

Team Relay  
Test #342  
Question #0  
Seat 1 – Theta

Team Relay  
Test #342  
Question #0  
Seat 2 – Alpha

Team Relay  
Test #342  
Question #0  
Seat 3 – Mu

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2023 MAΘ National Convention

Question #0

Seat 1 – Theta

**A** = the units digit of  $3^3$ .

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Question #0

Seat 2 – Alpha

**B** =  $\cos\left(\frac{A\pi}{4}\right)$ .

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Question #0

Seat 3 – Mu

**C** =  $\frac{d}{dx}(x^2)$  at  $x = \mathbf{B}$ .

Team Relay  
Test #342  
Question #1  
Seat 1 – Theta

Team Relay  
Test #342  
Question #1  
Seat 2 – Alpha

Team Relay  
Test #342  
Question #1  
Seat 3 – Mu

## Question #1

Seat 1 – Theta

**A** = the area of the ellipse with equation  $4x^2 - 8x + 9y^2 + 54y + 49 = 0$ .

## Question #1

Seat 2 – Alpha

**B** = the largest solution of  $\tan(2x) - \sqrt{3} = 0$  on the interval  $\left[0, \frac{A}{6}\right)$ .

## Question #1

Seat 3 – Mu

$$C = \int_0^B [x] dx$$

where  $[x]$  is the greatest integer less than  $x$ .

Team Relay  
Test #342  
Question #2  
Seat 1 – Theta

Team Relay  
Test #342  
Question #2  
Seat 2 – Alpha

Team Relay  
Test #342  
Question #2  
Seat 3 – Mu

## Question #2

Seat 1 – Theta

$A$  = the sum of the positive integral factors of 120.

## Question #2

Seat 2 – Alpha

$B$  = the second hand of a clock is  $(A - 350)$  centimeters long. Find the speed of the tip of the hand (in cm per second) in terms of  $\pi$ .

## Question #2

Seat 3 – Mu

An airplane flies at a constant altitude of 5 miles toward a point directly over an observer. The speed of the plane is 600 miles per hour. Find the rate (in radians per hour) at which the angle of elevation  $\theta$  is changing when the angle is  $B$ .

Team Relay  
Test #342  
Question #3  
Seat 1 – Theta

Team Relay  
Test #342  
Question #3  
Seat 2 – Alpha

Team Relay  
Test #342  
Question #3  
Seat 3 – Mu

## Question #3

Seat 1 – Theta

**A** = the term in the 2<sup>nd</sup> row, 2<sup>nd</sup> column of the cofactor matrix of  $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & 2 & -3 \end{bmatrix}$ .

## Question #3

Seat 2 – Alpha

**B** =  $\sin(2x)$  if  $\cos x = \frac{A+8}{13}$  and  $x$  is in quadrant IV.

## Question #3

Seat 3 – Mu

**C** = Find the volume of the solid formed by revolving the region bounded by  $f(x) = 2 - x^2$  and  $g(x) = 1$  about the line  $y = \left[-\frac{1}{B}\right]$  where  $[x]$  is the greatest integer less than  $x$ .



Team Relay  
Test #342  
Question #4  
Seat 1 – Mu

Team Relay  
Test #342  
Question #4  
Seat 2 – Theta

Team Relay  
Test #342  
Question #4  
Seat 3 – Alpha

## Question #4

Seat 1 – Mu

**A** = the number of ordered pairs of integers that satisfy the equation  $\frac{1}{m} + \frac{1}{n} = \frac{1}{10}$ .

## Question #4

Seat 2 – Theta

**B** = coefficient of the second term of  $(x - 3)^{4-13}$ .

## Question #4

Seat 3 – Alpha

**C** = the value of  $\cot 2\theta$  where  $\theta$  is the smallest positive angle to rotate the coordinate axes in order to eliminate the  $xy$  term of the graph  $7x^2 - \frac{B\sqrt{3}}{2}xy + (25 + B)y^2 - 16 = 0$ .

Team Relay  
Test #342  
Question #5  
Seat 1 – Mu

Team Relay  
Test #342  
Question #5  
Seat 2 – Theta

Team Relay  
Test #342  
Question #5  
Seat 3 – Alpha

## Question #5

Seat 1 – Mu

**A** = The value of  $k$  such that the area bounded by the line  $y = x$  and the graph of the parabola  $y = x^2 - x$  is cut in half by a line with equation  $x = k$ .

## Question #5

Seat 2 – Theta

**B** = the sum of the squares of the solutions of  $x^2 - \sqrt{27}x + (A + 12) = 0$ .

## Question #5

Seat 3 – Alpha

**C** = the cosine of the dihedral angle between the two planes given by  $x - 2y + \mathbf{B}z = 0$  and  $2x + 3y - 2z = 0$ .

Team Relay  
Test #342  
Question #6  
Seat 1 – Mu

Team Relay  
Test #342  
Question #6  
Seat 2 – Theta

Team Relay  
Test #342  
Question #6  
Seat 3 – Alpha

## Question #6

Seat 1 – Mu

**A** = the ordinate of the centroid of the region bounded by the  $x$  –axis and the graph  $y = 3x - x^2$ .

## Question #6

Seat 2 – Theta

**B** =  $a - b$  if  $\sum_{n=0}^{\infty} A \left( \frac{2}{2+\sqrt{3}} \right)^n = a + b\sqrt{3}$  for rational  $a, b$ .

## Question #6

Seat 3 – Alpha

**C** = the radius of the circle which circumscribes the triangle with points  $(2,0)$ ,  $(0,4)$ , and  $(20\mathbf{B}, 6)$ .

Team Relay  
Test #342  
Question #7  
Seat 1 – Alpha

Team Relay  
Test #342  
Question #7  
Seat 2 – Mu

Team Relay  
Test #342  
Question #7  
Seat 3 – Theta

## Question #7

Seat 1 – Alpha

**A** = the period of  $f(x) = \frac{\sin(67x) + \sin(x)}{\cos(67x) + \cos(x)}$ .

## Question #7

Seat 2 – Mu

**B** = the total distance traveled by the particle whose velocity is given by  $v(t) = \cos(2t)$  from  $t = 0$  to  $t = \frac{\pi^2}{A}$ .

## Question #7

Seat 3 – Theta

**C** = Smallest positive integer  $n$  such that

$$\sum_{k=1}^n [\sqrt{k}] > \mathbf{B}.$$

where  $[x]$  is the floor of  $x$ .



Team Relay  
Test #342  
Question #8  
Seat 1 – Alpha

Team Relay  
Test #342  
Question #8  
Seat 2 – Mu

Team Relay  
Test #342  
Question #8  
Seat 3 – Theta

## Question #8

Seat 1 – Alpha

**A** = the number of distinct solutions to  $4 \sin^2 \theta \cos \theta + \sqrt{3} \sin \theta - \sin 2\theta - \frac{\sqrt{3}}{2} = 0$  on the interval  $\theta \in [0, 2\pi)$ .

## Question #8

Seat 2 – Mu

**B** = the maximum value of  $f(x) = \frac{A}{\csc(x)} + \frac{A+1}{\sec(x)}$ .

## Question #8

Seat 3 – Theta

**C** = the minimum degree measure of each interior angle of a regular convex polygon with at least  $10\mathbf{B}$  diagonals.

Team Relay  
Test #342  
Question #9  
Seat 1 – Alpha

Team Relay  
Test #342  
Question #9  
Seat 2 – Mu

Team Relay  
Test #342  
Question #9  
Seat 3 – Theta

## Question #9

Seat 1 – Alpha

$\mathbf{A}$  = the largest real solution to  $f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24$ .

## Question #9

Seat 2 – Mu

$\mathbf{B}$  = smallest integer  $k$  such that  $3x^4 + 8x^3 - 48x^2 + k + \mathbf{A} \geq 0$  for all real  $x$ .

## Question #9

Seat 3 – Theta

$\mathbf{C} = r^3 + \frac{1}{r^3}$  if  $r$  is a root of  $x^2 - kx + 1 = 0$  where  $k$  is the sum of digits of  $|\mathbf{B}|$ .