All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means "None of the Above."

~~~~~~ *Good luck, and have fun!* ~~~~~~

1 Find the sum of the positive integral factors of 2023.

A. 2024 B. 2160 C. 2320 D. 2456 E. NOTA

2 Calculate $\varphi(2023)$, where $\varphi(n)$ is Euler's totient function.

A. 1536 B. 1632 C. 1734 D. 2022 E. NOTA

3 Find the number of positive integers n such that $\varphi(n) = 20 \cdot 23$.

A. 0 B. 2 C. 4 D. 6 E. NOTA

4 If A is the smallest integer greater than 2023 such that $A \equiv 2$ mod 5, $A \equiv 5$ mod 7, and $A \equiv 4$ mod 11, find A mod 19.

A. 2 B. 7 C. 11 D. 16 E. NOTA

5 Let (A, B) be the solution to the Pell equation $x^2 - 27y^2 = 1$ such that $A + B$ is minimized. Find AB mod 19.

A. 2 B. 7 C. 11 D. 16 E. NOTA

6 Radleigh can buy pencils for \$12 and sell them for \$16. After buying and selling pencils with Radleigh, Rick notices that the amount of money he has has changed by $\$X$. Find the sum of the possible values of X if $0 < X \leq 100$.

A. 1288 B. 1300 C. 2544 D. 2550 E. NOTA

7 When converted to base 10, which of the following base-19 numbers is divisible by 9?

A. $BEACHED$ B. $BIGHEAD$ C. $CABBAGE$ D. $EDIFICE$ E. NOTA
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For questions 8-9, you may use the following information.

The Legendre symbol is a multiplicative function whose arguments are a positive integer a and an odd prime p and is denoted $\left(\frac{a}{p}\right)$. If $a \equiv 0 \bmod p$, then $\left(\frac{a}{p}\right) = 0$. If $a \not\equiv 0 \bmod p$ and there exists an integer x such that $x^2 \equiv a$ modulo p $\left(\frac{a}{p}\right) = 1$. Otherwise, $\left(\frac{a}{p}\right) = -1$.

8 Let p and q are odd primes. Which of the following statements about the Legendre symbol is false?

A. $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{(p-1)(q-1)/4} = \begin{cases} -1 \text{ if } p \equiv q \equiv 3 \bmod 4 \\ 1 \text{ if otherwise} \end{cases}$

B. $\left(\frac{-1}{p}\right) = (-1)^{(p-1)/2} = \begin{cases} 1 \text{ if } p \equiv 1 \bmod 4 \\ -1 \text{ if } p \equiv 3 \bmod 4 \end{cases}$

C. $\left(\frac{2}{p}\right) = (-1)^{(p^2-1)/8} = \begin{cases} 1 \text{ if } p \equiv \pm 1 \bmod 8 \\ -1 \text{ if } p \equiv \pm 3 \bmod 8 \end{cases}$

D. For $p > 3$, $\left(\frac{3}{p}\right) = (-1)^{\lfloor (p-5)/6 \rceil} = \begin{cases} -1 \text{ if } p \equiv \pm 1 \bmod 12 \\ 1 \text{ if } p \equiv \pm 5 \bmod 12 \end{cases}$

E. NOTA

9 Evaluate: $9 \cdot \left(\frac{50}{83}\right) + 3 \cdot \left(\frac{29}{37}\right) + \left(\frac{2023}{19}\right)$.

A. -11 B. -7 C. -5 D. 7 E. NOTA

10 An Affine cipher $A(x) = ax + b$ encodes a message x by multiplying the value of each letter in the message by an integer constant a, adding another integer constant b, and taking this new value modulo 26, with the exception that 26 is used in place of 0. The value of a letter is given by its 1-indexed position in the alphabet, where $A = 1, B = 2, \ldots, Z = 26$. This operation is invertible modulo 26 if and only if $GCD(a, 26) = 1$.
Let $\#(x)$ be an operation that finds the sum of the 1-indexed positions of the letters in its argument. For example, $\#(TREVOR) = 20 + 18 + 5 + 22 + 15 + 18 = 98$. Find $\#\left(A^{-1}(RMJG)\right)$, where $A(n) = 3n + 2$.

A. 42 B. 50 C. 69 D. 74 E. NOTA

11 How many positive integer factors does 12! have?

A. 400 B. 420 C. 792 D. 1024 E. NOTA

12 When written in base 72, find the number of zeroes at the end of $2023!$.

A. 503 B. 671 C. 672 D. 1006 E. NOTA

13 Find the number of digits in the decimal expansion of 5^{23}.

A. 17 B. 18 C. 19 D. 20 E. NOTA

14 Find the number of solutions in the positive integers to $3x + 14y + 15z = 420$.

A. 117 B. 121 C. 126 D. 132 E. NOTA

15 Calculate 19^{2023} mod 23.

A. 1 B. 13 C. 17 D. 19 E. NOTA

16 A perfect number is a number whose positive integer factors sum to twice itself. Find the sum of the digits of the smallest perfect number divisible by 31.

A. 16 B. 19 C. 20 D. 22 E. NOTA

17 A recursive sequence $\{a_n\}_{n\geq 0}$ is given by $a_0 = -7$, $a_1 = -12$, and for all $n \geq 2$, $a_n - 5a_{n-1} + 6a_{n-2} = 0$. If $a_n = A \cdot B^n - C \cdot D^n$, find $\begin{vmatrix} A & B \\ C & D \end{vmatrix}$.

A. -23 B. -12 C. 24 D. 31 E. NOTA

18 The set $\{2N, 3N, 5N\}$ is equal to the set $\{A^2, B^3, C^5\}$. Find the remainder when the smallest possible value of N is divided by 7. (Remember that order does not matter in sets.)

A. 2 B. 3 C. 4 D. 5 E. NOTA

19 Trevor was born on February 19, so 219 is one of his favorite numbers. A math-magic trick he likes to perform is to have someone square this number, use a calculator to multiply this by a bunch of single digits, and read back every digit (except for one of them) in the resulting product in a random order. He would then tell the subject of the trick what digit they omitted. On one particular iteration of this trick, the subject reads back the set $\{9,6,9,4,2,0,9,1,9\}$. What number was omitted?

A. 3 B. 5 C. 7 D. 8 E. NOTA

20 Find the number of positive perfect cube divisors of $2^2 3^3 4^4 5^5 6^6 7^7 8^8 9^9 \cdot 10!$.

A. 640 B. 1084 C. 1683 D. 2992 E. NOTA

21 Find the number of integer values of n such that $\frac{336}{2n^2-n-15}$ is an integer.

A. 1 B. 3 C. 5 D. 6 E. NOTA

22 If $x \equiv 80 \bmod 480$, which of the following must be true?

I. $x \equiv 80 \bmod 120$
II. $x \equiv 8 \bmod 48$
III. $x \equiv 80 \bmod 960$

A. I ONLY B. I and II ONLY C. I and III ONLY D. II and III ONLY E. NOTA

23 4^{4^6} is equivalent to 2^{2^N} mod 2023. Which of the following is a possible value of N?

A. 3 B. 4 C. 5 D. 6 E. NOTA

24 Find the sum of the digits of $N = \sqrt[5]{481{,}170{,}140{,}857}$.

A. 9 B. 10 C. 11 D. 17 E. NOTA

25 An Egyptian fraction expansion of a rational number q is a sequence $\{a_1, a_2, \dots, a_n\}$ of distinct positive integers $a_1 < a_2 < \cdots < a_n$ such that $\sum_{i=1}^{n} \frac{1}{a_i} = q$. One algorithm for generating such sequences involves repeatedly using the identity $\frac{1}{k} + \frac{1}{k} = \frac{1}{k} + \frac{1}{k+1} + \frac{1}{k(k+1)}$ until there are no repeated denominators. Let $\{b_1, b_2, \dots, b_n\}$ be the sequence of denominators generated by using this algorithm on the fraction $\frac{3}{5}$, starting with the equation $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$. Find $\sum_{i=1}^{n} b_i$.

A. 12 B. 1051 C. 870744 D. 870749 E. NOTA

26 Find the number of positive integers that are divisors of at least one of 12^8, 14^{23}, or 21^{12}.

A. 859 B. 867 C. 876 D. 898 E. NOTA

27 Find the sum of the digits of the smallest positive integer that can be written as the sum of a perfect square and a positive perfect cube in two distinct ways.

A. 8 B. 11 C. 13 D. 19 E. NOTA

28 To convert a number N to balanced ternary, you must find the unique sequence $\{c_0, c_1, \dots, c_n\}$ such that $\sum_{i=0}^{n} c_i 3^i$, where each c_i is in the set $\{-1,0,1\}$. This sequence is represented with pluses (for $c_i = 1$), minuses (for $c_i = -1$), and zeroes (for $c_i = 0$) in decreasing powers of 3. For example, since $56 = 3^4 - 3^3 + 3^1 - 3^0$, 56 can be written as +−0+− in balanced ternary. Which of the following is the balanced ternary representation of 2023?

A. +0−+−+++ B. +0−+0−−+ C. +00−0+−+ D. +0−+0+0+ E. NOTA

29 Find the number of unordered sets of distinct positive integers $\{a, b, c, d\}$ that contain the number 2 and have the property that each member of the set divides the sum of the other three.

A. 6 B. 7 C. 8 D. 9 E. NOTA

30 A number is called "based" if it has the property that when decomposed into its prime factorization, the sum of the prime factors is equal to the sum of the exponents. For example, $320 = 2^6 5^1$ is based because $2 + 5 = 6 + 1$. Find the number of positive four-digit integers that are based.

A. 9 B. 8 C. 7 D. 6 E. NOTA