$[ABC]$ represents the area of ABC.

Good Luck!

1. Find the greatest prime number less than or equal to 2024. (I assure you, one of the answer choices is correct.)

   A. 2015 B. 2017 C. 2019 D. 2021 E. 2023

2. In Andrew's math class, homework is 20% of the grade, quizzes are 30%, participation is 10%, and the midterm and finals are 20% each. So far, Andrew has gotten a 90% in homework, 70% on the quizzes, 100% in participation and an 85% on the midterm. Given the final is graded as a whole number out of 100, what is the minimum score Andrew needs on the final to get at least a B (80%) in the class?

   A. 65 B. 70 C. 75 D. He can't E. NOTA

3. The maximum of the following expression for $\{A, B, C, D, E\} = \{3, 4, 5, 6, 7\}$ can be expressed as $\frac{m}{n}$ in simplest form.

$$\frac{A \cdot B - C}{D} + E$$

   Find $m + n$.

   A. 54 B. 55 C. 56 D. 57 E. NOTA

4. Triangle ABC has side lengths $AB = 10,\ AC = 6.$ Let points D, E lie on side BC such that AD is an angle bisector and AE is a median. If $DE = \frac{3}{2}$, what is BC?

   A. 8 B. 9 C. 10 D. 12 E. NOTA

5. Odd function $f(x)$ and even function $g(x)$ satisfy

$$f(x) + g(x) = \left(x + \frac{1}{x + 1}\right) 3^x.$$

   The value of $f(2) = \frac{m}{n}$ in simplest form. Find $m + n$.

   A. 34 B. 35 C. 36 D. 37 E. NOTA

6. Katie realizes that Mimo, her cat, is aging differently from her. Calculating a "cat age" is as following. The first cat year is 15 human years, the second cat year is 9 human years, and starting from the third cat year, each cat year is 4 human years. For example, when a cat is 4 "cat years old," it is 15 + 9 + 4 + 4 = 32 "human years old." Given Katie was exactly 16 when Mimo was just born (0 years old), how old is Katie when Mimo's human age is exactly twice Katie's human age? (Assume that cat ages/human ages are always whole numbers)

A. 22 B. 24 C. 26 D. 28 E. NOTA

7. Erick has a lot of Pandas. If he groups them in groups of 6, one will be left out. If he groups them in groups of 7, two will be left out. If he groups them in groups of 11, 6 will be left out. If he has less than 500 Pandas, how many does he have?

A. 289 B. 457 C. 467 D. 489 E. NOTA

8. The expected product of two dice rolls (Assume that the dice are both fair, six-sided dice) can be written as $\frac{m}{n}$ in simplest form. What is $m+n$?

A. 49 B. 51 C. 53 D. 55 E. NOTA

9. Find the remainder when $1+12+123+\cdots+123456789$ is divided by 9.

A. 3 B. 4 C. 5 D. 6 E. NOTA

For $10-11$, let $f(x)=\frac{4x^2+8x+7}{2x+1}$.

10. Find $[f(2024)]$ where $[x]$ is the greatest integer less than or equal to x.

A. 4047 B. 4048 C. 4049 D. 4050 E. NOTA

11. What is the smallest positive value in the range of $f(x)$?

A. $\frac{17}{3}$ B. 6 C. $\frac{19}{3}$ D. $\frac{20}{3}$ E. NOTA

12. A right triangle with 2-digit integer side lengths has the property that the length of the hypotenuse has reversed digits of a length of a leg. What is the perimeter of the triangle?

A. 154 B. 156 C. 158 D. 160 E. NOTA

13. The graph of the following in the xy-plane exhibits a

$$y = \sqrt{-x^2 + 4x - 3} + \sqrt{-x^2 + 7x - 12}$$

A. semi-ellipse B. point C. semi-circle D. hyperbola E. NOTA

14. Erick has 3 pandas, who take $\log_2(2), \log_3(2)$, and $\log_4(2)$ hours each to eat 1 bamboo treat. Let K be the number of hours it takes for them to together eat 10 bamboo treats. Which of the following is closest to K? (Note that $\log_{10} 2 \approx 0.3010$ and $\log_{10} 3 \approx 0.4771$)

A. 1.6 B. 1.8 C. 2.0 D. 2.2 E. NOTA

15. Let n be the smallest positive integer such that adding a '1' in front of the number (i.e. $35 \to 135$) is equivalent to $8n + 1$. Find the sum of the digits of n.

A. 12 B. 18 C. 24 D. 27 E. NOTA

16. Prime numbers a, b, c satisfy

$$a + b + c = 22,$$
$$ab + bc + ca = 131.$$

Find the sum of the digits of abc.

A. 7 B. 8 C. 10 D. 11 E. NOTA

17. What is the number of not necessarily distinct prime factors of $2^{20} - 2^{11} + 1$? For example, $12 = 2 \cdot 2 \cdot 3$ has 3 not necessarily distinct prime factors. (Hint: try to factor the equation)

A. 6 B. 8 C. 9 D. 10 E. NOTA

18. Given that the 3 lines $x + 2y = 5, 2x + y = 4, 4x + ky = 6$ split the xy-plane into 6 regions, compute the sum of all possible values for k.

A. 1 B. 10 C. 11 D. 12 E. NOTA

19. Triangle ABC has angle bisector AD. The line parallel to AD through B intersects line AC at point E. If $BD = 8, CD = 6, BE = 14$, what is AC^2?

A. 63 B. 72 C. 108 D. 135 E. NOTA

20. The sequence A_n is recursively defined as the following. $A_1 = 2, A_n = 4A_{n-1} - 1$. A_n can be written as $p \cdot q^n + r$ for all positive integers n. $p + q + r$ can be written as $\frac{m}{n}$ in simplest form. Find $m + n$.

A. 11 B. 12 C. 13 D. 23 E. NOTA

21. Let r be a root of $x^2 - 4x + 1 = 0$. What is $r^4 + r^{-4}$?

A. 194 B. 224 C. 225 D. 256 E. NOTA

22. Consider a sequence of five positive integers a_0, a_1, a_2, a_3, a_4 that satisfy the conditions $a_0 = 1, a_4 = 2^3 \cdot 3^5$, and a_{i+1} is divisible by a_i for all $0 \leq i \leq 3$. Find the number of possible sequences.

A. 840 B. 980 C. 1120 D. 1440 E. NOTA

23. The rotated ellipse $x^2 - 2xy + 2y^2 = 9$ is inscribed in rectangle $ABCD$, which has sides parallel to the x and y axes. $[ABCD] = p\sqrt{q}$ in simplest radical form. Find $p + q$.

A. 11 B. 19 C. 20 D. 38 E. NOTA

24. $\frac{5}{18_{10}} = 0.\overline{a_1 a_2 \ldots a_n}$ in base 7 where n is minimal.
Find $a_1 + a_2 + \cdots + a_n + n$ in base 10.

A. 10 B. 11 C. 12 D. 13 E. NOTA

25. Erick has 4 pandas in his back yard who are all awake. Every turn, he randomly chooses a panda and makes it fall asleep. If it is already asleep, nothing happens. The probability that all the pandas are asleep after 5 turns can be written as $\frac{a}{2^b}$ in simplest form. What is $a + b$?

A. 15 B. 17 C. 19 D. 21 E. NOTA

26. Erick wants to play with his pandas, so he bought a toy for them! He has three pandas, and he puts one on each of the vertices of equilateral triangle ABC. Then he goes to the circumcenter of ABC and places the toy randomly inside triangle ABC. When Erick blows his whistle, him and his 3 pandas run towards the toy. Given that Erick and all 3 of his pandas have equal speeds, the probability that Erick reaches the toy before any panda does can be written in the form $\frac{m}{n}$. Find $m + n$.

A. 3 B. 4 C. 5 D. 7 E. NOTA

27. Erick is playing a game with his 3 pandas. The pandas are holding the cards 0, 1, 2. Erick starts with the number 16. Every turn, he chooses a random card and multiplies his number by the card he picked. If the number he has is a perfect cube, Erick wins and the game ends. (Note that 0 is a perfect cube). Let $E = \frac{m}{n}$ be the expected number of turns it takes until the game ends, where m, n are relatively prime positive integers. Find $m + n$.

A. 5 B. 7 C. 11 D. 13 E. NOTA

28. The polynomial $f(x) = x^4 + ax^3 + bx^2 + ax + 1$ has 3 distinct positive real roots. Given $a + b = \frac{11}{5}$, the sum of the 3 distinct roots can be written in the form $\frac{m}{n}$ where m, n are relatively prime positive integers. Compute $m + n$.

A. 21 B. 22 C. 23 D. 24 E. NOTA

29. The value of $|x|$ that minimizes

$$f(x) = \sqrt{x^2 + 8x + 17} + \sqrt{x^2 - 4x + 53}$$

can be written in the form $\frac{m}{n}$ in simplest form. Find $m + n$.

A. 15 B. 16 C. 17 D. 18 E. NOTA

30. Congratulations for making it to the end of the test! Let's end with an easy question. How many integers become a 2-digit integer when squared?

A. 4 B. 5 C. 6 D. 7 E. NOTA