Good Luck! :)

1. Jeffrey has 28 coins, all quarters and nickels. The value of his coins amounts to $3.20. He has Q quarters and N nickels. What is product of Q and N?

   A. 115 B. 132 C. 160 D. 171 E. NOTA

2. Mike has a bag of 26 marbles. He asks his buddy, Will, to pick 2 marbles from the bag without replacement. In the bag, there are 9 red marbles, 4 green marbles, 5 yellow marbles, and the rest are orange. What is the probability that the two marbles Will picked are orange?

   A. $\frac{28}{325}$ B. $\frac{14}{169}$ C. $\frac{56}{325}$ D. $\frac{28}{169}$ E. NOTA

3. Pedro alone can paint a fence in 12 hours. At 8:00 AM, Pedro begins to paint a fence. After one hour, Pedro's brother Juan arrives and starts painting the same fence. At 11:00 AM, their younger brother Paul shows up and starts to paint with them. If working alone, Juan can paint the entire fence in 8 hours, and Paul can paint the entire fence in 6 hours. Assuming that they work at constant independent rates, how many hours will it take for the fence to be completely painted since Pedro started working?

   A. $\frac{17}{9}$ B. $\frac{8}{3}$ C. $\frac{35}{9}$ D. $\frac{13}{3}$ E. NOTA

4. Alberto's two best athletes are Galen and Mo. To prepare them for the Olympic games, he has them run a 400m race around a 400m track. Mo runs at a constant 5 m/s. Galen, on the other hand, ate too much Taco Bell the day before and can only run 4 m/s. Mo, knowing that he will easily win, decides that he will continue to run around the track at the same speed after he reaches the finish line. Assuming that they start at the same time and run the shortest distance, how many more meters will Mo have run when Galen finishes the race?

   A. 100 B. 120 C. 150 D. 200 E. NOTA

5. Ian was sleeping in class when Mr. Moody drops an object on his head. He looks up, flabbergasted, and asks "What is this?". Mr. Moody replies: "I'll save you the time and tell you that a solid has 20 faces and 30 vertices. How many edges does it have, Ian?" he asks. Ian, because he is built different, answers the question correctly immediately. What was Ian's answer?

A. 12 B. 24 C. 48 D. 50 E. NOTA

6. Terry is at the point (-2, 5) on the coordinate plane and is sprinting towards his house at the point (6, 9), which is on fire. There is a river on the line $y = x$ where Terry needs to fetch water in order to put out the fire. What is the shortest distance Terry can travel in order to put out the fire?

A. 11 B. $\sqrt{122}$ C. $\sqrt{123}$ D. $2\sqrt{31}$ E. NOTA

7. Mike, Will, Caleb, and Dustin are playing a game with a fair 3-sided dice whose sides are numbered 4, 2, and 0. In this game, if a player rolls a 0, they win. If not, they pass the dice to the next player. Due to their stubbornness, they will only stop once a player wins. If Mike goes first, Will second, Caleb third, and Dustin fourth, what is the probability that Caleb wins the game?

A. $\frac{4}{65}$ B. $\frac{12}{65}$ C. $\frac{4}{19}$ D. $\frac{27}{65}$ E. NOTA

8. Sara takes out a $100,000 loan to buy a limited-edition Ferrari. The interest rate given by the bank is 30%, compounded every 4 months. If she owed nothing to the bank prior, how much does she owe the bank one year after taking out the loan?

A. 130,000 B. 133,100 C. 146,410 D. 271,828 E. NOTA

9. Eliud wants to build a rectangular farm for his cows. He has 400 units of fencing to enclose a specified area for his cows, and on his property, there is a very long river that he will use for one side of the fence. What is the maximum area that Eliud can enclose for his cows?

A. $\frac{160,000}{9}$ B. 10000 C. 15000 D. 20000 E. NOTA

10. Jake is standing on top of a 100ft tall building. He sticks his arm out, and drops a bowling ball. The ball has a special property where it rebounds to ten percent of its previous height after every drop. If the ball is left to bounce for an infinite amount of time, what distance will it travel, in feet?

A. $\frac{1000}{9}$ B. $\frac{1100}{9}$ C. 150 D. 200 E. NOTA

11. Craig is hosting a track meet in his hometown of Pfafftown, North Carolina. His financial analyst, Cooper, estimates that if ticket prices were $10, thirty-eight thousand people would attend the meet. He also estimates that for every one dollar that ticket prices go up, 1500 less people will attend the meet. If the ticket price is an integer, what price should Craig charge per ticket in order to maximize revenue?

A. 14 B. 15 C. 16 D. 17 E. NOTA

12. Dixit is writing a super program that spits out a function given its roots. For instance, if he inputs 2, some possible outputs are $x-2$ and x^2-4. However, if a root imaginary, the output will have real coefficients. For instance, if the input is i, some possible outputs are x^2+1 and $2x^2+2$. Which of the following is a possible equation the program will spit out when Dixit inputs 1, $2-i$, and $1+3i$?

A. $x^5-7x^4+29x^3-73x^2+100x-50=0$
B. $x^5+7x^4+37x^3-24x^2+100x-50=0$
C. $x^5-4x^4+15x^3-27x^2+40x-50=0$
D. $x^5-14x^4-7x^3+18x^2-100x+50=0$
E. NOTA

13. Philip has 5 liters of a mixture of 40% goo and 60% water. However, his friend Aidan secretly pours in pure goo so that the mixture becomes 80% goo. Philip, who is now out of pure water, goes to the grocery store to get a type of drink called Gooba, which is 50% goo and 50% water, and plans on making the final mixture 60% goo. How many liters of Gooba does Philip need to make the desired final mixture?

A. 20 B. 30 C. 40 D. 50 E. NOTA

14. Emma is running along the line $6x - 8y = 15$. Suddenly, she realizes that she forgot her water bottle at school, which is at the point $(-5,6)$.What is the shortest possible distance between Emma and her water bottle?

A. $\frac{63\sqrt{61}}{61}$ B. $\frac{63}{10}$ C. $\frac{93\sqrt{61}}{61}$ D. $\frac{93}{10}$ E. NOTA

15. Michael is throwing a huge party at his mansion because he got all 5's on his AP exams. He invites 20 different friend groups to the party, and each friend group has 5 members. No person is in 2 or more different friend groups. Every person at the party, including Michael, shakes hands with everyone except with those in their friend group. Michael is not part of any friend group. How many handshakes occur at the party?

A. 9500 B. 9600 C. 4750 D. 4850 E. NOTA

16. Ivan ties up his goat Peter with a 25-meter rope to a corner on the outside of his rectangular fenced ranch, which is 20 meters long and 10 meters wide. If the rope does not stretch, what is the area of Peter's roaming space?

A. $\frac{1875\pi}{4}$ B. 525π C. $\frac{2125\pi}{4}$ D. 625π E. NOTA

17. George is about to start studying for his history final. Just now, his friend Jakob called and asked to play Dungeons and Dragons (DND) in his mom's basement. George's score on the history test is directly proportional to the number of hours he spends studying, the square root of the number of hours he sleeps, and inversely proportional to the number of hours he spends playing DND. It is currently 5 PM, and George has to wake up at 7 AM. If George studies for 2 hours, plays DND for 8 hours, and sleeps for 4 hours, he will get 60 points on the exam. If he sleeps for 9 hours, plays DND for 4 hours, and spends the rest of his time studying, how many points will he receive on the final exam.

A. 90 B. 155 C. 180 D. 270 E. NOTA

18. Usain, Justin, and Andre are having head-to-head 200m races. They each run at constant rates. Usain beats Justin by 20m and Justin beats Andre by 15 meters. By how many meters does Usain beat Andre?

A. 31 B. 33.5 C. 35 D. 36.5 E. NOTA

19. A giant cold pool is 20 meters wide, 50 meters long, and 10 meters high. Shrek the Giant, who wants to cool off, lowers himself into the pool at a constant rate of $55\ m^3/s$. After 55 seconds, the pool starts to overflow. What was the initial height of the water measured from the bottom of the pool before Shrek entered?

A. $\frac{121}{80}$ B. $\frac{679}{80}$ C. $\frac{121}{40}$ D. $\frac{279}{40}$ E. NOTA

20. Every time Vlad opens his computer, it gives him a math problem to which he has to solve in order to unlock it. Today, when he opened his computer, it asked him "Find the sum of the cubes of the distinct roots of $x^6 - 5x^5 - 36x^4 + 170x^3 - 91x^2 - 165x + 126$." What answer must Vlad give in order to unlock his computer?

A. 154 B. 156 C. 586 D. 588 E. NOTA

21. In a mathathlon, contestants run a marathon where at the end of every mile, they stop to solve a math problem. At the end of mile 23, Yared was give the following problem:

$$Find\ the\ number\ of\ integers\ that\ satisfy\ \left|25 - \left|6 + |8 - x|\right|\right| < 13$$

If Yared finished the mathathlon, what answer did he give?

A. 11 B. 25 C. 50 D. 63 E. NOTA

22. Timmy wrote a program that gives the roots of an equation given the input of a polynomial in the form of $x^2 + ax + b$, where a and b are positive real numbers. In one case, Timmy input $x^2 + px + 12$, and the program gave one value q. What is the value is $pq + p + q$?

A. $24 - 6\sqrt{3}$ B. $24 + 6\sqrt{3}$
C. $-24 - 2\sqrt{3}$ D. $-24 + 2\sqrt{3}$ E. NOTA

23. Albert, Ben, Caroline, Danielle, and Emily are going to watch a movie at the movie theatre. They have five consecutive seats in the same row reserved. Albert and Ben insist on sitting next to each other, while Caroline and Emily have some beef and refuse to sit next to each other. In how many ways can the five friends be seated to watch the movie?

A. 12 B. 24 C. 36 D. 48 E. NOTA

24. Mr. Payne has cows and chickens in his barn. However, due to a meltdown in his nuclear power plant, half of his cows become special and grew a 5^{th} leg. If there are 72 animal heads and 249 animal legs in the barn, what is the positive difference between the number of special cows and the number of chickens in the barn? (Assume Mr. Payne initially had an even number of cows)

A. 6 B. 9 C. 12 D. 15 E. NOTA

25. An isosceles trapezoid has bases of lengths 9 and 16. The length of its height is the geometric mean of the lengths of its bases. This isosceles trapezoid is then rotated about its longer base. What is the volume of the resulting figure?

A. 1296π B. 3312π C. 1632π D. 1968π E. NOTA

26. Miguel has 100 grams of the radioactive substance ooblekium. In 32.1 seconds, it will have decayed so that he will only have 12.5 grams of the substance left. What is the half life of ooblekium, in seconds?

A. 10.7 B. $\frac{642}{35}$ C. 21.4 D. $\frac{963}{70}$ E. NOTA

27. Josh is dropping off a secret package from a plane. The plane is flying horizontally at 800 ft/s and is 10,000 ft above the surface. The package has a special property: once it is dropped, it immediately reaches terminal velocity of 128 ft/s. Josh is trying to drop the package at a certain rendezvous point on the ground. At what horizontal distance away in feet from the rendezvous point should Josh drop off the package?

A. 7875 B. 15625 C. 31250 D. 62500 E. NOTA

28. Eighteen points lie on quadrilateral EPIC: the four vertices E, P, I, and C; 2 other points on EP; 3 other points on PI; 4 other points on IC; and 5 other points on CE. How many triangles with positive area can be constructed whose vertices are among the eighteen points?

A. 747 B. 757 C. 812 D. 816 E. NOTA

29. In McDonalds High School, incoming freshman take at least one of the three STEM classes available for their grade level: Algebra II, Chemistry, and Biology. 54 students are taking Algebra II, and 32 are taking Biology. Five students are taking both Chemistry and Biology only. The number of students taking both Algebra II and Biology only is a third of the amount taking Biology only. 3 students are taking all 3 classes, and there are 99 incoming freshmen at McDonalds High. How many incoming freshmen are taking Chemistry only at McDonalds High School?

A. 6 B. 8 C. 18 D. 37 E. NOTA

30. The test average for Mrs. Smith's summer reading test for her fourth period was a 76. However, the day after the test, one of her students, Albert, got switched into her fourth period. With his score of 100, the test average of her fourth period rose to an 82. How many students were originally in her fourth period?

A. 3 B. 5 C. 7 D. 9 E. NOTA