Mu School Bowl
Test #812
Question #0

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Question #0

#0 Mu School Bowl
MAΘ National Convention 2023

Let X be the binary representation of a number such that

$$23_{20} + 20_{23} + 23_{2023} + 2023_4 = X_2$$

How many times does the digit 1 appear in X?

#0 Mu School Bowl
MAΘ National Convention 2023

Let X be the binary representation of a number such that

$$23_{20} + 20_{23} + 23_{2023} + 2023_4 = X_2$$

How many times does the digit 1 appear in X?

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Question #1

#1 Mu School Bowl
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Let $\mathcal{L}_1$ be the line tangent to the graph of $y = 2x^3 - 2x - 4$ at $x = 1$.

Let $\mathcal{L}_2$ be the line tangent to the graph of $y = \frac{2}{x^2+1}$ at $x = 1$.

Let $\mathcal{L}_3$ be the line tangent to the graph of $y = \sin^2(\pi x) + x + 4$ at $x = -1$.

$\mathcal{L}_1$ and $\mathcal{L}_2$ intersect at the point A. $\mathcal{L}_1$ and $\mathcal{L}_3$ intersect at the point B. $\mathcal{L}_2$ and $\mathcal{L}_3$ intersect at the point C.

Find the area of triangle ABC.

#1 Mu School Bowl
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$\mathcal{L}_1$ and $\mathcal{L}_2$ intersect at the point A. $\mathcal{L}_1$ and $\mathcal{L}_3$ intersect at the point B. $\mathcal{L}_2$ and $\mathcal{L}_3$ intersect at the point C.

Find the area of triangle ABC.

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Question #2

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Question #2

#2 Mu School Bowl
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Let A be the sum of the real solutions to the relation $x = \log_2(x + \log_2(x + \log_2(x + \cdots \log_2(x)) \ldots))$

Let B be the number of values of θ in $[0, \pi)$ for which

$$\frac{-1+\sqrt{3}}{2} = \frac{\cos(2023\theta)}{1 + \frac{\cos(2023\theta)}{1 + \frac{\cos(2023\theta)}{1 + \cdots}}}$$

Find $A + B$.

#2 Mu School Bowl
MAΘ National Convention 2023

Let A be the sum of the real solutions to the relation $x = \log_2(x + \log_2(x + \log_2(x + \cdots \log_2(x)) \ldots))$

Let B be the number of values of θ in $[0, \pi)$ for which

$$\frac{-1+\sqrt{3}}{2} = \frac{\cos(2023\theta)}{1 + \frac{\cos(2023\theta)}{1 + \frac{\cos(2023\theta)}{1 + \cdots}}}$$

Find $A + B$.

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Question #3

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Question #3

#3 Mu School Bowl
MAΘ National Convention 2023

Let $A = \int_0^\infty \frac{\ln(x)}{x^2+2024x+2023}\,dx$

Let $B = \int_1^2 e^{-x^2}\,dx$

Let $C = \int_{1/e}^{1/e^4} \sqrt{\ln\left(\frac{1}{x}\right)}\,dx$

Let $D = \int_0^\infty \frac{\ln(x)}{2023x^2+2024x+1}\,dx$

Find $(A + B + C + D)e^5$.

#3 Mu School Bowl
MAΘ National Convention 2023

Let $A = \int_0^\infty \frac{\ln(x)}{x^2+2024x+2023}\,dx$

Let $B = \int_1^2 e^{-x^2}\,dx$

Let $C = \int_{1/e}^{1/e^4} \sqrt{\ln\left(\frac{1}{x}\right)}\,dx$

Let $D = \int_0^\infty \frac{\ln(x)}{2023x^2+2024x+1}\,dx$

Find $(A + B + C + D)e^5$.

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Question #4

#4 Mu School Bowl
MAΘ National Convention 2023

Note that the domain of all variables in this question are all real numbers.

Let A be the maximum value of $6\sin(2023x) + 8\cos(2023x)$

Let B be the maximum value of $2023 + 18x - 3x^2$

Let C be the maximum value of xyz given that $20x + 2y + 3z = 6$ and $x, y, z > 0$

Let D be the maximum value of $3x + 6y + 22z$ given that $x^2 + y^2 + z^2 = 25$

Find $(A + B + D)C$.

#4 Mu School Bowl
MAΘ National Convention 2023

Note that the domain of all variables in this question are all real numbers.

Let A be the maximum value of $6\sin(2023x) + 8\cos(2023x)$

Let B be the maximum value of $2023 + 18x - 3x^2$

Let C be the maximum value of xyz given that $20x + 2y + 3z = 6$ and $x, y, z > 0$

Let D be the maximum value of $3x + 6y + 22z$ given that $x^2 + y^2 + z^2 = 25$

Find $(A + B + D)C$.

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Question #5

#5 Mu School Bowl
MAΘ National Convention 2023

Let A be the minimum value of $y = \frac{e^{x^2}}{x}$ for positive x.

Let B be the maximum value of $y = \frac{x+3}{x^2+3x+4}$.

Let C be the maximum slope of a tangent line to the curve $y = e^{-\frac{1}{18}(x-2)^2}$.

Let D be the number of relative extrema on the graph of $y = (x-2)^2(x-3)^3(x-4)^4$

Find $\frac{A}{C} + B + D$.

#5 Mu School Bowl
MAΘ National Convention 2023

Let A be the minimum value of $y = \frac{e^{x^2}}{x}$ for positive x.

Let B be the maximum value of $y = \frac{x+3}{x^2+3x+4}$.

Let C be the maximum slope of a tangent line to the curve $y = e^{-\frac{1}{18}(x-2)^2}$.

Let D be the number of relative extrema on the graph of $y = (x-2)^2(x-3)^3(x-4)^4$

Find $\frac{A}{C} + B + D$.

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Question #6

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Question #6

Anagh is standing motionless at the origin, and Alan is motionless at the point (4,0).

At time T, Srijan is at the point $(-2,2)$ and is moving upward parallel to the positive y-axis at a rate of 7 units per minute.

At time T, Luke is at the point (5,1) and is moving rightward parallel to the positive x-axis at a rate of 5 units per minute.

Let A be the rate of change of the distance between Srijan and Luke at time T.

Let B be the rate of change of the tangent of the angle formed by the x-axis and the line connecting Anagh and Luke at time T.

Let C be the rate of change of the area of the quadrilateral formed by Anagh, Alan, Luke, and Srijan at time T.

Find $10(A + B + C)$

#6 Mu School Bowl
MAΘ National Convention 2023

Anagh is standing motionless at the origin, and Alan is motionless at the point (4,0).

At time T, Srijan is at the point $(-2,2)$ and is moving upward parallel to the positive y-axis at a rate of 7 units per minute.

At time T, Luke is at the point (5,1) and is moving rightward parallel to the positive x-axis at a rate of 5 units per minute.

Let A be the rate of change of the distance between Srijan and Luke at time T.

Let B be the rate of change of the tangent of the angle formed by the x-axis and the line connecting Anagh and Luke at time T.

Let C be the rate of change of the area of the quadrilateral formed by Anagh, Alan, Luke, and Srijan at time T.

Find $10(A + B + C)$

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Question #7

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Question #7

#7 Mu School Bowl
MAΘ National Convention 2023

Let $A = \int_0^{\pi} \frac{\sin(x)}{\cos^2(x)+1}\,dx$

Let $B = \int_0^{\frac{\pi}{3}} \pi \sin(x)\cos(\pi\, cos(x))\,dx$

Let $C = \int_{-\infty}^{\infty} e^{-|x|}dx$

Let $D = \int_0^{\infty} x^2 e^{-x^3}\,dx$

Find $(A + B\pi)CD$

#7 Mu School Bowl
MAΘ National Convention 2023

Let $A = \int_0^{\pi} \frac{\sin(x)}{\cos^2(x)+1}\,dx$

Let $B = \int_0^{\frac{\pi}{3}} \pi \sin(x)\cos(\pi\, cos(x))\,dx$

Let $C = \int_{-\infty}^{\infty} e^{-|x|}dx$

Let $D = \int_0^{\infty} x^2 e^{-x^3}\,dx$

Find $(A + B\pi)CD$

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Question #8

#8 Mu School Bowl
MAΘ National Convention 2023

The values a_1 and a_3 are chosen uniformly at random with replacement from the set $\{\pm 1, \pm 2, \pm 3\}$

Let A be the probability that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse. (In other words, when graphed on the Cartesian plane, the graph is an ellipse with positive area).

Let B be the probability that the area contained by the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is less than or equal to 2023π, given that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse.

Find $\frac{1}{A} + B$.

#8 Mu School Bowl
MAΘ National Convention 2023

The values a_1 and a_3 are chosen uniformly at random with replacement from the set $\{\pm 1, \pm 2, \pm 3\}$

Let A be the probability that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse. (In other words, when graphed on the Cartesian plane, the graph is an ellipse with positive area).

Let B be the probability that the area contained by the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is less than or equal to 2023π, given that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse.

Find $\frac{1}{A} + B$.

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Question #9

#9 Mu School Bowl
MAΘ National Convention 2023

Let $f_1(x) = kx(\alpha - x)$. If $\int_0^a f_1(x)dx = 1$ and $\int_0^a xf_1(x)dx = 2$, find $A = \int_0^a x^2 f_1(x)dx - 4$. Express A as a numerical value <u>not</u> in terms of k or α.

Let $f_2(x) = \frac{1}{x^2}$. Sharvaa flips a fair coin, and picks $f_1(x)$ if it is heads and $f_2(x)$ if it is tails. The probability that Sharvaa attends practice is $\int_3^a f_1(x)dx$ if he picks $f_1(x)$, or $\int_3^\infty f_2(x)dx$ if he picks $f_2(x)$. If Sharvaa does indeed attend practice, let B be the probability that $f_1(x)$ was chosen. Express B as a numerical value <u>not</u> in terms of k or α.

Find AB.

#9 Mu School Bowl
MAΘ National Convention 2023

Let $f_1(x) = kx(\alpha - x)$. If $\int_0^a f_1(x)dx = 1$ and $\int_0^a xf_1(x)dx = 2$, find $A = \int_0^a x^2 f_1(x)dx - 4$. Express A as a numerical value <u>not</u> in terms of k or α.

Let $f_2(x) = \frac{1}{x^2}$. Sharvaa flips a fair coin, and picks $f_1(x)$ if it is heads and $f_2(x)$ if it is tails. The probability that Sharvaa attends practice is $\int_3^a f_1(x)dx$ if he picks $f_1(x)$, or $\int_3^\infty f_2(x)dx$ if he picks $f_2(x)$. If Sharvaa does indeed attend practice, let B be the probability that $f_1(x)$ was chosen. Express B as a numerical value <u>not</u> in terms of k or α.

Find AB.

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Question #10

#10 Mu School Bowl
MAΘ National Convention 2023

Consider the following lines in three-dimensional Cartesian space:

$$\text{Line } \mathcal{L}_1: x+1 = \frac{y-1}{2} = \frac{z-3}{2}$$

$$\text{Line } \mathcal{L}_2: \frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{6}$$

Let D_1 be the minimum distance between the point (2,2,3) and $\mathcal{L}_1$.
$D_1^2 = \frac{m}{n}$ in simplest form, $A = m + n$.

Let D_2 be the minimum distance between $\mathcal{L}_1$ and $\mathcal{L}_2$.
$D_2^2 = \frac{m}{n}$ in simplest form. $B = m + n$.

Find $A + B$.

#10 Mu School Bowl
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Consider the following lines in three-dimensional Cartesian space:

$$\text{Line } \mathcal{L}_1: x+1 = \frac{y-1}{2} = \frac{z-3}{2}$$

$$\text{Line } \mathcal{L}_2: \frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{6}$$

Let D_1 be the minimum distance between the point (2,2,3) and $\mathcal{L}_1$.
$D_1^2 = \frac{m}{n}$ in simplest form, $A = m + n$.

Let D_2 be the minimum distance between $\mathcal{L}_1$ and $\mathcal{L}_2$.
$D_2^2 = \frac{m}{n}$ in simplest form. $B = m + n$.

Find $A + B$.

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Question #11

#11 Mu School Bowl
MAΘ National Convention 2023

A differentiable function $f(x)$ attains the following values:

$\boldsymbol{x} =$	0	1	2	3	4	5	6
$\boldsymbol{f(x)} =$	20	23	71	63	32	73	19

Let L be the approximation of $\int_0^6 f(x)dx$ using a left-handed Riemann sum with three equal subintervals.

Let R be the approximation of $\int_0^6 f(x)dx$ using a right-handed Riemann sum with three equal subintervals.

Let M be the approximation of $\int_0^6 f(x)dx$ using a midpoint Riemann sum with three equal subintervals.

Let S be the approximation of $\int_0^6 f(x)dx$ using Simpson's rule with six equal subintervals.

Find $R + L + 5M - 6S$.

#11 Mu School Bowl
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A differentiable function $f(x)$ attains the following values:

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Let L be the approximation of $\int_0^6 f(x)dx$ using a left-handed Riemann sum with three equal subintervals.

Let R be the approximation of $\int_0^6 f(x)dx$ using a right-handed Riemann sum with three equal subintervals.

Let M be the approximation of $\int_0^6 f(x)dx$ using a midpoint Riemann sum with three equal subintervals.

Let S be the approximation of $\int_0^6 f(x)dx$ using Simpson's rule with six equal subintervals.

Find $R + L + 5M - 6S$.

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Question #12

#12 Mu School Bowl
MAΘ National Convention 2023

Let $A = \lim_{x\to 3} \frac{x^3-3x^2+4x-12}{x^2+x-12}$

Let $B = \lim_{x\to \infty} \frac{x^2-3x^3+4x-12}{x^2+x^3-12}$

Let $C = \lim_{x\to 0}(1+2023x)^{\frac{2}{x}}$

Let $D = \lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$

Find $70A + B + \ln(C) + D$

#12 Mu School Bowl
MAΘ National Convention 2023

Let $A = \lim_{x\to 3} \frac{x^3-3x^2+4x-12}{x^2+x-12}$

Let $B = \lim_{x\to \infty} \frac{x^2-3x^3+4x-12}{x^2+x^3-12}$

Let $C = \lim_{x\to 0}(1+2023x)^{\frac{2}{x}}$

Let $D = \lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$

Find $70A + B + \ln(C) + D$

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Question #13

#13 Mu School Bowl
MAΘ National Convention 2023

Let $A = \sum_{n=1}^{K} 2023$

Let $B = \sum_{n=1}^{2023} n$

Let $C = \sum_{n=1}^{2023} n^2$

Let $D = \sum_{n=1}^{2023} n^3$

Find the smallest positive value of K so that $\gcd\left(A, \frac{BC}{D}\right) > 1$

#13 Mu School Bowl
MAΘ National Convention 2023

Let $A = \sum_{n=1}^{K} 2023$

Let $B = \sum_{n=1}^{2023} n$

Let $C = \sum_{n=1}^{2023} n^2$

Let $D = \sum_{n=1}^{2023} n^3$

Find the smallest positive value of K so that $\gcd\left(A, \frac{BC}{D}\right) > 1$

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#14 Mu School Bowl
MAΘ National Convention 2023

The partial fraction decomposition of $\frac{25}{(x-2)^2(x^2+1)}$ is $\frac{Ax+B}{x^2+1}+\frac{C}{x-2}+\frac{D}{(x-2)^2}$.

Find $A+B+C+D$.

#14 Mu School Bowl
MAΘ National Convention 2023

The partial fraction decomposition of $\frac{25}{(x-2)^2(x^2+1)}$ is $\frac{Ax+B}{x^2+1}+\frac{C}{x-2}+\frac{D}{(x-2)^2}$.

Find $A+B+C+D$.