Good Luck! :)

1. Evaluate the following Boolean algebra expression:

$$1 + 1 \times 1 + 0.$$

A. 0 B. 1 C. 2 D. 110 E. NOTA

2. How many sets S satisfy $\{1,2,3\} \subset S \subseteq \{1,2,3,4,5,6,7\}$?

A. 7 B. 8 C. 15 D. 16 E. NOTA

3. Find the sum of the last three digits of 99^{17}.

A. 1 B. 4 C. 24 D. 27 E. NOTA

4. Which of the following are countable?

I: Set of positive integers less than 10000
II: Set of ordered pairs of integers
III: Set of ordered pairs of rational numbers
IV: Set of real numbers from 0 to 1
V: Points on the unit circle

A. I B. I, II C. I, II, III D. I, II, III, IV E. NOTA

5. Functions $f(x), g(x)$ satisfy $f(x)g(x) = 0$ for all integers x. Let A be the set of values of x such that $f(x) = 0$, and let B be the set of values of x such that $g(x) = 0$. How many of the following are true?

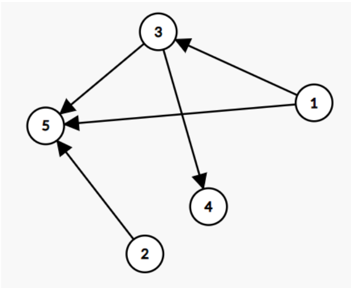
I: A and B are both infinite sets.
II: At least one of A and B are a finite set.
III: At least one of A and B are an infinite set.
IV: If A is a finite set, B is an infinite set.
V: If A is an infinite set, B is a finite set.

A. 0 B. 1 C. 2 D. 3 E. NOTA

6. How many ordered triplets (a, b, c) of positive integers exist such that $3a + 2b + c = 30$?

A. 58 B. 59 C. 60 D. 61 E. NOTA

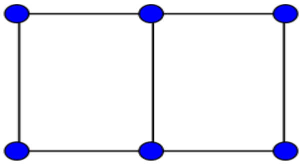
7. What is the adjacency matrix of the graph below? ($A_{i,j}$ indicates an edge from i to j.)



A. $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ E. NOTA

8. The Domino graph is shown below. This graph has which of the following?

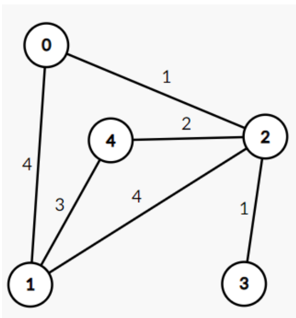


I: Hamiltonian Path
II: Hamiltonian Circuit
III: Euler Path
IV: Euler Circuit

A. I, II B. III, IV C. I, II, III D. I, III, IV E. NOTA

9. Which of the following are a field over addition and multiplication?
I: The set of integers
II: The set of rational numbers
III: The set of real numbers from 0 to 1
IV: The set of real numbers
V: The set of complex numbers in the first quadrant of the complex plane
A. II, IV B. I, II, IV C. I, II, IV, V D. II, III, IV, V E. NOTA

10. What is the diameter of the following graph?



A. 4 B. 5 C. 6 D. 8 E. NOTA

11. Simplify the following Boolean expression:

$$\neg\Big(\big(a \wedge (b \rightarrow c)\big) \vee \neg d\Big)$$

A. $\big(\neg a \wedge (b \vee \neg c)\big) \vee d$ B. $\neg a \vee \big((b \wedge \neg c) \wedge d\big)$
C. $\neg a \vee \big((\neg b \vee \neg c) \wedge d\big)$ D. $\neg a \vee (\neg b \wedge \neg c) \vee d$ E. NOTA

12. How many surjective functions $f: \{1,2,3,4,5\} \rightarrow \{1,2,3\}$ exist such that $nf(n) < 10$ for $1 \leq n \leq 5$?
A. 19 B. 30 C. 31 D. 38 E. NOTA

13. Let N be the smallest two-digit integer such that N^2 and N have the same sum of digits in both base 5 and base 10. N is equal to ab_8 Find $a + b$ in base 10.
A. 9 B. 10 C. 11 D. 12 E. NOTA

14. A full binary tree with 1001 nodes has a minimum of m levels and a maximum of M levels. Find $M - m$.

A. 491 B. 492 C. 991 D. 992 E. NOTA

15. Which of the following graphs has the largest chromatic number? (Select E if there is a tie for the maximum)

A. Cube graph (Q_3) B. $K_{3,3}$ C. K_3 D. C_6 E. NOTA

16. Let S be the set of positive integers less than 50. Let T_n be the set of integers k such that $k \in S$ and either $\frac{n}{k}$ or $\frac{k}{n}$ is an integer. For how many positive integers $n \in S$ is $|T_n| = 4$?

A. 6 B. 7 C. 8 D. 9 E. NOTA

(Questions 17-18) Define set S_k to be the set of positive integral factors of $N^k = 12^k$ for positive integer k. For example, S_1 is the set of positive integral factors of $N^1 = 12^1$. Namely $S_1 = \{1, 2, 3, 4, 6, 12\}$.

17. Which of the following are true if a, b are positive integers?

I: $S_a \cup S_b = S_{\max(a,b)}$.

II: $S_a \cap S_b = S_{\min(a,b)}$.

III: If $b > a$, $|S_b| - |S_a| > 3(a + b)$.

IV: If $b > a$, $|S_b| - |S_a| > 9(b - a)$.

A. I, II B. I, II, III C. I, II, IV D. I, II, III, IV E. NOTA

18.
$$\lim_{k\to\infty} \sum_{x\in(S_k\cap S_1^c)} \frac{1}{x^2} = \frac{m}{n}$$

in simplest form. Find $m + n$.

A. 5 B. 11 C. 13 D. 21 E. NOTA

19. What is the crossing number of $K_{3,4}$?

A. 0 B. 1 C. 2 D. 3 E. NOTA

20. Which of the following graphs are planar?
I: $K_{3,3}$
II: $K_{2,4}$
III: K_6
IV: C_5
V: cube graph (Q_3)
A. I, II B. II, IV C. IV, V D. III, IV, V E. NOTA

21. How many lattice points are on the circle $x^2 + y^2 = 2025^2$?
A. 12 B. 20 C. 28 D. 32 E. NOTA

22. Erick and his 2 pandas are playing rock-paper-scissors to qualify for the world rock-paper-scissors championship. The rules of 3-animals rock-paper-scissors game is as following: It is a draw if all 3 animals have same hands or all 3 have different hands. If two animals out of the three wins, the two play each other in a 1 on 1 rock-scissor-paper match. Let E be the expected number of games they need to play to determine a single winner. If $E = \frac{m}{n}$ in simplest form, find $m + n$.
A. 5 B. 7 C. 11 D. 13 E. NOTA

23. Erick flips a coin until one outcome shows up two more times than the other outcome. He finishes flipping the coin when there are 5 heads and 3 tails. In how many different ways can this happen?
A. 6 B. 8 C. 10 D. 12 E. NOTA

24. In Love Island, there are 6 people living together. Every night, each person chooses a person they like. If everyone is equally likely to get chosen, the expected number of couples (when A picks B and B picks A) on a night is equal to $\frac{m}{n}$ in simplest form. Find $m + n$. (You cannot choose yourself.)
A. 8 B. 10 C. 12 D. 14 E. NOTA

25. Refer to the previous question for information about Love Island. One day, a new person joins the island. This person is very attractive, so the probability he gets chosen is twice of everyone else. The expected number of couples is equal to $\frac{m}{n}$ in simplest form. Find $m+n$.

A. 77 B. 78 C. 79 D. 80 E. NOTA

26. In equilateral triangle ABC, Amy puts a rock on each of the vertices, 2 rocks on side AB, 3 rocks on side BC, and 4 rocks on side CA, with a total of 12 rocks. Then, Andrew draws a line segment connecting every pair or rocks. What is the maximum number of intersections of line segments that occur in the interior of triangle ABC?

A. 189 B. 210 C. 252 D. 273 E. NOTA

27. Find the remainder when $12!\left(\frac{1}{1\cdot 12}+\frac{1}{2\cdot 11}+\frac{1}{3\cdot 10}+\frac{1}{4\cdot 9}+\frac{1}{6\cdot 7}\right)$ is divided by 13.

A. 1 B. 3 C. 8 D. 12 E. NOTA

28. Let $N = 2^{50}\cdot 5^{48}\cdot(10^6+1)\cdot(10^4+1)$. The number $\frac{1}{N}$ when written as a repeating decimal has a non-repeating part of length a and a repeating part of length b. Find $a+b$.

A. 60 B. 62 C. 72 D. 74 E. NOTA

29. Evaluate

$$\sum_{a=1}^{\infty}\sum_{b=a}^{\infty}\frac{1}{a^2b^2}$$

given $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}, \sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}$.

A. $\frac{\pi^4}{72}$ B. $\frac{7\pi^4}{360}$ C. $\frac{\pi^4}{45}$ D. $\frac{\pi^4}{40}$ E. NOTA

30. Congratulations for making it until the end! For how many integers n is $\frac{n(n+1)}{2}$ a 2 digit-integer?

A. 9 B. 10 C. 11 D. 12 E. NOTA