

# MAΘ



## Speed Math

## Test #402

Name: \_\_\_\_\_

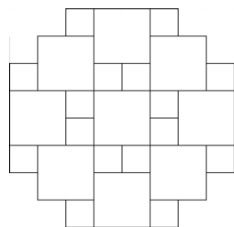
ID Number: \_\_\_\_\_

School: \_\_\_\_\_

Division (circle one):

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- \_\_\_\_\_ 1. Is it true that for any positive real numbers  $a, b, c, \frac{a+b+c}{3} \geq \sqrt[3]{abc}$ ? (True/False)
- \_\_\_\_\_ 2. Evaluate:  $\sum_{i=2}^{\infty} \frac{1}{i^2-1}$
- \_\_\_\_\_ 3. Evaluate:  $\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{vmatrix}$
- \_\_\_\_\_ 4. Let  $r_1, r_2, \dots, r_{2022}$  be the roots of the polynomial  $x^{2022} + x^2 + x + 1$ . Find the value of  $\sum_{i=1}^{2022} r_i^{2021}$
- \_\_\_\_\_ 5. Find the units digit of  $\sum_{i=1}^{10} i^i$ .
- \_\_\_\_\_ 6. Find the number of trailing zeros of  $2022!$
- \_\_\_\_\_ 7. Find the number of positive integers less than 60 that are relatively prime to 30.
- \_\_\_\_\_ 8. Find  $2^{3^5} \pmod{30}$
- \_\_\_\_\_ 9. How many rectangles are in the following figure:



- \_\_\_\_\_ 10. Suppose we have square  $ABCD$  with side length 1. Now, we fold the paper such that  $B$  now lies on the midpoint of side  $CD$ . Let  $N$  be the point where the crease intersects side  $BC$ . Compute the ratio  $\frac{BN}{NC}$ .
- \_\_\_\_\_ 11. How many positive integers are not part of a Pythagorean triple (for some integer  $x$ , there does not exist integers  $y$  and  $z$  such that  $x^2 + y^2 = z^2$  or  $y^2 + z^2 = x^2$ )?
- \_\_\_\_\_ 12. In triangle  $ABC$ , we have  $\overline{AB} = 6$  and  $\overline{AC} = 8$ . Let the angle bisector of angle  $A$  intersect side  $BC$  at point  $D$ . Then we also have  $\overline{BD} = 3$  and  $\overline{CD} = 4$ . What is  $\overline{AD}$ ?

Mr. Lu has some 6-sided dice. However, he is having some troubles getting his mind around expected values. Help him find the answers to the following problems:

- \_\_\_\_\_ 13. On average, how many times must a 6-sided die be rolled until a 6 turns up?
- \_\_\_\_\_ 14. On average, how many times must a 6-sided die be rolled until a 6 turns up twice in a row?
- \_\_\_\_\_ 15. On average, how many times must a 6-sided die be rolled until a 6 and then a 5 turns up?
- \_\_\_\_\_ 16. On average, how many times must a 6-sided die be rolled until a 6 turns up three times in a row?

A **24 puzzle** involves using addition, subtraction, multiplication, and division among four numbers (each used exactly once) to compute 24. For example, with the numbers 1, 2, 3, 4, you can make 24 with  $1 \cdot 2 \cdot 3 \cdot 4$ .

- \_\_\_\_\_ 17. Find a solution for the following 24: 4, 4, 4, 4
- \_\_\_\_\_ 18. Find a solution for the following 24: 4, 7, 7, 7
- \_\_\_\_\_ 19. Find a solution for the following 24: 5, 5, 11, 11
- \_\_\_\_\_ 20. Find a solution for the following 24: 1, 5, 11, 11

Find the next term in the **sequence** indicated by the   :

- \_\_\_\_\_ 21.  $J, F, M, A, M, J, \_$
- \_\_\_\_\_ 22.  $Q, W, E, R, T, \_$
- \_\_\_\_\_ 23.  $H, H, L, B, B, C, \_$
- \_\_\_\_\_ 24.  $A, D, I, P, \_$

- \_\_\_\_\_ 25. How many times is the number 1 printed on this side of the page (including question number)?