

Answer choice “E. NOTA” means “None of the Above” answers are correct. Good luck!

1. Evaluate $\lim_{x \rightarrow -\infty} (\sqrt[3]{3x^2 + x^3 + 37} - \sqrt[3]{6x^2 + x^3 + 65})$.
A. 1 B. -1 C. 0 D. DNE E. NOTA
2. Evaluate $\lim_{n \rightarrow \infty} \frac{n!e^n}{n^{n+\frac{1}{2}}}$.
A. $\sqrt{2\pi}$ B. $\sqrt{\pi}$ C. π D. DNE E. NOTA
3. Approximate $33^{\frac{1}{5}}$ using the tangent line to $y = \sqrt[5]{x}$ at $x = 32$.
A. $\frac{43}{5}$ B. $\frac{81}{40}$ C. $\frac{161}{80}$ D. $\frac{159}{80}$ E. NOTA
4. If $y = \sqrt{2x + \sqrt{2x + \sqrt{2x + \dots}}}$, find y' at $x = \frac{3}{8}$.
A. 0 B. 1 C. $\frac{2}{3}$ D. $\frac{1}{2}$ E. NOTA
5. Evaluate $\lim_{x \rightarrow \infty} \frac{2(\sin x)^3}{\cos x + x^3 - 1}$.
A. 0 B. 1 C. 2 D. $\frac{2}{3}$ E. NOTA
6. Using two iterations of Newton's Method and initial point $x_0 = 1$, find an approximation of a root of $y = 5x^4 - 8x^3$.
A. $\frac{1}{4}$ B. $\frac{49}{304}$ C. $\frac{8}{5}$ D. $\frac{202321}{1920064}$ E. NOTA

7. Find the tangent line to $y = 3x \sin^{-1} x$ at $x = \frac{1}{2}$.
- A. $y - \frac{\pi}{4} = \left(\frac{\pi}{2} + \sqrt{3}\right)x - \frac{\sqrt{3}}{2}$ B. $y = \left(\frac{\pi}{2} + \sqrt{3}\right)\left(x - \frac{1}{2}\right)$
C. $y = \left(\frac{\pi}{2} + \sqrt{3}\right)x - \frac{\sqrt{3}}{2}$ D. $y = \left(\frac{\pi}{2} + \sqrt{3}\right)x - \frac{\sqrt{3}}{2} - \frac{\pi}{2}$ E. NOTA
8. Evaluate $\lim_{x \rightarrow 0^+} (2024 \sin x)^{\tan x}$.
- A. 1 B. 0 C. e D. 2024 E. NOTA
9. Find the tangent line to $y = x^3 + x^2 + x + 1$ at the point $(1, 4)$.
- A. $y = 6x - 2$ B. $y = -6x + 2$
C. $y = 6x - 10$ D. $y = -6x + 10$ E. NOTA
10. Using the third-degree Taylor polynomial centered at $x = 0$, estimate the value of $f\left(\frac{1}{10}\right)$, if $f(x) = \tan^{-1} x$.
- A. $\frac{29}{300}$ B. $\frac{299}{300}$ C. $\frac{299}{3000}$ D. $\frac{2999}{3000}$ E. NOTA
11. If a particular company must produce at least 200 bottles of juice, and the cost of the company in relation to the bottles of juice produced is determined by the function $C(x) = 2a^3 - 15a^2 + 36a + 2024$, where a is the number of bottles of juice in addition to the minimum production, find the number of bottles that would minimize the company's cost.
- A. 202 B. 203 C. 200 D. 3 E. NOTA
12. Evaluate $\lim_{i \rightarrow \infty} \sum_{n=1}^i \frac{1}{3n+6i}$.
- A. $\frac{1}{3} \ln \frac{1}{3}$ B. $\frac{1}{6} \ln 3$ C. $\frac{1}{3} \ln \frac{2}{3}$ D. $\frac{1}{3} \ln \frac{3}{2}$ E. NOTA

13. Find the radius of convergence of the Maclaurin series of the of the function $y = \tan x$.
- A. $\frac{\pi}{2}$ B. π C. 1 D. Always converges E. NOTA
14. Evaluate $\lim_{x \rightarrow 0} \frac{\int_0^{\sin x} e^t \sinh t \, dt}{2x}$.
- A. 0 B. $\frac{1}{2}$ C. 1 D. DNE E. NOTA
15. Find $\lim_{h \rightarrow 0} \frac{f\left(\frac{\sqrt{2}}{2}-2h\right)-f\left(\frac{\sqrt{2}}{2}+3h\right)}{h}$, if $f(x) = 4 \sin^{-1}(x^2)$.
- A. $-40\sqrt{6}$ B. $40\sqrt{6}$ C. $-\frac{40\sqrt{6}}{3}$ D. $\frac{40\sqrt{6}}{3}$ E. NOTA
16. Sam is admiring the painting of Mona Lisa, but he does not realize. The 4-meter tall painting hangs on a wall, with the bottom of the painting 2 meters off the ground. When Sam is standing, his eyes are 150 centimeters above the ground. How far from the wall (in meters) should he stand to maximize his viewing angle of the painting and its mysterious smile?
- A. $\sqrt{6}$ B. $\frac{3}{2}$ C. $\sqrt{2}$ D. $3\sqrt{2}$ E. NOTA
17. Find $\frac{dx}{dy}$ at $(1, -2)$, if $2x^2 + y^2 = -3xy$.
- A. -2 B. 2 C. $\frac{1}{2}$ D. $-\frac{1}{2}$ E. NOTA
18. For $r = \cot \theta + \theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.
- A. $-\frac{\pi}{8+2\pi^2}$ B. $\frac{\pi}{4+\pi^2}$ C. $\frac{-\pi}{\pi+8}$ D. $-\frac{\pi}{2}$ E. NOTA

19. Find $\frac{d^2y}{dx^2}$ at $x = 1$, if $x^{\frac{5}{2}} + y^{\frac{5}{2}} = 2$.
- A. -3 B. $-\frac{3}{2}$ C. 3 D. 0 E. NOTA
20. Let $f(x) = \sqrt{x + \sqrt{x + 3}}$, find $f'(6)$.
- A. $\frac{1}{36}$ B. $\frac{7}{6}$ C. $\frac{1}{6}$ D. $\frac{7}{36}$ E. NOTA
21. If the juice mentioned in a question 11 is produced in bottles with shapes of inverted cones, find the rate of increase in volume of juice in the bottles, in cubic inches per second, when the depth of the juice is 3 inches and increasing at 1 foot per minute. EZ examined the bottles and discovered the bottles have a diameter of 1 foot and a depth of 2 feet.
- A. 9π B. $\frac{9\pi}{20}$ C. $\frac{9\pi}{80}$ D. $\frac{9\pi}{16}$ E. NOTA
22. Evaluate $\lim_{x \rightarrow 0} (\sqrt{x^3 + \sqrt{x}} \cdot \cos(\pi x))$.
- A. 0 B. 1 C. π D. DNE E. NOTA
23. Evaluate $\lim_{x \rightarrow 27} \frac{x-27}{\sqrt[3]{x}-3}$.
- A. 27 B. 9 C. $\frac{1}{3}$ D. 3 E. NOTA
24. Using the epsilon-delta definition of a limit for the limit $\lim_{x \rightarrow 4} (5x + 4) = 24$ and $\varepsilon = \frac{1}{2024}$, find the least upper bound for δ that works for the given value of ε .
- A. $\frac{1}{2024}$ B. $\frac{1}{10120}$ C. $\frac{5}{2024}$ D. $-\frac{5}{2024}$ E. NOTA

25. Evaluate $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3n^2i - 7ni^2 + 1}{n^4}$.
- A. $\frac{1}{6}$ B. $-\frac{1}{6}$ C. $\frac{1}{3}$ D. $-\frac{5}{6}$ E. NOTA
26. A shape is defined by the parametric equations: $x = t^3 + t^2 - 2$ and $y = \ln t$. What is the value of $\frac{d^2y}{dx^2}$ at $t = 1$?
- A. $-\frac{13}{25}$ B. $-\frac{13}{125}$ C. $\frac{1}{5}$ D. $\frac{13}{5}$ E. NOTA
27. Using four iterations of Euler's method, estimate $y(1)$ using four equal intervals using a starting point of $(0, 1)$, if $\frac{dy}{dx} = 2y + x$.
- A. $\frac{3}{2}$ B. $\frac{115}{32}$ C. $\frac{357}{64}$ D. $\frac{179}{32}$ E. NOTA
28. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 + 2x^2y^2 + y^4}{x^2 + y^2 + x^4y^4}$.
- A. 0 B. 1 C. $\frac{4}{3}$ D. DNE E. NOTA
29. Find $f'(1)$, if $f(x) = \frac{\sqrt{x}e^{\sqrt{x+3}}}{\tan x}$.
- A. $\left(\frac{e^2}{\tan 1}\right)\left(\frac{1}{2} + \frac{(\sec 1)^2}{\tan 1}\right)$ B. $\left(\frac{e^2}{\tan 1}\right)\left(\frac{3}{4} + \frac{(\sec 1)^2}{\tan 1}\right)$
C. $\left(\frac{e^2}{\tan 1}\right)\left(\frac{3}{4} - \frac{(\sec 1)^2}{\tan 1}\right)$ D. $\left(\frac{e^2}{\tan 1}\right)\left(\frac{1}{2} - \frac{(\sec 1)^2}{\tan 1}\right)$ E. NOTA
30. Find $f'\left(\frac{1}{2}\right)$, if $f(x) = \ln(1 + 4x^2)$.
- A. 2 B. $\frac{1}{2}$ C. 1 D. 4 E. NOTA