For all questions, answer E. NOTA means none of the above answers is correct. The symbol $[ABC]$ signifies the area of polygon ABC. Good luck!

1. Find the number of terminating zeroes when $2022!$ in base 10 is expressed in base 12.

A. 182 B. 503 C. 670 D. 1007 E. NOTA

2. 3 circles of radius 1 are arranged such that each pair is externally tangent. How many ways are there to color these circles red, green, orange, and blue? Rotations and reflections of the same coloring are considered identical, and a color can be used more than once.

A. 10 B. 20 C. 27 D. 64 E. NOTA

3. Farmer Sam, located at the point (1,4), is trying to get water for his pet squirrel, located at the point (6,5). Unfortunately, the squirrel is very picky, and requires water from a lake that goes along $y = -x + 1$. Find the square of the minimum distance Farmer Sam needs to travel to get water from the lake and give it to his squirrel.

A. 26 B. 106 C. $90 + 48\sqrt{2}$ D. $\frac{1274}{9}$ E. NOTA

4. Given $\sin(x) + \cos(x) = \frac{5}{4}$, find $\sin^3(x) + \cos^3(x)$.

A. $\frac{10}{64}$ B. $\frac{1}{2}$ C. $\frac{35}{64}$ D. $\frac{115}{128}$ E. NOTA

5. Find the magnitude of $\sum_{n=0}^{\infty}\left(4 \cdot \left(\frac{1+2i}{3}\right)^n\right)$, where $i = \sqrt{-1}$.

A. 4 B. 5 C. $3\sqrt{2}$ D. $9 + 3\sqrt{5}$ E. NOTA

6. Find the sum of the cubes of the roots of $P(x) = x^3 - 7x^2 - 9x + 12$.

A. 343 B. 469 C. 496 D. 520 E. NOTA

7. Triangle ABC has $AB = 13$, $BC = 14$, and $CA = 15$, and let H be the orthocenter of ABC. The radius of the circle that passes through the midpoints of AH, BH, and CH can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

A. 81 B. 65 C. 50 D. 47 E. NOTA

8. Given that $3x + 2y = 16$, with $x, y > 0$, find the maximum possible value of xy.

A. 8 B. $\frac{256}{25}$ C. $\frac{32}{3}$ D. 16 E. NOTA

9. Given that $3x + 2y = 16$, with $x, y > 0$, find the maximum possible value of x^3y.

A. 64 B. $\frac{65536}{625}$ C. $\frac{2048}{27}$ D. 128 E. NOTA

10. Helena loves dogs! She currently lives with 9 dogs – dachshund twins, border collie triplets, and golden retriever quadruplets. She is planning on taking them on a road trip, in a van with her in the front, and 3 rows of 3 in the back. How many ways are there to arrange the dogs, given that each breed's members are indistinguishable, and the dachshunds are in different rows?

A. 315 B. 420 C. 945 D. 1260 E. NOTA

11. Let x and y be real numbers such that $x^2 + y^2 = 50$. Find the minimum value of xy.

A. 0 B. -7 C. -12 D. -25 E. NOTA

12. Consider the graph of $f(x) = \sqrt[3]{x - 6}$. Find the x-coordinate of the point on the graph of $f(x)$ that is closest to the graph of $f^{-1}(x)$.

A. -6 B. -3 C. $6 - 3\sqrt{3}$ D. $6 + 3\sqrt{3}$ E. NOTA

13. Find the area of the region bounded by the locus of points in the complex plane that satisfy $|z-6|+|z+8i|=26$.

A. 169π B. 156π C. 144π D. 144 E. NOTA

14. Find the maximum value of a 2×2 determinant of a matrix with the following distinct entries:

- The third pentagonal number
- The sum of the squares of the first 4 positive integers
- The largest perfect number less than 100 (defined as a number that is the sum of its proper divisors)
- The sum of the first 4 triangular numbers

A. 600 B. 240 C. -200 D. 1080 E. NOTA

15. How many positive integers less than 2100 have the property that it is divisible by 3 or 5 but not 7?

A. 839 B. 840 C. 959 D. 960 E. NOTA

16. Let $f(x)$ be the polynomial function with the minimum degree that satisfies $f(0)=-2$, $f(1)=1, f(2)=4, f(3)=13$, and $f(4)=34$. Compute $\int_0^4 f(x)\,dx$.

A. 16 B. 32 C. 34 D. 52 E. NOTA

17. How many values of x satisfy $\{x\}x^2=\lfloor x\rfloor$ in the interval $1\le x\le 2022$? $\{x\}$ is the fractional part of x and $\lfloor x\rfloor$ is the greatest integer less than x.

A. 2020 B. 2021 C. 2022 D. 2023 E. NOTA

18. How many ways are there to arrange the letters in OLIVIALEE such that no two adjacent letters are the same? ILOVEALEI is one acceptable permutation, but OLIVIALEE is not.

A. 32738 B. 21960 C. 15768 D. 5040 E. NOTA

For questions 19 – 21, consider a point P inside square $ABCD$ such that $BP = 4$, $CP = 1$, and $DP = \sqrt{14}$.

19. Find the length of AP.

A. 4 B. 5 C. $\sqrt{29}$ D. $3 + \sqrt{14}$ E. NOTA

20. Find $[CDP]$. (Hint: $\angle BCP + \angle DCP = 90°$. Try moving $\triangle BCP$ somewhere else to make another right angle!)

A. $\sqrt{14}$ B. $\frac{\sqrt{14}}{2}$ C. $\sqrt{7}$ D. $\frac{\sqrt{7}}{2}$ E. NOTA

21. Draw square $ABCD$ and draw every segment connecting P to the vertices. An ant starts at A, and every minute walks to a point connected to it through a drawn segment. In how many ways can the ant get to C in exactly 3 minutes? (For example, after one minute, the ant can be on B, D, or P, but not C, as segment AC is not drawn.)

A. 4 B. 8 C. 12 D. 16 E. NOTA

22. Three distinct single digit positive integers, $A, B,$ and $C,$ are chosen at random with equal probability. Find the expected value of $C - A$.

A. 2 B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. 0 E. NOTA

23. Three single digit positive integers, A, B, and C, with $A < B < C$, are chosen at random with equal probability. Find the expected value of $C - A$.

A. 5 B. $\frac{16}{3}$ C. $\frac{80}{21}$ D. 3 E. NOTA

24. Simplify : $\frac{(a^2-3^2-c^2)^2-4(3c)^2}{(a^2-c^2-6a+9)(a^2+3a+3c-c^2)}$

A. 1 B. $\frac{a+c+3}{a+c}$ C. $\frac{a-c+3}{a-c}$ D. $\frac{a+c-9}{a+c+9}$ E. NOTA

25. An isosceles triangle has a perimeter of 50. The sum of the length of the base and the height is 31. What is the sum of all the possible values for the height of this triangle?
A. 15 B. 25 C. 31 D. 50 E. NOTA

26. Hexagon $SQUARE$ is equiangular, such that $SQ = 4$, $QU = 6$, $UA = 8$, and $ES = 10$. Find $[SQUARE]$.
A. $54\sqrt{3}$ B. $57\sqrt{3}$ C. $60\sqrt{3}$ D. $63\sqrt{3}$ E. NOTA

27. Point X is 9 units from the center of a circle of diameter 30. How many different chords of the circle contain X and have integer lengths?
A. 3 B. 6 C. 7 D. 12 E. NOTA

28. Consider the graph of $f(x) = x^2$. Points A and B are randomly selected on the graph of $f(x)$, where each x-coordinated is selected uniformly on $[0, 6]$. If O is the origin, compute the expected area of ΔBOA.
A. 6 B. $\frac{36}{5}$ C. 12 D. $\frac{72}{5}$ E. NOTA

29. Let $\{A_0, A_1, A_2, A_3, \ldots\}$ be a sequence that satisfies $A_n = 5A_{n-1} - 6A_{n-2} + 6$ for $n \geq 2$. Given that $A_0 = 12$ and $A_1 = 26$, find the remainder when A_{2023} is divided by 10.
A. 0 B. 2 C. 4 D. 6 E. NOTA

30. Consider a circle with 4 equally spaced points. Jeremy starts at one of these points and moves to a point adjacent to the point he is on with equal probability every second. Find the probability that after 2022 seconds Jeremy is in the same point as the beginning.
A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. 1 E. NOTA