1. Evaluate

$$\lim_{x\to 0}\left(1+\frac{1}{x^2}\right)^{\tan x}$$

A. 1 B. e C. e^2 D. DNE E. NOTA

2. Compute

$$\lim_{x\to 0}\frac{\sqrt{x^4+16x^3}}{\sin^2 x}$$

A. 2 B. 4 C. ∞ D. DNE E. NOTA

3. If $f(x)$ satisfies the first order differential equation $f'(x)=x\ln f(x)$ and has initial condition $f(1)=2$, then compute the value of

$$\lim_{x\to 1}\frac{f(x)-2}{x^2-1}$$

A. $\ln\sqrt{2}$ B. $\ln 2$ C. $\ln 4$ D. DNE E. NOTA

4. Let $f(x)$ be a function such that $f^{(n)}(0)=n$ for all positive integers n and $f(0)=1$. You are given that

$$\lim_{x\to 0}\frac{f(x)-e^x}{x^k}=L$$

for a positive integer k and for a finite and nonzero real value L. Compute kL.

A. 1 B. 3 C. 6 D. DNE E. NOTA

5. Compute $\int_{-1}^{2}\frac{d}{dx}(|x|^2)\,dx$.

A. 1 B. 3 C. 5 D. Undefined E. NOTA

6. Compute

$$\lim_{x\to 0}\frac{\sin x\int_0^x \cos t^2\,dt}{x^2}$$

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 1 D. DNE E. NOTA

7. Compute the value of

$$\lim_{n\to\infty}\frac{1}{n}\lim_{x\to -1}\frac{\sum_{i=0}^{n}x^i}{1+x}$$

A. 0 B. $\frac{1}{2}$ C. 1 D. DNE E. NOTA

8. Compute

$$\int_0^1 \frac{1-2x\arctan x}{(1+x^2)^2}\,dx$$

A. $\frac{\pi^2}{36}$ B. $\frac{\pi}{8}$ C. $\frac{\pi^2}{16}$ D. $\frac{\pi}{4}$ E. NOTA

9. The line $y=k$ is tangent to the curve

$$f(x)=\left(\frac{1}{\arcsin x}+\frac{1}{\arccos x}\right)^{-1}$$

What is the minimum possible value of $[k]$, where $[x]$ is the greatest integer function of x?

A. -3 B. -1 C. 0 D. 2 E. NOTA

10. Compute $\lim_{x\to 0^+} x^{2x}$.

A. 0 B. 1 C. 2 D. 4 E. NOTA

11. Let $f(x)$ be a function such that $e^x f(x)+1=x^x$. What is the value of

$$\lim_{x\to 0} f(x)$$

A. 0 B. $\frac{1}{e}$ C. 1 D. DNE E. NOTA

12. The function $f(x)=x\ln^2 x$ has a local maxima, b local minima, and c points of inflection. What is $a+b+c$?

A. 2 B. 3 C. 4 D. 5 E. NOTA

13. A function $f(x)$ has 3rd degree Taylor Polynomial $P(x) = 1 - x^2 + 3x^3$ around $x = 0$. Which of the following describes $f(x)$ at $x = 0$?

A. Real root B. Local maximum
C. Local minimum D. Point of inflection E. NOTA

14. If $f^{-1}(x) = e^x + x - 1$, what is the value of $f'(0)$?

A. $\frac{1}{2}$ B. $\frac{1}{\ln 4}$ C. 1 D. $\frac{1}{\ln 2}$ E. NOTA

15. If $f(x) = \sum_{i=0}^{2022} x^i$, then compute the value of $[\log_2 f''(2)\ ]$, where $[x]$ is the greatest integer function of x.

A. 2022 B. 2032 C. 2042 D. 2052 E. NOTA

16. The line $y = a_1 x - 4$ is tangent to the graph of $f(x) = x^2 - a_2 x + 12$. What is the sum of all distinct possible values of $a_1 + a_2$?

A. -8 B. -4 C. 0 D. 8 E. NOTA

17. Let $T_n(x)$ be the nth order (n nonzero terms) Taylor polynomial approximation of the function $f(x) = xe^x$. Which of the following is closest to the value of

$$T'_{2023}(0.1) - T_{2023}(0.1)$$

A. 1.09 B. 1.10 C. 1.11 D. 1.12 E. NOTA

18. Compute the maximum possible value of $\sin x \cos^3 x + \cos x \sin^3 x$ over the interval $[0, \pi]$.

A. $\frac{1}{2}$ B. $\frac{\sqrt{2}}{2}$ C. $\frac{\sqrt{3}}{2}$ D. 1 E. NOTA

19. How many critical points does the function $f(x) = 1 + x \ln x + x \ln^2 x$ have for x a positive real number?

A. 0 B. 1 C. 2 D. 3 E. NOTA

20. Which of the following intervals contains a root of $f(x) = x^3 + x - 2023$?

A. $[10, 11)$ B. $[11, 12)$ C. $[12,13)$ D. $[13,14)$ E. NOTA

21. Caroline selects n (not necessarily distinct) positive integers with sum equal to 50. If her intent was to maximize the product of these integers, what is the value of n?

A. 10 B. 17 C. 18 D. 25 E. NOTA

22. Let $f(x) = xe^x$. For what value of x is the quantity $f(x) + f'(x)$ minimized?

A. -2 B. $-\frac{3}{2}$ C. -1 D. $-\frac{1}{2}$ E. NOTA

23. The tangent line to the curve $x^2 + y^2 = 1$ at $(\frac{3}{5}, \frac{4}{5})$ intersects the ellipse $25x^2 + y^2 = 25$ at a point in the first quadrant (m, n). Compute the value of $3m + 4n$.

A. 3 B. 4 C. 5 D. 6 E. NOTA

24. A hot air balloon currently 100 feet in the air is floating up at a rate of 20 feet per second. From an observing point on the ground 75 feet horizontally away, what is the rate of change of the angle of observation from the ground to the balloon, in radians per second?

A. $\frac{2}{25}$ B. $\frac{12}{125}$ C. $\frac{3}{25}$ D. $\frac{17}{125}$ E. NOTA

25. The polar graph $r = 1 + \sin\theta$ is plotted in the Cartesian plane. What is the minimum value of any y coordinate on this graph?

A. $-\frac{1}{2}$ B. $-\frac{1}{4}$ C. $-\frac{1}{8}$ D. $-\frac{1}{16}$ E. NOTA

26. The solution to the differential equation $2xy\frac{dy}{dx} + y^2 = \cos x$ containing the point $(0,1)$ also contains which of the following points?

A. $(0, -1)$ B. $\left(\frac{\pi}{2}, -1\right)$ C. $\left(-\frac{\pi}{2}, 1\right)$ D. $\left(\frac{\pi}{2}, 1\right)$ E. NOTA

27. The graph of the function $f(x) = 1 - 2x + x^3$ is rotated by $\frac{\pi}{4}$ counterclockwise. How many critical points does this rotated graph contain?

A. 0 B. 1 C. 2 D. 3 E. NOTA

For questions 28-30, let $f(x)$ be a function with power series representation

$$f(x) = \sum_{n=0}^{\infty} \frac{a_n}{n+1} x^n$$

Where $\{a_n\}$ is a sequence of real numbers indexed by the non-negative integers.

28. For this question only, assume that the interval of convergence of $f(x)$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$. Compute the value of

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|.$$

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 1 D. 2 E. NOTA

29. For this question only, let $a_n = F_n$, where $\{F_n\}$ is the sequence of Fibonacci numbers satisfying $F_0 = 0,\ F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Compute

$$\lim_{x\to 0} \frac{1 - \cos x}{f(2x) - x}$$

A. $\frac{3}{8}$ B. $\frac{1}{2}$ C. $\frac{5}{8}$ D. $\frac{3}{4}$ E. NOTA

30. For this question only, let $f(0) = 1$ and let $f(x)$ satisfy the first order differential equation $f'(x) = f(x)$. What is the value of a_3?

A. $\frac{2}{3}$ B. 1 C. $\frac{3}{2}$ D. 2 E. NOTA