Mu School Bowl
Test #
Question #0

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Question #0

Let $X$ be the binary representation of a number such that

$$23_{20} + 20_{23} + 23_{2023} + 2023_4 = X_2$$

How many times does the digit 1 appear in $X$?

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**Solution:**

$23_{20} + 20_{23} + 23_{2023} + 2023_4 = 2(20) + 3 + 2(23) + 2(2023) + 3 + 2(64) + 2(4) + 3 = 43 + 46 + 4049 + 139 = 4277 = 4096 + 181 = 4096 + 128 + 32 + 16 + 4 + 1 = 1000010110101_2$ which has $\boxed{6}$ ones.

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Question #1

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Let $\mathcal{L}_1$ be the line tangent to the graph of $y = 2x^3 - 2x - 4$ at $x = 1$.

Let $\mathcal{L}_2$ be the line tangent to the graph of $y = \frac{2}{x^2+1}$ at $x = 1$.

Let $\mathcal{L}_3$ be the line tangent to the graph of $y = \sin^2(\pi x) + x + 4$ at $x = -1$.

$\mathcal{L}_1$ and $\mathcal{L}_2$ intersect at the point $A$. $\mathcal{L}_1$ and $\mathcal{L}_3$ intersect at the point $B$. $\mathcal{L}_2$ and $\mathcal{L}_3$ intersect at the point $C$.

Find the area of triangle $ABC$.

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**Solution:**

$\mathcal{L}_1$: $y' = 6x^2 - 2 \rightarrow m = 4 \rightarrow y + 4 = 4(x - 1) \rightarrow y = 4x - 8$.
$\mathcal{L}_2$: $y' = -\frac{4x}{(x^2+1)^2} \rightarrow m = -1 \rightarrow y - 1 = -(x - 1) \rightarrow y = -x + 2$.
$\mathcal{L}_3$: $y' = 2\pi \sin(\pi x)\cos(\pi x) + 1 \rightarrow m = 1 \rightarrow y - 3 = x + 1 \rightarrow y = x + 4$.

$A$: $4x - 8 = -x + 2 \rightarrow x = 2, y = 0$.
$B$: $4x - 8 = x + 4 \rightarrow x = 4, y = 8$.
$C$: $-x + 2 = x + 4 \rightarrow x = -1, y = 3$.

The area is $\frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ 4 & 8 & 1 \\ -1 & 3 & 1 \end{vmatrix} = \frac{1}{2}(16 + +0 + 12 + 8 - 6 - 0) = \boxed{15}$

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Question #2

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Question #2

Let $A$ be the sum of the real solutions to the relation $x = \log_2(x + \log_2(x + \log_2(x + \cdots \log_2(x)) \ldots))$

Let $B$ be the number of values of $\theta$ in $[0, \pi)$ for which

$$\frac{-1+\sqrt{3}}{2} = \frac{\cos(2023\theta)}{1 + \frac{\cos(2023\theta)}{1 + \frac{\cos(2023\theta)}{1 + \cdots}}}$$

Find $A + B$.

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**Solution:**

$\boldsymbol{A}$: $x = \log_2(x + \log_2(x + \log_2(x + \cdots))) = \log_2(x + x) = \log_2(2x) \rightarrow 2^x = 2x \rightarrow x = 1,2 \rightarrow A = 3$.

$\boldsymbol{B}$: $\frac{-1+\sqrt{3}}{2} = \frac{\cos(2023\theta)}{1+\frac{\cos(2023\theta)}{1+\frac{\cos(2023\theta)}{1+\cdots}}} = \frac{\cos(2023\theta)}{1+\left(\frac{-1+\sqrt{3}}{2}\right)} = \frac{\cos(2023\theta)}{\frac{1+\sqrt{3}}{2}} \rightarrow \cos(2023\theta) = \left(\frac{1+\sqrt{3}}{2}\right)\left(\frac{-1+\sqrt{3}}{2}\right) = \frac{1}{2}$.

This equivalent to asking for the number of solutions of

$$\cos(x) = \frac{1}{2}, x \in [0, 2023\pi)$$

This equation has a solution at $Q1,4$ or one solution every $\pi$. Thus, there are $2023$ solutions.

**The final answer** is $3 + 2023 = \boxed{2026}$

Mu School Bowl
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Question #3

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Question #3

Let $A = \int_0^\infty \frac{\ln(x)}{x^2+2024x+2023} dx$

Let $B = \int_1^2 e^{-x^2} dx$

Let $C = \int_{1/e}^{1/e^4} \sqrt{\ln\left(\frac{1}{x}\right)} dx$

Let $D = \int_0^\infty \frac{\ln(x)}{2023x^2+2024x+1} dx$

Find $(A + B + C + D)e^5$.

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**Solution:**

Using the substitution $u = \frac{1}{x} \to -\frac{1}{u^2} du = dx$ on $A$ yields $A = \int_0^\infty \frac{\ln(x)}{x^2+2024x+2023} dx \to \int_\infty^0 \frac{\ln\left(\frac{1}{u}\right)}{\frac{1}{u^2}+\frac{2024}{u}+2023} \frac{-1}{u^2} du = -\int_0^\infty \frac{\ln(u)}{1+2024u+2023u^2} dx = -D$. Therefore $A + D = 0$.

Note that if we let $f(x) = e^{-x^2}$, then $C = \int_{f(1)}^{f(2)} f^{-1}(x)dx$. If we let $x = f(u) \to dx = f'(u)du$ then $C = \int_1^2 f^{-1}(f(u))f'(u)du = \int_1^2 uf'(u)du = [u\,f(u)]_{u=1}^{u=2} - \int_1^2 f(u)du = 2f(2) - f(1) - B$ by integration by parts. So $B + C = 2f(2) - f(1) = \frac{2}{e^4} - \frac{1}{e}$.

So $(A + B + C + D)e^5 = \boxed{2e - e^4}$

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Question #4

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Question #4

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Note that the domain of all variables in this question are all real numbers.

Let $A$ be the maximum value of $6\sin(2023x) + 8\cos(2023x)$

Let $B$ be the maximum value of $2023 + 18x - 3x^2$

Let $C$ be the maximum value of $xyz$ given that $20x + 2y + 3z = 6$ and $x, y, z > 0$

Let $D$ be the maximum value of $3x + 6y + 22z$ given that $x^2 + y^2 + z^2 = 25$

Find $(A + B + D)C$.

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**Solution:**

For $A$, the amplitude is $\sqrt{6^2 + 8^2} = 10$.

For $B$, the maximum occurs at the vertex which is $-\frac{b}{2a} = -\frac{18}{-6} = 3$. That makes the maximum value $2023 + 18 * 3 - 27 = 2050$

For $C$, we use AM-GM so $(xyz)^{\frac{1}{3}} \le \frac{x+y+z}{3} \to 20x * 2y * 3z \le \frac{(20x+2y+3z)^3}{3^3} = 8 \to xyz \le \frac{1}{15}$

For $D$, we use Cauchy-Schwartz so $\boldsymbol{u} \cdot \boldsymbol{v} \le \|\boldsymbol{u}\|\|\boldsymbol{v}\| \to 3x + 6y + 22z =< x, y, z >\cdot< 3,6,22 >\le \sqrt{x^2 + y^2 + z^2}\sqrt{3^2 + 6^2 + 22^2} = 5 * 23 = 115$

So the answer is $\frac{10+2050+115}{15} = \boxed{145}$

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Question #5

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Question #5

**#5 Mu School Bowl**
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Let $A$ be the minimum value of $y = \frac{e^{x^2}}{x}$ for positive $x$.

Let $B$ be the maximum value of $y = \frac{x+3}{x^2+3x+4}$.

Let $C$ be the maximum slope of a tangent line to the curve $y = e^{-\frac{1}{18}(x-2)^2}$.

Let $D$ be the number of relative extrema on the graph of $y = (x-2)^2(x-3)^3(x-4)^4$

Find $\frac{A}{C} + B + D$.

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**Solution:**

**For *A***, $y = \frac{e^{x^2}}{x} \to y' = 2x\frac{e^{x^2}}{x} - \frac{e^{x^2}}{x^2} = e^{x^2}\left(2 - \frac{1}{x^2}\right) = 0 \to x = \frac{1}{\sqrt{2}} \to y = \sqrt{2e}$
**For *B***, $y = \frac{x+3}{x^2+3x+4} \to y' = \frac{x^2+3x+4-(x+3)(2x+3)}{(x^2+3x+4)^2} = \frac{x^2+3x+4-2x^2-9x-9}{(x^2+3x+4)^2} = \frac{-x^2-6x-5}{(x^2+3x+4)^2} = \frac{-(x+1)(x+5)}{(x^2+3x+4)^2} = 0$. The maximum occurs at $x = -1 \to y = \frac{2}{1-3+4} = 1$.
**For *C***, the maximum slope will occur at a point of inflection going from concave up to down. $y = e^{-\frac{1}{18}(x-2)^2} \to y' = -\frac{1}{9}(x-2)e^{-\frac{1}{18}(x-2)^2} \to y'' = -\frac{1}{9}e^{-\frac{1}{18}(x-2)^2} + \frac{1}{81}(x-2)^2 e^{-\frac{1}{18}(x-2)^2} = 0 \to \frac{1}{81}e^{-\frac{1}{18}(x-2)^2}(-9 + (x-2)^2) = 0 \to x - 2 = \pm 3 \to x = -1{,}5$. The slope is clearly negative at $x = 5$and positive at $x = -1$ so the maximum slope is $-\frac{1}{9}(-3)e^{-\frac{1}{18}(-3)^2} = \frac{1}{3\sqrt{e}}$
**For *D***, $y = (x-2)^2(x-3)^3(x-4)^4 \to y' = 2(x-2)(x-3)^3(x-4)^4 + 3(x-2)^2(x-3)^2(x-4)^4 + 4(x-2)^2(x-3)^3(x-4)^3 = (x-2)(x-3)^2(x-4)^3(2(x-3)(x-4) + 3(x-2)(x-4) + 4(x-2)(x-3)) = (x-2)(x-3)^2(x-4)^3(2x^2 - 14x + 24 + 3x^2 - 18x + 24 + 4x^2 - 20x + 24) = (x-2)(x-3)^2(x-4)^3(9x^2 - 52x + 72)$. Since $52^2 - 9(72) > 0$, the last term has two real roots. Only roots of odd multiplicity will be extrema, so $D = 4$. (Or just graph it out given the root behavior!)
**The final answer** is $\frac{\sqrt{2e}}{\frac{1}{3\sqrt{e}}} + 1 + 4 = \boxed{3e\sqrt{2} + 5}$

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Question #6

Anagh is standing motionless at the origin, and Alan is motionless at the point $(4,0)$.

At time $T$, Srijan is at the point $(-2,2)$ and is moving upward parallel to the positive $y$-axis at a rate of 7 units per minute.

At time $T$, Luke is at the point $(5,1)$ and is moving rightward parallel to the positive $x$-axis at a rate of 5 units per minute.

Let $A$ be the rate of change of the distance between Srijan and Luke at time $T$.

Let $B$ be the rate of change of the tangent of the angle formed by the $x$-axis and the line connecting Anagh and Luke at time $T$.

Let $C$ be the rate of change of the area of the quadrilateral formed by Anagh, Alan, Luke, and Srijan at time $T$.

Find $10(A+B+C)$



**Solution:**

The distance squared between them at time $T$ is $D^2 = (-2-x_l)^2 + (y_s - 1)^2 = 50$. The rate of change will be given by $2DD' = -2(-2-x_l)x_l' + 2(y_s-1)y_s' \rightarrow D' = \frac{-(-2-5)(5)+(2-1)(7)}{\sqrt{50}} = \frac{42}{5\sqrt{2}} = \frac{21\sqrt{2}}{5} = A$

From right triangle trigonometry, $\tan(\theta) = \frac{1}{x_l} \rightarrow \frac{d(\tan(\theta))}{dt} = -\frac{x_l'}{x_l^2} = -\frac{5}{25} = -\frac{1}{5} = B$
From the shoelace formula, the area is given by $\mathrm{A} = x_l y_s + 6 \rightarrow \mathrm{A}' = x_l' y_s + x_l y_s' = (5)(2) + (5)(7) = 45 = C$

The final answer is $10\left(\frac{21\sqrt{2}}{5} - \frac{1}{5} + 45\right) = 42\sqrt{2} - 2 + 450 = \boxed{42\sqrt{2} + 448}$

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Question #7

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Question #7

## #7 Mu School Bowl
## MAΘ National Convention 2023

Let $A = \int_0^{\pi} \frac{\sin(x)}{\cos^2(x)+1} dx$

Let $B = \int_0^{\frac{\pi}{3}} \pi \sin(x) \cos(\pi \, cos(x)) \, dx$

Let $C = \int_{-\infty}^{\infty} e^{-|x|} dx$

Let $D = \int_0^{\infty} x^2 e^{-x^3} dx$

Find $(A + B\pi)CD$

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## MAΘ National Convention 2023

**Solution:**

$A = \int_0^{\pi} \frac{\sin(x)}{\cos^2(x)+1} dx$. Let $u = \cos(x) \to du = -\sin(x)\, dx$. Then $\int_0^{\pi} \frac{\sin(x)}{\cos^2(x)+1} dx = -\int_1^{-1} \frac{1}{u^2+1} du = [-\arctan(u)]_1^{-1} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$B = \int_0^{\frac{\pi}{3}} \pi \sin(x) \cos(\pi \, cos(x)) \, dx$. Let $u = cos(x) \to -du = \sin(x)\, dx$. Then $\int_0^{\frac{\pi}{3}} \pi \sin(x) \cos(\pi \, cos(x)) \, dx = -\pi \int_1^{\frac{1}{2}} \cos(\pi u) \, du = [\sin(\pi u)]_{\frac{1}{2}}^{1} = -1$.

$C = \int_{-\infty}^{\infty} e^{-|x|} dx = 2 \int_0^{\infty} e^{-x} dx = 2[-e^{-x}]_0^{\infty} = 2$.

$D = \int_0^{\infty} x^2 e^{-x^3} dx$. Let $u = -x^3 \to -\frac{1}{3} du = x^2 dx$. Then $\int_0^{\infty} x^2 e^{-x^3} dx = \int_0^{-\infty} -\frac{1}{3} e^u du = -\frac{1}{3}[e^u]_0^{-\infty} = \frac{1}{3}$

The final answer is $\left(\frac{\pi}{2} - \pi\right)(2)\left(\frac{1}{3}\right) = \boxed{-\frac{\pi}{3}}$

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Question #8

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Question #8

The values $a_1$ and $a_3$ are chosen uniformly at random with replacement from the set $\{\pm 1, \pm 2, \pm 3\}$

Let $A$ be the probability that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse.
(In other words, when graphed on the Cartesian plane, the graph is an ellipse with positive area).

Let $B$ be the probability that the area contained by the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is less than or equal to $2023\pi$, given that the graph of $a_1x^2 + 4xy + a_3y^2 = 2023$ is a non-degenerate ellipse.

Find $\frac{1}{A} + B$.

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**Solution:**

***A***: To be an ellipse, the discriminant $B^2 - 4AC = 16 - 4a_1a_3 < 0 \to 4 < a_1a_3$. To avoid imaginary ellipses, we also need $\delta \cdot a_3 = \begin{vmatrix} a_1 & 2 & 0 \\ 2 & a_3 & 0 \\ 0 & 0 & -2023 \end{vmatrix} \cdot a_3 = -2023a_3(a_1a_3 - 4) < 0$. Since $a_1a_3 - 4 > 0$ this means $a_3 > 0 \to a_1 > 0$ since $0 < 4 < a_1a_3$. The only possible values that fit these criteria are $a_1 = a_3 = 3$, $a_1 = 2$ & $a_3 = 3$, or $a_1 = 3$ & $a_3 = 2$ The probability is therefore $\frac{3}{6^2} = \frac{1}{12} = A$.

***B***: The area enclosed by $Lx^2 + Mxy + Ny^2 = 1$ is $\frac{2\pi}{\sqrt{4LN - M^2}}$. Therefore the area in this curve is $\frac{2023\pi}{\sqrt{a_1a_3 - 4}}$. The maximum value this could ever attain is $2023\pi$, so the area will always be less than or equal to $2023\pi$ and the probability is thus $1 = B$.

**Final Answer:** $\frac{1}{A} + B = 12 + 1 = \boxed{13}$.

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Question #9

Let $f_1(x) = kx(\alpha - x)$. If $\int_0^\alpha f_1(x)dx = 1$ and $\int_0^\alpha xf_1(x)dx = 2$, find $A = \int_0^\alpha x^2 f_1(x)dx - 4$. Express $A$ as a numerical value <u>not</u> in terms of $k$ or $\alpha$.

Let $f_2(x) = \frac{1}{x^2}$. Sharvaa flips a fair coin, and picks $f_1(x)$ if it is heads and $f_2(x)$ if it is tails. The probability that Sharvaa attends practice is $\int_3^\alpha f_1(x)dx$ if he picks $f_1(x)$, or $\int_3^\infty f_2(x)dx$ if he picks $f_1(x)$. If Sharvaa does indeed attend practice, let $B$ be the probability that $f_1(x)$ was chosen. Express $B$ as a numerical value <u>not</u> in terms of $k$ or $\alpha$.

Find $AB$.

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**Solution:**

$\int_{-\infty}^{\infty} f_1(x)dx = 1 = k\int_0^\alpha \alpha x - x^2 dx = k\left[\frac{\alpha}{2}x^2 - \frac{1}{3}x^3\right]_0^\alpha = \frac{\alpha^3}{6}k \rightarrow k = \frac{6}{\alpha^3}$. Further, $E[X] = 2 = k\int_0^\alpha \alpha x^2 - x^3 dx = k\left[\frac{\alpha}{3}x^3 - \frac{1}{4}x^4\right]_0^\alpha = \frac{\alpha^4}{12}k \rightarrow k = \frac{24}{\alpha^4} = \frac{6}{\alpha^3} \rightarrow \alpha = 4, k = \frac{3}{32}$. So the variance is $\left(\frac{3}{32}\int_0^4 4x^3 - x^4 dx\right) - 4 = \frac{3}{32}\left[x^4 - \frac{1}{5}x^5\right]_0^4 - 4 = \frac{4}{5}$

$P(f_1|Sharvaa\ attends) = \frac{P(f_1)P(Sharvaa\ attends|f_1)}{P(f_1)P(Sharvaa\ attends|f_1)+P(f_2)P(Sharvaa\ attends|f_2)}$ by Bayes' Law. Since $P(f_1) = P(f_2) = 0.5$, this is equivalent to $\frac{P(Sharvaa\ attends|f_1)}{P(Sharvaa\ attends|f_1)+P(Sharvaa\ attends|f_2)}$. Since $P(Sharvaa\ attends|f_1) = \frac{3}{32}\int_3^4 4x - x^2 dx = \frac{5}{32}$ and $P(Sharvaa\ attends|f_2) = \int_3^\infty \frac{1}{x^2}dx = \frac{1}{3}$, this results in $B = \frac{\frac{5}{32}}{\frac{5}{32}+\frac{1}{3}} = \frac{15}{47}$.

The final answer is $\frac{4}{5} \cdot \frac{15}{47} = \boxed{\frac{12}{47}}$

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Question #10

Consider the following lines in three-dimensional Cartesian space:

$$\text{Line } \mathcal{L}_1 : x+1 = \frac{y-1}{2} = \frac{z-3}{2}$$

$$\text{Line } \mathcal{L}_2 : \frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{6}$$

Let $D_1$ be the minimum distance between the point (2,2,3) and $\mathcal{L}_1$.
$D_1^2 = \frac{m}{n}$ in simplest form, $A = m + n$.

Let $D_2$ be the minimum distance between $\mathcal{L}_1$ and $\mathcal{L}_2$.
$D_2^2 = \frac{m}{n}$ in simplest form. $B = m + n$.

Find $A + B$.

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**Solution:**

$\mathcal{L}_1$ goes through the point $P_1$: $(-1,1,3)$ and has directional vector $\overrightarrow{v_1} = \langle 1,2,2 \rangle$. $\mathcal{L}_2$ goes through the point $P_2(1,3,-2)$ and has directional vector $\overrightarrow{v_2} = \langle 2,3,6 \rangle$.
If we define $Q$: (2,2,3) then the distance $A$ is given by $D_1 = \frac{\|\overrightarrow{v_1} \times \overrightarrow{P_1Q}\|}{\|\overrightarrow{v_1}\|} = \frac{\|\langle 1,2,2 \rangle \times \langle -3,-1,0 \rangle\|}{\|\langle 1,2,2 \rangle\|} = \frac{1}{3}\left\| \begin{vmatrix} \hat{\imath} & j & k \\ 1 & 2 & 2 \\ -3 & -1 & 0 \end{vmatrix} \right\| = \frac{\|\langle 2,-6,5 \rangle\|}{3} = \frac{\sqrt{65}}{3}$. So $D_1^2 = \frac{65}{9}$

The distance between these lines is given by $d = \frac{|(\overrightarrow{v_1} \times \overrightarrow{v_2}) \cdot \overrightarrow{P_1P_2}|}{\|\overrightarrow{v_1} \times \overrightarrow{v_2}\|}$. $\overrightarrow{v_1} \times \overrightarrow{v_2} = \begin{vmatrix} \hat{\imath} & j & k \\ 1 & 2 & 2 \\ 2 & 3 & 6 \end{vmatrix} = \langle 6,-2,-1 \rangle$.
$\overrightarrow{P_1P_2} = \langle -2,-2,5 \rangle$. So $D_2 = \frac{|-12+4-5|}{\sqrt{36+4+1}} = \frac{13}{\sqrt{41}}$ and $D_2^2 = \frac{169}{41}$

Therefore, the final answer is

$$65 + 9 + 169 + 41 = \boxed{284}$$

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Question #11

A differentiable function $f(x)$ attains the following values:

| $\boldsymbol{x} =$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{f(x)} =$ | 20 | 23 | 71 | 63 | 32 | 73 | 19 |

Let $L$ be the approximation of $\int_0^6 f(x)dx$ using a left-handed Riemann sum with three equal subintervals.

Let $R$ be the approximation of $\int_0^6 f(x)dx$ using a right-handed Riemann sum with three equal subintervals.

Let $M$ be the approximation of $\int_0^6 f(x)dx$ using a midpoint Riemann sum with three equal subintervals.

Let $S$ be the approximation of $\int_0^6 f(x)dx$ using Simpson's rule with six equal subintervals.

Find $R + L + 5M - 6S$.

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**Solution:**

The best way to approach this question is to note that if $T$ is the trapezoidal approximation with three equal subintervals, then $3S = 2M + T \rightarrow 6S = 4M + 2T$. Furthermore, $2T = R + L$. Therefore $6S = 4M + R + L$ so $R + L + 5M - 6S = M$. So we only need to find $M = 2\big(f(1) + f(3) + f(5)\big) = 2(23 + 63 + 73) = 2(159) = \boxed{318}$

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Question #12

# #12 Mu School Bowl
# MAΘ National Convention 2023

Let $A = \lim_{x\to 3} \frac{x^3-3x^2+4x-12}{x^2+x-12}$

Let $B = \lim_{x\to\infty} \frac{x^2-3x^3+4x-12}{x^2+x^3-12}$

Let $C = \lim_{x\to 0}(1+2023x)^{\frac{2}{x}}$

Let $D = \lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$

Find $70A + B + \ln(C) + D$

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**Solution:**

$A = \lim_{x\to 3} \frac{x^3-3x^2+4x-12}{x^2+x-12} = \lim_{x\to 3} \frac{(x-3)(x^2+4)}{(x-3)(x+4)} = \frac{13}{7}.$

$B = \lim_{x\to\infty} \frac{x^2-3x^3+4x-12}{x^2+x^3-12} = -3.$

$C = \lim_{x\to 0}(1+2023x)^{\frac{2}{x}} = \lim_{x\to\infty}\left(1+\frac{2023}{x}\right)^{2x} = e^{4046}.$

$D = \lim_{x\to 9} \frac{x-9}{\sqrt{x}-3} = \lim_{x\to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3} = \lim_{x\to 9}(\sqrt{x}+3) = 6.$

The final answer is $70\left(\frac{13}{7}\right) - 3 + 4046 + 6 = \boxed{4179}$

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## #13 Mu School Bowl
## MAΘ National Convention 2023

Let $A = \sum_{n=1}^{K} 2023$

Let $B = \sum_{n=1}^{2023} n$

Let $C = \sum_{n=1}^{2023} n^2$

Let $D = \sum_{n=1}^{2023} n^3$

Find the smallest positive value of $K$ so that $\gcd\left(A, \frac{BC}{D}\right) > 1$

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**Solution:**

$A = \sum_{n=1}^{K} 2023 = 2023K = 7 \cdot 17^2 \cdot K.$

$B = \sum_{n=1}^{2023} n = \frac{2023(2024)}{2}.$

$C = \sum_{n=1}^{2023} n^2 = \frac{2023(2024)(4047)}{6}.$

$D = \sum_{n=1}^{2023} n^3 = \frac{2023^2 2024^2}{4}.$

Since $\frac{BC}{D} = \frac{\left(\frac{2023(2024)}{2}\right)\left(\frac{2023(2024)(4047)}{6}\right)}{\frac{2023^2 2024^2}{4}} = \frac{4047}{3} = 1349 = 19 * 71$, $K = \boxed{19}$ will be the smallest positive value of $K$ so that $A$ is not relatively prime to $\frac{BC}{D}$.

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The partial fraction decomposition of $\frac{25}{(x-2)^2(x^2+1)}$ is $\frac{Ax+B}{x^2+1}+\frac{C}{x-2}+\frac{D}{(x-2)^2}$.

Find $A+B+C+D$.

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**Solution:**

$\frac{25}{(x-2)^2(x^2+1)}=\frac{Ax+B}{x^2+1}+\frac{C}{x-2}+\frac{D}{(x-2)^2} \rightarrow 25=(Ax+B)(x-2)^2+C(x-2)(x^2+1)+D(x^2+1)$

When $x=2, 25=5D \rightarrow D=5$. When $x=0, 25=4B-2C+5 \rightarrow C=2B-10$. When $x=1,\ 25=A+B+(2B-10)(-1)(2)+5(2) \rightarrow 25=A-3B+30 \rightarrow -5=A-3B \rightarrow -35=7A-21B$. When $x=3,\ 25=3A+B+(2B-10)(1)(10)+5(10) \rightarrow 25=3A+21B-50 \rightarrow 75=3A+21B$.

Adding the last two together gives $40=10A \rightarrow A=4$ which gives $-5=4-3B \rightarrow B=3$ which gives $C=-4$. Adding them together results in $4+3-4+5=\boxed{8}$

**ANSWERS**

0. 6
1. 15
2. 2026
3. $2e - e^4$
4. 145
5. $3e\sqrt{2} + 5$
6. $42\sqrt{2} + 448$
7. $-\frac{\pi}{3}$
8. $13$
9. $\frac{12}{47}$
10. 284
11. 318
12. 4179
13. 19
14. 8