

1. D Decompose the velocity vector into vertical and horizontal components. There is only acceleration in the negative vertical direction of magnitude g . The horizontal component is $v\cos(\Theta)$ and the vertical is $v\sin\Theta$. We need to first solve for how long the object is in the air. Using the definition of acceleration, we get that the time to reach the apex is $(v\sin\Theta)/g$. Multiplying by 2 to get the total air time, we get time = $(2 v\sin\Theta)/g$. Multiply this by the horizontal velocity to get $(2 v^2 \sin\Theta \cos\Theta)/g$, which is also equivalent to answer choice by double angle identity
2. A Plug into $1-(Q_{\text{cold}}/Q_{\text{hot}})$. $1-600/1000$ is $2/5$.
3. B The particle performs simple harmonic oscillation in both the x and y directions independently. If the period of oscillations in the y direction is T , then the period of oscillations in the x direction is $21T/20$. The minimum possible period is T , while the maximum possible period is the lowest common multiplier of T and $21T/20$, i.e. $21T$. Then the desired ratio is 21
4. C Charge is conserved since the battery is discharged. Capacitance is directly proportional to the reciprocal of the separation between plates. Capacitance thus is cut in half. Since capacitance is cut in half, voltage must double. Potential is capacitance times voltage squared over two. Potential doubles.
5. B Construct a picture as follows: Draw the 5 m/s vector and the 10 m/s vector such that the arrows point to the same point. Connect the tails of the vectors. This will represent the acceleration vector. Taking any point along the acceleration vector and connecting it to the point that connects the 5 and the 10 vector will depict the instantaneous velocity of the Chell at some point along the path. Thus, the smallest vector would be the altitude of the right triangle, or $2\sqrt{5}$ m/s
6. A This is a simple infinite series question. Using the reciprocal of the sum of reciprocal rule for parallel branches. We can say that $x = 2R + \frac{1}{\frac{1}{R} + \frac{1}{x}}$ so multiplying everything by $(R+x)$ and rearranging gives you $x^2 - 2Rx - 2R^2 = 0$ Solving for x , you get A.
7. B Since the process is adiabatic, the change in internal energy is equal to the work done on the gas. We know that work is $-PdV$, but P is related to V as a result of thermodynamic processes. We can take $PV=nRT$ to try to come up with the relationship, alongside the equation for the internal energy of a gas, $fPV/2$ or $fnRT/2$. Differentiating both equation, you get $dU=fnRdt/2=fd(PV)/2=f(PdV + VdP)/2$. Now we can implement our previous equations related to work and internal energy to get $-PdV=fPdV/2+fVdP/2$. Factor out a $-PdV$ after rearranging terms to get $-\frac{(f+2)dV}{2V} = \frac{f dP}{2P}$. Integrate both sides from the initial V and P to the final V and P to get $\ln\left(\frac{P(f)}{P(i)}\right) = -\frac{f+2}{f}\ln\left(\frac{V(f)}{V(i)}\right)$ and substitute in the adiabatic index and exponentiate both sides to get $\left(\frac{P(f)}{P(i)}\right) = \left(\frac{V(f)}{V(i)}\right)^{-a}$. Reciprocate and rearrange the terms to see that PV^a is a constant.

8. C Set up $m g \sin(\Theta) - \text{friction} = ma$. Torque is equal to both $I\alpha$ and Fr , where the F in this case would be friction. Solve for friction to be $I\alpha/r$, which is also equal to $\frac{\beta m r^2 a}{r^2}$. Moving the a terms to both sides and solving for a gives us answer choice C.
9. A Multiply the circumference of the hole before it expands by the coefficient of thermal expansion and the change in temperature to get its change 3.76π , then add that back to the original to get 203.76π so X rounds to 204.
10. B We can think of this as similar to how a spring works, thus we need to find the effective spring constant. The mass of block is proportional to ρh and the cross sectional area which will factor out. The buoyant force is equal to that area times $-\rho_0 g \Delta h$. So our analogous k constant is equal to area times $\rho_0 g$. Plugging this into the period for the simple harmonic motion of a spring simplifies to B.
11. E Definition: crackle has units m/s^5 so its derivative with respect to time is m/s^6
12. A Using $V_f^2 - V_i^2 = 2ad$, we can find what the final velocity of the ball is when it hits the target. Substituting this in, we find that the final velocity is 20 m/s. This means the ball has been in the air for 3 seconds (using the definition of acceleration). Thus the ball has traveled 150 meters horizontally. Since the ball collides elastically with the target, the ball's path has essentially been flipped in the horizontal direction. So we find the total range and subtract out the 150 meters already traveled. We solved for the equation in question 1, so we plug in the values to get that the total range is 500 meters. Thus the ball travels another 350 meters, stopping 200 meters behind Kejin.
13. C In order for the average speed to clock out at $50/3$ m/s, the car needs to travel 10 meters in $3/5$ of a second. Plugging this into $x = 0.5at^2 + vt$ gives us that a must be $100/9$ meters per second.
14. D Plug this into the equation from question 1.
15. B By Newton's third law, it is equivalent to find the force of the particle on the shell. The pressure exerted on the shell is $P = Gm\sigma/R^2$. Now consider a closed hemisphere filled with a gas of pressure P . Since the hemisphere cannot move on its own, the net force on the flat face is equal to the net force on the curved face, which is $\pi R^2 P$. Then the force is $\pi(Gm\sigma)$.
16. D This is basically Gauss's law disguised as a gravity question. No enclosed mass is the same as no charge enclosed so the answer is 0.
17. D We will solve this the same way we do a Gauss's law question. The electric field is the exact same meaning as acceleration (F/m is the same as F/q for the two formulas). Thus the gravitational acceleration $g dA = (\text{mass enclosed}) / (\text{constant that is analogous to epsilon})$. We need the mass enclosed so we say that the mass enclosed is equal to the integral of the density with respect to the volume. We can differentiate volume in terms of the radius of the planet in order to be able to integrate the density since it is a function of the radius. If we say the density is equal to ρ , the integrand of the equation would be $4\pi r^3 dr$. We want to integrate from 0 to R in order to find a in terms of M and other known constants. We get that $M = \pi \rho R^4$ so $a = \frac{M}{\pi R^4}$. Integrate from 0 to r this time and substitute it into our pseudo Gauss's law equation.

We get that $g = \frac{M\pi r^4}{\pi R^4(4\pi r^2)e}$. The G constant is analogous to the k constant, which is $4\pi e$. Thus the final answer is D

18. A The final frequency is proportional to $\frac{v \pm v_{obs}}{v \mp v_{sou}}$ with the sign depending on the direction of the movements of the source and the observer. In this scenario, the frequency is $525 \cdot \frac{340-10}{340-30} \approx 493$
19. C Using the perpendicular axis theorem, the moment of inertia through the center of the disk is equivalent to the sum of the two moment of inertias along 2 perpendicular diameters, which is what we wanted. Thus the moment of inertia through the diameter is one half the inertia through the center of the disk.
20. D $V = L \frac{di}{dt} + \frac{q}{c}$. Take the derivative on both sides with respect to time to get $\frac{dV}{dt} = L \frac{d^2i}{dt^2} + \frac{i}{c}$. The left side is zero since the voltage is a constant. Rearrange the right side to get $\frac{d^2i}{dt^2} = -\frac{i}{LC}$. This is similar to simple harmonic motion, where we have a quantity is directly proportional to its second derivative. Thus we can plug this into the formula for simple harmonic motion to get answer choice D.
21. D Power is equal to force times velocity or mv . This implies acceleration varies with velocity. If the kinetic energy is multiplied by a factor of 6, the velocity increases by a factor of $\sqrt{6}$ and the acceleration decreases by a factor of $1/\sqrt{6}$. Thus we get answer choice
22. B We need to use relativistic coordinates in order to solve this problem. Let k represent the proportion of the rubber band length the bug has traveled in time t . dk is equal to the distance the beetle has traveled over the total length of the band, or $\frac{ds}{L(t)} = dk$. Integrating both sides gives us $1 = \int_0^T \frac{ds}{L(t)}$. The integrand can be represented as $\frac{vdt}{L+ut}$. Using an "r substitution" (just u sub but I already used u in a variable oops) we can say that $r = L+ut$ and $dr = udt$. Rearranging the integral, we get $1 = \frac{v}{u} \int_0^T \frac{dr}{r}$. This is equal to $\frac{u}{v} = \ln\left(\frac{uT+L}{L}\right)$. Solving for T , you get B
23. C Power is equal to V^2/R . Since neither changes for the 6 ohm resistor branch, power output is the same.
24. A I assert that the solution to the problem is simply finding the maximum number of intersections n lines can have on a graph. Plot a graph of distance x versus time. Any time a line intersects, this implies a collision happens between the two beads. When the two beads collide, the velocities swap, meaning they essentially swap what line they are represented by on the graph. Do it for the 2, 3, 4 bead case and it becomes very obvious that this works.
25. D First collision $v = mv/3m$. Second collision $v = mv/4m$. third collision $v = mv$. Multiply by the mass to get impulse to the block. Thus it is D.
26. A $1000(4.32)(100-T) = 6000(2.16)(T-50) \rightarrow 2(100-T) = 6(T-50)$. $T = 62.5$
27. C Take the harmonic mean of the two speeds. This gives you C
28. A Jack's distance increases by the same factor as Jae's. Thus divide $4/5$ by $5/6$ to get A
29. D The velocities simply swap since the masses are equal.

30. D Net force = ma so $bmg - mg = ma + bma$. So $a = (b-1)/(b+1)g$. So $mg + ma = T$. $T = \frac{2mgb}{b+1}$