All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means "None of the Above."

~~~~~~ *Good luck, and have fun!* ~~~~~~

1. The first term of an infinite geometric series (assume $r \neq 0$) is 1. Find the range of possible finite values of the sum of all of the terms in this series.

   A. $(0, \infty)$ B. $\left(\frac{1}{2}, \infty\right)$
   C. $(0,1) \cup (1, \infty)$ D. $\left(\frac{1}{2}, 1\right) \cup (1, \infty)$ E. NOTA

2. The second term of an infinite geometric series is 1. The possible finite values of the sum of all of the terms in this series is the range $(-\infty, m) \cup [n, \infty)$. Find $m + n$.

   A. $\frac{8}{3}$ B. 3 C. $\frac{7}{2}$ D. 4 E. NOTA

3. The third term of an infinite… *just kidding!* For an infinite sequence $\{a_n\}_{n\geq 1}$ of positive integers, $\{a_{2n-1}, a_{2n}, a_{2n+1}\}$ is a geometric sequence, and $\{a_{2n}, a_{2n+1}, a_{2n+2}\}$ is an arithmetic sequence. If $a_1 = 4$ and $a_2 = 6$, find $\sum_{n=1}^{18} a_n$.

   A. 612 B. 712 C. 822 D. 943 E. NOTA

4. Let h_n equal the harmonic mean of the set $\{1,2,4,8, \ldots, 2^n\}$. As n grows large, h_n is asymptotic to which of the following?

   A. 0 B. $\frac{1}{2}$ C. $\frac{n}{2}$ D. n E. NOTA

5. The initial proportion p_0 of lions on an island with only lions and gazelles on it is $\frac{1}{8675309}$. For all future generations of the island, the proportion changes logistically as lions eat gazelles or die of starvation according to the equation $p_{n+1} = \frac{9}{4} p_n (1 - p_n)$. Find $\lim_{n\to\infty} p_n$.

   A. 0 B. $\frac{4}{9}$ C. $\frac{5}{9}$ D. 1 E. NOTA
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6. Determine the convergence of the series $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n}$.

A. Absolutely convergent B. Bifurcating
C. Conditionally convergent D. Divergent E. NOTA

7. Determine the convergence of the series $\sum_{n=1}^{\infty} \frac{(n+1)\sqrt{n} - n\sqrt{n+1}}{(-1)^n(n+2)}$.

A. Absolutely convergent B. Bifurcating
C. Conditionally convergent D. Divergent E. NOTA

8. If $f(x) = \tan x$, then $f'(0) = 1$, $f'''(0) = 2$, $f^{(5)}(0) = 16$, and $f''(0) = f^{(4)}(0) = 0$. Use the degree-5 Maclaurin series for $\tan x$ to approximate $15 \tan 1$.

A. 22 B. 23 C. 24 D. 27 E. NOTA

9. Let $(x - 1 - i)^8 = a_8x^8 + a_7x^7 + \cdots + a_1x + a_0$. Find $|a_6 + a_2|$.

A. 0 B. 168 C. 256 D. 313 E. NOTA

10. Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$.

A. $\sqrt{\pi/2}$ B. $\pi/2$ C. $\pi^2/4$ D. ∞ E. NOTA

11. If $\lim_{x\to 0} \frac{x^3 + x^2 - \ln^2(1-x)}{x^2(\cos x - 1)} = \frac{A}{B}$, find $A + B$.

A. 7 B. 11 C. 17 D. 35 E. NOTA

12. Determine the coefficient of the x^3 term in the Maclaurin series expansion of $\sqrt{1+x}$.

A. $-\frac{1}{8}$ B. $\frac{1}{16}$ C. $-\frac{1}{16}$ D. $\frac{1}{8}$ E. NOTA

13. Determine the coefficient of the x^6 term in the Maclaurin series expansion of $\ln(\sec x)$.

A. $\frac{1}{90}$ B. $\frac{1}{45}$ C. $\frac{1}{30}$ D. $\frac{1}{18}$ E. NOTA

14. Evaluate: $\lim_{n\to\infty} \sqrt[n]{\frac{1}{n!}\sum_{m=1}^{n} m^m}$. *Hint: the sum is bounded by n^n and $n \cdot n^n$.*

A. 1 B. $\sqrt{2\pi}$ C. e D. $e\sqrt{2\pi}$ E. NOTA

15. Determine the numerator of the simplified fraction equal to the coefficient of the x^3 term in the Maclaurin series expansion of $\frac{1}{x^2-5x+6}$.

A. 31 B. 57 C. 65 D. 97 E. NOTA

16. Find the number of three-element strictly increasing arithmetic sequences of positive integers whose third element is 100.

A. 49 B. 50 C. 51 D. 100 E. NOTA

17. Evaluate: $\sqrt{1980+\sqrt{1980+\sqrt{1980+\cdots}}}$.

A. $\sqrt{2024}$ B. 45 C. $\sqrt{2112}$ D. 46 E. NOTA

18. Find the square root of the sum of the first 100 positive perfect cubes.

A. 5050 B. 10100 C. 50500 D. 101000 E. NOTA

19. The roots of $x^3 - 15\sqrt{3}x^2 + qx - 345\sqrt{3} = 0$ form an arithmetic progression. Find q.

A. -219 B. -57 C. 57 D. 219 E. NOTA

20. Evaluate: $\sum_{n=0}^{\infty}\frac{(-1)^n(2024\pi)^{2n+1}}{(2n+1)!}$.

A. -1 B. 0 C. 1 D. DNE E. NOTA

21. The sum of the first 11 terms of an increasing arithmetic sequence with positive integer terms is 2024. Find the number of possible first terms of this sequence.

A. 36 B. 37 C. 183 D. 184 E. NOTA

22. Let $\{a_n\}_{n\in\mathbb{N}}$ be a sequence such that $a_n = \sqrt[2n]{1+n}$. Find $\lim_{n\to\infty} a_n$.

A. 1 B. $\sqrt{e}$ C. e D. e^2 E. NOTA

23. Evaluate: $\tan\left(\lim_{n\to\infty}\sum_{k=0}^{n}\frac{n}{n^2+(n+k)^2}\right)$.

A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. NOTA

24. Find $\frac{1}{2025!}\int_0^1\left(\left(\prod_{i=1}^{2024}(x+i)\right)\left(\sum_{i=1}^{2024}\frac{1}{x+i}\right)\right)dx$. *Hint: start with the indefinite integral.*

A. $\frac{1}{2025}$ B. $\frac{2024}{2025}$ C. 1 D. 2025 E. NOTA

25. For a sequence $\{a_n\}_{n\in\mathbb{N}}$, $a_n = \frac{8n^2}{n^3+512}$. Find the value of n for which a_n is maximized.

A. 8 B. 9 C. 10 D. 11 E. NOTA

26. Determine the interval of convergence for the series $\sum_{n=0}^{\infty}\frac{(-1)^n(x+4)^{2n}}{9^n}$.

A. $(-7,-1)$ B. $[-7,-1]$ C. $(-13,5)$ D. $[-13,5]$ E. NOTA

27. Find $\sum_{n=0}^{\infty} \frac{1}{i^n n!}$, where $i = \sqrt{-1}$ and $s = \sin 1$ and $c = \cos 1$.

A. $-s - ic$ B. $-s + ic$ C. $is - c$ D. $is + c$ E. NOTA

28. Let $f(x) = \sum_{n=0}^{\infty} \frac{nx^{n-1}}{(n+1)!}$. Evaluate $\int_0^1 f(x)\,dx$.

A. $e - 2$ B. $e - 1$ C. e D. $e + 1$ E. NOTA

29. Evaluate: $\int_0^{1/2} \left(\frac{x}{x + \frac{x}{x + \frac{x}{x + \cdots}}} + \frac{1}{1-x} \right) dx$. *Hint: Find the inverse of the continued fraction.*

A. $\ln 2 + \frac{1}{4}$ B. $\frac{7}{8}$ C. $\ln 2 + \frac{3}{8}$ D. $\frac{9}{8}$ E. NOTA

30. A *worm* is a finite list of non-negative integers. The rightmost (last) element of this list is called the *head*. If the head is not $\{0\}$, then the worm has an *active segment* consisting of the largest contiguous block of elements that includes the head and numbers that are not less than the head. The *reduced active segment* is the active segment, but the head has been decremented by 1. For example, the active segment of the worm $\{3,1,2,3,2\}$ is $\{2,3,2\}$, and the reduced active segment is $\{2,3,1\}$. A worm evolves according to the following rules.

In generation t, if the head of the worm is $\{0\}$, delete the head. Otherwise, replace the active segment by $t + 1$ copies of the reduced active segment. Every worm eventually evolves into the empty list $\{\}$, and the number of generations this takes is known as the *lifetime of the worm*. The evolution of the worm $\{1,1\}$ is shown to the right.

Find the lifetime of the worm $\{2\}$.

Evolution of $\{1, 1\}$

Gen.	Worm
0	(1 1)
1	1 0 1 0
2	1 0 (1)
3	1 0 0 0 0 0
4	1 0 0 0 0
5	1 0 0 0
...	...
8	(1)
9	0 0 0 0 0 0 0 0 0 0
10	0 0 0 0 0 0 0 0 0
...	...
18	0
19	[Lifetime = 19]

- The active segments are shown in parentheses.
- Braces and commas are removed for readability.

A. 33 B. 44 C. 47 D. 51 E. NOTA