1. A We can rewrite the limit as $\lim_{x\to 0} e^{\tan x \ln(1+\frac{1}{x^2})} = e^{\frac{\ln\left(1+\frac{1}{x^2}\right)}{\cot x}}$, and by L'Hopital on the exponent, we find that it has limit 0, which implies that the answer is 1.
2. D We can verify that the limit exists from the positive side, but does not on the negative side, so the limit itself does not exist.
3. A We first check and see that the direct substitution results in 0/0. We then apply L'Hospitals to get $\frac{f'(x)}{2x} = \frac{\ln f(x)}{2}$ as the limit expression. Since $f(1) = 2$, we see the limit is $\ln\sqrt{2}$.
4. A We see that the lowest power term that exists in the difference between Taylor expansions of f and e^x is the x^2 term, where f has a $\frac{2}{2} = 1$ coefficient and e^x has a $\frac{1}{2}$ coefficient. So $k = 2$ and the limit comes out to $1 - \frac{1}{2} = \frac{1}{2}$. This makes the product 1.
5. B Note that $|x|^2 = x^2$, so this is just $(2^2) - (-1)^2 = 3$.
6. C We can either L'Hopital's twice, or we can use Taylor Expansions and compare the x^2 terms. Doing this, we see that $\sin x = x - O(x^3)$ and $\int_0^x \cos x^2\, dx = x - O(x^5)$, so $\sin x \int_0^x \cos x^2\, dx = x^2 - O(x^4)$, which makes the limit 1 by the ratio of coefficients.
7. D We see that this limit has drastically different behaviors for odd n and even n, so this limit will not converge.
8. B Recognize the numerator of the integral as a quotient rule expression, where $(1 + x^2)f'(x) - 2xf(x) = 1 - 2x \arctan x \Rightarrow f(x) = \arctan x$. Therefore, this integral is $\frac{\arctan x}{x^2+1}$ evaluated at 1 and 0, or $\frac{\pi}{8} - 0 = \frac{\pi}{8}$.
9. C We can rewrite this as $\left(\frac{\arcsin x + \arccos x}{\arcsin x \arccos x}\right)^{-1} = \left(\frac{\pi}{2 \arcsin x \arccos x}\right)^{-1} = \frac{2}{\pi} \arcsin x \arccos x$. We can differentiate to get $\frac{2}{\pi}\frac{1}{\sqrt{1-x^2}} \arccos x - \frac{2}{\pi}\frac{1}{\sqrt{1-x^2}} \arcsin x$. To find horizontal tangents, we need this to be 0, or $\arccos x = \arcsin x$, which occurs only at $x = \frac{1}{\sqrt{2}}$. The function at this value is $\frac{\pi}{8}$, which has floor 0.
10. B We can use a similar trick as in question 1 to see that the limit is 1, but only from this direction.
11. D We are effectively trying to compute $\lim_{x\to 0} \frac{x^x - 1}{e^x}$, but the limit of x^x does not exist from the negative direction, so this does not exist.
12. B $f'(x) = \ln^2 x + 2 \ln x \Rightarrow f''(x) = \frac{2 \ln x}{x} + \frac{2}{x}$. f' changes signs at $\ln x = 0, \ln x = -2$, so these are mins/maxes, and f'' changes signs at $\ln x = -1$, so this is an inflection point.
13. B The first derivative is 0 and the second derivative is negative from the Taylor polynomial, so the point is a local maximum.
14. A If we apply the function f to both sides we get $x = f(e^x + x - 1) \Rightarrow 1 = f'(e^x + x - 1)(e^x + 1)$. Note that $x = 0$ is the only root to $e^x + x - 1 = 0$ (verify since its derivative is $e^x + 1 > 0$, so it can only have one root), so we have $1 = 2f'(0) \Rightarrow f'(0) = \frac{1}{2}$.

15. C Note that $(1-x)f(x) = 1 - x^{2023} \Rightarrow f(2) = 2^{2023} - 1$. We also know by taking the derivative of the expression that $(1-x)f'(x) - f(x) = -2023x^{2022} \Rightarrow f(2) + f'(2) = 2023 \cdot 2^{2022}$. We can differentiate again to get that $(1-x)f''(x) - 2f'(x) = -2023 \cdot 2022x^{2021} \Rightarrow f''(2) + 2f'(2) = 2023 \cdot 2022 \cdot 2^{2021}$. Subtracting two of the second equation from the third gets $f''(2) - 2f(2) = 2023 \cdot 2022 \cdot 2^{2021} - 2023 \cdot 2^{2023} = 2023 \cdot 2014 \cdot 2^{2021}$. Adding two of the first equation gives $f''(2) = (2023 \cdot 2014 + 8) \cdot 2^{2021} - 2$, which makes the log floor $2021 + \log_2(2023 \cdot 2014 + 8) = 2021 + 21 = 2042$.

16. C We need the graph to look like a perfect square after taking away the linear approximation, so $x^2 - a_2x + 12 - (a_1x - 4) = x^2 - (a_1 + a_2)x + 16$ needs to be a square for this to be tangent. Therefore, $a_1 + a_2 = \pm 8$, which makes the sum 0.

17. C The 2023rd iteration will be very close to the true value. So $f'(x) - f(x) = (x+1)e^x - xe^x = e^x$. We need to evaluate $e^{0.1}$, so we will now use the Taylor polynomial to get that $e^{0.1} = 1 + 0.1 + 0.005 + 0.0003333.. + \cdots$, which is closer to 1.11 than it is to 1.10.

18. A We can factor this as $\sin x \cos x\,(\cos^2 x + \sin^2 x) = \sin x \cos x$ which has maximum value $\frac{1}{2}$.

19. C We can differentiate to get $f'(x) = \ln^2(x) + 3\ln(x) + 2 = (\ln(\mathrm{x}) + 1)(\ln(x) + 2)$. Our critical points are at $x = \frac{1}{e}$ and $x = \frac{1}{e^2}$ since the derivative is zero at both points and both points lie in the domain of $f(x)$. For positive x, there are 2 critical points.

20. C We can apply the IVT by noting that $f(12) < 0$ and $f(13) > 0$ and that f is a polynomial function so it is continuous.

21. B The continuous analogue of this problem (making assumption of equality) is to maximize the function $f(x) = x^{\frac{50}{x}}$, which one can verify to have global maximum value at $x = e$. We can check $x = 2$ and $x = 3$ to see which integer is better to use as the main source, we see that $2^{25} < 2 \cdot 3^{16}$ by taking the 8th root of both sides. Thus, $n = 1 + 16 = 17$.

22. B $f(x) + f'(x) = (2x+1)e^x$. We can differentiate to get the derivative of this expression is $(2x+3)e^x$, so the minimum occurs at $x = -\frac{3}{2}$.

23. C Note that the tangent line to the curve is $3x + 4y = 5$. This means that at any point on the line, $3x + 4y$ is constant, and so this intersection is no different and has value 5.

24. B
$$\tan\theta = \frac{x}{75} \Rightarrow \sec^2\theta \frac{d\theta}{dt} = \frac{1}{75}\frac{dx}{dt} \Rightarrow \frac{d\theta}{dt} = \frac{1}{75}\,20\,\frac{9}{25} = \frac{12}{125}.$$

25. B We can multiply both sides by r to get that $r^2 = r + r\sin\theta \Rightarrow y = r^2 - r$. We see by differentiating that the minimum y occurs at $r = \frac{1}{2}$, where $y = -\frac{1}{4}$. Note that r is and must be nonnegative.

26. A Note that the LHS is a product rule expression, so integrating out gives us that $xy^2 = \sin x + C$. Since it contains $(0,1)$, we know that $C = 0$. Therefore, we have that $(0,-1)$ is also contained.

27. E To find critical points of the rotated graph, we need points that have tangent lines that will be vertical or horizontal after rotation. In other words, points where the

slope is -1 or 1 in the old graph. Solving $-2+3x^2 = -1$ yields two solutions and $-2+3x^2 = 1$ yields two more distinct solutions, so the new graph would have 4 critical points.

28. D The radius of convergence is $\frac{1}{2}$, so this limit must be 2 by the ratio test.

29. A Here we can use up to the x^2 term in each power series to get that $\lim_{x\to 0}\frac{1-(1-\frac{x^2}{2}+O(x^4))}{\left(x+\frac{4x^2}{3}+O(x^3)\right)-x} = \frac{3}{8}$.

30. A The initial condition and differential equation imply that $f(x) = e^x$. Therefore, $\frac{a_n}{n+1} = \frac{1}{n!} \Rightarrow a_n = \frac{n+1}{n!}$, so $a_3 = \frac{4}{6} = \frac{2}{3}$.