frac{d^2y}{dx^2} = \frac{1}{2(t-1)^2(2t-2)}$. When $t = 2$, this equals $\frac{1}{4}$.

17. D Note that $x = 1 - \cos 2t$ and $y = \sin 2t$. Then $(1 - x)^2 + y^2 = 1$, which is a circle with radius 1 and thus a circumference of 2π.

18. D $\frac{dx}{dt} = 6t^2$ and $\frac{dy}{dt} = 6t$. $2\pi \int_0^1 3t^2\sqrt{36t^4 + 36t^2}\,dt = 36\pi \int_0^1 t^3\sqrt{t^2+1}\,dt$. $u = t^2 + 1$ gives $18\pi \int_1^2 (u-1)\sqrt{u}\,du = 18\pi\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right)\Big]_1^2 = \frac{24+24\sqrt{2}}{5}\pi$. $24 + 24 + 2 + 5 = 55$.

19. C The first three derivatives of $y = \tan x$ are $y' = \sec^2 x$, $y'' = 2\tan x \sec^2 x$, and $y''' = 2\sec^4 x + 4\tan^2 x \sec^2 x$, which evaluated at $\frac{\pi}{4}$ are 2, 4, and 16. Thus, the Taylor series is $T(x) = 1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3$ and $T\left(\frac{\pi}{4} + 1\right) = 1 + 2 + 2 + \frac{8}{3} = \frac{23}{3}$. Note that even though $\frac{\pi}{4} + 1 > \frac{\pi}{2}$, the Taylor polynomial is a cubic whose domain is all real numbers.

20. B From $\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$, we have $\tan a \tan b = \frac{\tan a - \tan b}{\tan(a-b)} - 1$. This causes the inner sum to telescope. $\sum_{m=1}^{30}\sum_{n=1}^{60} \tan\frac{mn\pi}{61}\tan\frac{m(n+1)\pi}{61} = \sum_{m=1}^{30}\sum_{n=1}^{60}\left(\frac{\tan\frac{m(n+1)\pi}{61} - \tan\frac{mn\pi}{61}}{\tan\frac{m\pi}{61}} - 1\right) = \sum_{m=1}^{30}\left(\frac{\tan\frac{61m\pi}{61} - \tan\frac{m\pi}{61}}{\tan\frac{m\pi}{61}} - 60\right) = \sum_{m=1}^{30} -61 = -1830$. The sum of the digits of 1830 is 12.

21. C By the Limit-Comparison test, the series is equivalent to $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$, which is conditionally convergent.

22. D $y\,dy = \frac{dx}{x}$, so $y^2 = 2\ln x + C$. Using the given shows $C = 1$ and $y = \sqrt{2\ln x + 1}$. Setting this equal to 3 gives $a = e^4$ and $\sqrt{a} = e^2 = 7.389$ ....

23. A The characteristic equation is $y^2 - 3y + 2 = 0$, which has roots 1 and 2. Thus, $f(x) = Ae^x + Be^{2x}$. Plugging in points gives $A + B = 2e - 1$ and $Ae + Be^2 = e^2$. This gives $A = 2e$ and $B = -1$ so $f(x) = 2e^{x+1} - e^{2x}$ and $f(2) = 2e^3 - e^4$.

24. C The LHS can be recognized as the derivative of $(x^2+1)y$ using the Product Rule; alternatively, it is the step after multiplying by the integrating factor when solving $\frac{dy}{dx}+\frac{2x}{x^2+1}y=\frac{x^3}{x^2+1}$. Integrating both sides, $(x^2+1)y=\frac{x^4}{4}+C$. Plugging in the point $(0,2)$ gives $C=2$, so $y=\frac{x^4+8}{4(x^2+1)}$. When $x=2$, $y=\frac{6}{5}$. $6+5=11$.

25. A $\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=\frac{1}{S-c}+\frac{1}{S-a}+\frac{1}{S-b}=\frac{(e_1-a)(e_1-b)+(e_1-b)(e_1-c)+(e_1-c)(e_1-a)}{(e_1-a)(e_1-b)(e_1-c)}=\frac{3e_1^2-2e_1(a+b+c)+(ab+bc+ca)}{e_1^3-e_1^2(a+b+c)+e_1(ab+bc+ca)-abc}=\frac{e_1^2+e_2}{e_1e_2-e_3}$. By Vieta's, $e_1=-3$, $e_2=2$, and $e_3=-4$. Evaluating, the expression is $-\frac{11}{2}$.

26. A The limit is possible via l'Hospital's or by setting $u=x-1$, where $\lim_{u\to 0}\frac{(u+1)^{u+1}-u-1}{\ln(u+1)-u}=\lim_{u\to 0}\frac{\left(1+u+u^2+O(u^3)\right)-u-1}{\left(u-\frac{u^2}{2}+O(u^3)\right)-u}=\lim_{x\to 1}\frac{u^2+O(u^3)}{-\frac{u^2}{2}+O(u^3)}=-2$.

27. D By the Bounds Trick, the integral equals $\int_0^{\pi/2}\frac{\cos\theta}{1+\sqrt{\sin 2\theta}}\,d\theta$. Summing the integrals gives $2I=\int_0^{\pi/2}\frac{\sin\theta+\cos\theta}{1+\sqrt{2\sin\theta\cos\theta}}\,d\theta=\int_0^{\pi/2}\frac{\sin\theta+\cos\theta}{1+\sqrt{1-(\sin\theta-\cos\theta)^2}}\,d\theta$, and $u=\sin\theta-\cos\theta$ gives $\int_{-1}^{1}\frac{du}{1+\sqrt{1-u^2}}$. By symmetry, the original integral equals $\int_0^1\frac{du}{1+\sqrt{1-u^2}}$, and $u=\sin\varphi$ gives $\int_0^{\pi/2}\frac{\cos\varphi}{1+\cos\varphi}\,d\varphi=\int_0^{\pi/2}\frac{\cos\varphi-\cos^2\varphi}{1-\cos^2\varphi}\,d\varphi=\int_0^{\pi/2}(\cot\varphi\csc\varphi-\cot^2\varphi)\,d\varphi=-\csc\varphi+\cot\varphi+\varphi]_0^{\pi/2}=\frac{\pi-2}{2}$.

28. D Note that $\sin\theta=\cos\theta$, so the slope is $\frac{r'(\theta)+r(\theta)}{r'(\theta)-r(\theta)}$. $r\left(\frac{\pi}{4}\right)=\frac{\pi}{2\sqrt{2}}$, and since $r'(\theta)=2\cos\theta-2\theta\sin\theta$, $r'\left(\frac{\pi}{4}\right)=\sqrt{2}\left(1-\frac{\pi}{4}\right)$. Plugging in to the normal slope gives $\frac{\pi-2}{2}$.

29. B $\sinh 2x=\frac{e^{2x}-e^{-2x}}{2}$. The Laplace transform of this with $s=3$ is $\frac{1}{2}\int_0^{\infty}(e^{-t}-e^{-5t})\,dt=\frac{1}{2}\left(1-\frac{1}{5}\right)=\frac{2}{5}$.

30. D Inspired by the flavor text, draw a perpendicular from B to AD and call the intersection point E. Because ΔABD is isosceles, $AE=ED$. Let $AE=x$, $EB=y$, and $DC=z$. Then $x^2+y^2=25$ and $(x+z)^2+y^2=49$ as well as $2x+z=9$. Subtracting, $2xz+z^2=24$. Factoring, $2xz+z^2=z(2x+z)=9z$, so $z=\frac{8}{3}$. Plugging back into $2x+z=9$ gives $x=\frac{19}{6}$. The desired ratio is $\frac{2x}{z}=\frac{19}{8}$.