NOTA stands for "None Of These Answers." The set of natural numbers does not contain 0. Good luck, and have fun!

1. Which of these represents a discrete quantity?
   A. Time elapsed since this test began, in seconds
   B. Liters of water you drank today
   C. WiFi signal strength
   D. Permutations of a sequence of letters
   E. NOTA

For the next two questions, use the following information:

Consider the following algorithm, which takes an integer as input, and outputs either "Fizz," "Buzz," "FizzBuzz," or a number:

```
function fizzbuzz(integer i) {
    if (i % 3 == 0 and i % 5 == 0)
        output "FizzBuzz"
    else if (i % 3 == 0)
        output "Fizz"
    else if (i % 5 == 0)
        output "Buzz"
    else
        output i
}
```

Note that a % b means the remainder when a is divided by b. For example, $7 \% 3 = 1$. This algorithm is run for inputs of 1 to 100 inclusive, and the output for each run printed to a new line of text.

2. In the 100 lines of output produced, how many times does the character "z" appear?
   A. 94 B. 106 C. 118 D. 130 E. NOTA

3. From the 100 lines of output produced, how many do not contain any alphabetic characters?
   A. 41 B. 47 C. 53 D. 59 E. NOTA

4. How many of these relations $f : \mathbb{N} \to \mathbb{R}^+$ are injective?
   I. $f(x) = (x - 2020)^2$ II. $f(x) = x^3$ III. $f(x) = \sin(x) + 1$ IV. $f(x) = 1/x$
   A. 1 B. 2 C. 3 D. 4 E. NOTA

For the next two questions, use the following information:

Within a universe of nonnegative integers, let A be the set of divisors of 20240, and let B be the set whose base-2 representation contains 10 digits or fewer.

5. Which of these is in $A^C \cap B$?

A. 8 B. 40 C. 880 D. 1024 E. NOTA

6. What is the cardinality of $A \cap B$?

A. 31 B. 32 C. 33 D. 34 E. NOTA

7. Which of these is the logical negation of this statement?

"All cats are devious."

A. No cats are devious.
B. There exists a cat that is devious.
C. All cats are not devious.
D. There exists a cat that is not devious.
E. NOTA

8. Which symbolic statement is a correct logical representation of this statement?

"There's a matching lid for every pot."

(Assume a commutative binary relation $Match(x, y)$ that denotes whether x and y match.)

A. $\forall Pot\,.\,\exists Lid\,.\,Match(Pot, Lid)$
B. $\forall Lid\,.\,\exists Pot\,.\,Match(Pot, Lid)$
C. $\exists Lid\,.\,\forall Pot\,.\,Match(Pot, Lid)$
D. $\exists Pot\,.\,\exists Lid\,.\,Match(Pot, Lid)$
E. NOTA

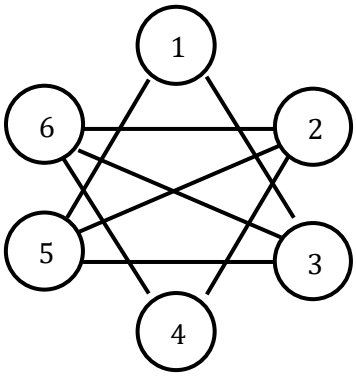
9. Simplify the Boolean expression $a \wedge (a \to b) \wedge (b \to c)$.

A. True B. False C. c D. $a \wedge b \wedge c$ E. NOTA

10. Simplify the Boolean expression $(a \vee \neg b \vee \neg c) \wedge (a \vee \neg b \vee c) \wedge (a \vee b \vee \neg c)$.

A. $(a \wedge b) \vee c$ B. $a \vee (\neg b \wedge c)$ C. $(\neg a \wedge b) \vee c$ D. $a \vee (b \wedge \neg c)$ E. NOTA

For the next three questions, use the following undirected graph:



11. Which of these is the adjacency matrix for the graph?

A. $\begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ C. $\begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$

E. NOTA

12. Which of these is the matrix N has elements N_{ij} equal to the number of walks containing exactly two edges from vertex i to vertex j?

A. $\begin{bmatrix} 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 & 1 & 1 \\ 1 & 0 & 3 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 & 0 & 1 \\ 1 & 1 & 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 1 & 0 & 3 \end{bmatrix}$ B. $\begin{bmatrix} 2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 3 & 0 & 2 & 2 & 2 \\ 2 & 0 & 3 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 & 0 & 2 \\ 2 & 2 & 2 & 0 & 3 & 0 \\ 0 & 2 & 2 & 2 & 0 & 3 \end{bmatrix}$ C. $\begin{bmatrix} 2 & 1 & 2 & 0 & 2 & 1 \\ 1 & 3 & 2 & 2 & 1 & 2 \\ 2 & 2 & 3 & 1 & 2 & 1 \\ 0 & 2 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 3 & 2 \\ 1 & 2 & 1 & 2 & 2 & 3 \end{bmatrix}$ D. $\begin{bmatrix} 2 & 1 & 1 & 0 & 1 & 1 \\ 1 & 3 & 2 & 1 & 0 & 1 \\ 1 & 2 & 3 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 & 3 & 2 \\ 1 & 1 & 0 & 1 & 2 & 3 \end{bmatrix}$

E. NOTA

13. What is the crossing number of the graph?

A. 1 B. 3 C. 6 D. 11 E. NOTA

14. For some integers a, b, $2a + 3b$ is a multiple of 13. How many of the following must also be a multiple of 13?

I. $a - 5b$ II. $3a + 2b$ III. $7a + 4b$ IV. $a^4 + b^4$ V. $a^4 - b^4$

A. 1 B. 2 C. 3 D. 4 E. NOTA

15. What are the last two digits of 12^{2023}?
A. 08 B. 28 C. 48 D. 68 E. NOTA

16. Let S be the set of lattice points in the first quadrant where neither coordinate exceeds 9. In other words, $S = \{(x, y) | x, y \in \mathbb{N} \text{ and } x, y \leq 9\}$. How many ways are there to select 4 points from S such that they are the vertices of a non-square rectangle whose sides are parallel to the x and y axes.
A. 784 B. 1008 C. 1092 D. 1296 E. NOTA

17. Michelle is eyeing four different types of mochi muffins. In how many ways can she choose a subset of types to buy, if she definitely wants at least one?
A. 10 B. 12 C. 15 D. 16 E. NOTA

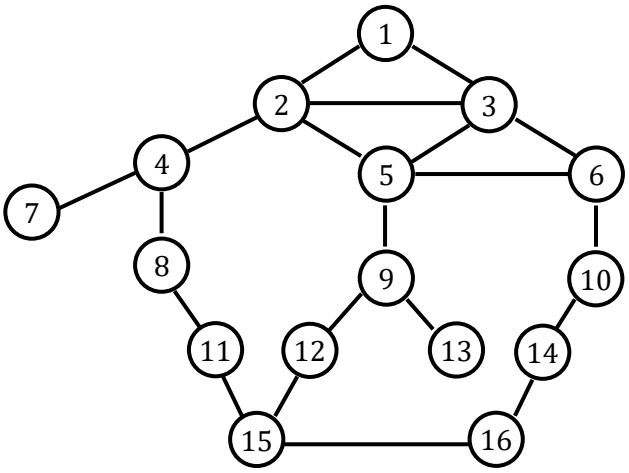
18. Adam is distributing six indistinguishable beakers to his three labmates. In how many ways can he do this, if some labmates might receive more than one, and some might receive none at all?
A. 10 B. 15 C. 21 D. 36 E. NOTA

19. Richard and Super Radz are playing a special game of pick-up-Cheetos. They start with a pile of N $(N > 0)$ Cheetos and take turns picking up exactly 1, 2, or 3 Cheetos. Whoever takes the last Cheeto wins. Under what condition does the player who goes first have a strategy that will allow him to guarantee a win?
A. $N \equiv 0 \; mod \; 4$
B. $N \equiv 0 \; mod \; 2$, or $N \equiv 0 \; mod \; 3$
C. $N \equiv 1, 2,$ or $3 \; mod \; 4$
D. $N \equiv 1 \; mod \; 2$, or $N \equiv 1$ or $2 \; mod \; 3$
E. NOTA

20. Paloma and Leigh build a randomized triangle by independently choosing points $A(a, 0)$ and $B(0, b)$, and connecting them with $C(0,0)$. Paloma chooses a to be a nonnegative integer where the probability of choosing a to equal i is $\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^i$ for all $i \geq 0$. Leigh chooses b to be a nonnegative integer where the probability of choosing b to equal i is $\left(\frac{3}{7}\right)\left(\frac{4}{7}\right)^i$ for all $i \geq 0$. What is the expected area of triangle ABC?
A. $4/3$ B. $4/21$ C. $7/2$ D. 2 E. NOTA

21. Nora and Dale play the high-low game, where Nora chooses an integer from 1 to 100 inclusive, and Dale tries to guess the number. After each guess, Nora tells Dale whether the guess was correct, too high, or too low. If Dale plays optimally, he can guarantee never having to guess more than N numbers, including the correct number. What is N?
A. 6 B. 7 C. 8 D. 9 E. NOTA

For the next six questions, use the following graph:



22. Which of these is false about the graph?
    A. The graph is complete.
    B. The graph has a cycle.
    C. The graph is undirected.
    D. The graph is not a tree.
    E. NOTA

23. How many walks of length 4 are there that start at vertex 1 and end at vertex 3?
    A. 10 B. 12 C. 14 D. 16 E. NOTA

24. A breadth-first search is performed starting from node 1 until node 16 is visited. How many nodes are visited in the process, including nodes 1 and 16? At each step, if there are multiple nodes neighboring a given node, add them to the to-visit list such that the lowest-numbered nodes will be visited first.
    A. 9 B. 11 C. 13 D. 15 E. NOTA

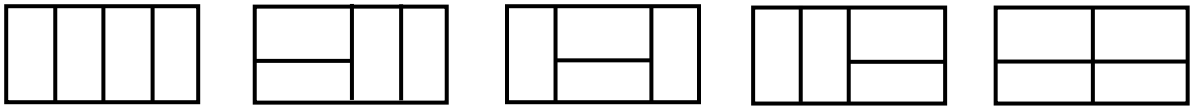
25. How many bridges are there in the graph?
    A. 0 B. 1 C. 2 D. 3 E. NOTA

26. Note that the given graph is planar. Find the minimum number of edges that can be added such that the graph is no longer planar.
    A. 2 B. 3 C. 4 D. 5 E. NOTA

27. How many nodes are at level n in a complete binary tree with k levels, where level 1 is the root and $1 \leq n < k$?

A. $2n$ B. 2^{n-1} C. 2^n D. $2^n - 1$ E. NOTA

28. There are five distinct ways to tile an oriented 2×4 checkerboard with only 1×2 dominoes. Here, "oriented" means arrangements that are $180°$ rotations of each other are counted separately, as in the second and fourth arrangements:



Under the same condition, how many distinct ways are there to tile a 2×10 checkerboard with only 1×2 dominoes?

A. 89 B. 101 C. 144 D. 225 E. NOTA

29. Theoretical computer scientists often use recurrences to analyze the number of operations performed in a divide-and-conquer algorithm. For example, an algorithm takes an $n \times n$ matrix, where n is a power of 2, squares the matrix (which takes n^3 operations in a naïve algorithm), and recurses eight times on $\left(\frac{n}{2}\right) \times \left(\frac{n}{2}\right)$ submatrices. The number of operations on an input of size n is described by the recurrence below with $T(1) = 2$.

$$T(n) = 8T\left(\frac{n}{2}\right) + n^3$$

If $T(n)$ can be expressed in the form $u + vn^w + xn^y \log_2 n$, then what is $u + v + w + x + y$?

A. 6 B. 7 C. 8 D. 9 E. NOTA

30. A fast food place sells chicken nuggets in packages of 8, 9, and 15. Find the largest quantity of chicken nuggets that cannot be purchased in some combination of those packages.

A. 21 B. 22 C. 23 D. 26 E. NOTA