

- What is the volume of a regular tetrahedron with side length  $a$ ?  
A.  $\frac{a^3\sqrt{3}}{6}$       B.  $\frac{a^3\sqrt{2}}{4}$       C.  $\frac{a^3\sqrt{2}}{12}$       D.  $\frac{a^3\sqrt{5}}{24}$       E. NOTA
- A regular hexagon with radius 6 and an irregular icosagon are placed such that the area that is inside the icosagon, but outside the hexagon is the same as the area inside the hexagon, but outside the icosagon. What is the area of the icosagon?  
A.  $54\sqrt{3}$       B.  $9\sqrt{3}$       C.  $36\sqrt{5}$       D.  $18\sqrt{5}$       E. NOTA
- Jake fills a cup that is in the shape of a cone with the vertex pointed down. The cup has a height of 12 a radius of 4 at the top. Right now, the cup has  $\frac{1}{3}$  of the volume of water it can hold in it. What is the height of the water in the cup right now?  
A.  $\frac{4\sqrt{3}}{3}$       B.  $4\sqrt{3}$       C.  $4\sqrt[3]{9}$       D.  $4\sqrt[3]{3}$       E. NOTA
- Given that the area of an isosceles triangle with vertex angle  $72^\circ$  and base length 1 is  $x$ , what is the volume of a right pentagonal prism that has height 12 and a base in the shape of a regular pentagon with side length 8?  
A.  $7680x$       B.  $960x$       C.  $1920x$       D.  $3840x$       E. NOTA
- In trapezoid SEAN, S is at  $(-3, 4)$ , E is at  $(-3, 0)$ , A is at  $(3, 0)$ , and N is at  $(5, 4)$ . What is the volume of the solid formed when SEAN is fully rotated around the x-axis?  
A.  $\frac{352}{3}\pi$       B.  $\frac{416}{3}\pi$       C.  $32\pi$       D.  $128\pi$       E. NOTA
- Olivia has a  $60^\circ$  sector cut out of a paper circle of radius 6. She forms a cone by pushing the edges together. Amy has a  $120^\circ$  sector cut out of the same paper circle. She forms a cone in a similar manner. What is the ratio of the volume of Olivia's cone to the volume of Amy's cone?  
A.  $\frac{\sqrt{70}}{32}$       B.  $\frac{16\sqrt{70}}{35}$       C.  $\frac{1}{2}$       D.  $\frac{2\sqrt{35}}{5}$       E. NOTA

7. An ellipse is defined by the equation  $\frac{(x-4)^2}{16} + \frac{(y+1)^2}{50} = 1$ . What is the area of the rectangle with vertices at the end points of the latera recta?
- A.  $\frac{64\sqrt{17}}{5}$       B.  $\frac{16\sqrt{17}}{5}$       C.  $50\sqrt{34}$       D.  $\frac{25\sqrt{34}}{2}$       E. NOTA
8. In triangle ABC, cevians AD and AE are drawn such that D and E are on side BC and  $BD:DE:EC = 1:2:3$ . The midsegment parallel to BC is drawn, which intersects AB at F, AD at G, AE at H, and AC at I. What is the ratio of the area of triangle AFG to the area of quadrilateral HICE?
- A.  $\frac{1}{3}$       B.  $\frac{1}{6}$       C.  $\frac{1}{9}$       D.  $\frac{1}{12}$       E. NOTA
9. Square ABCD has side length 5 and square EFGH has side length 7. They overlap in such a way that the area shared between ABCD and EFGH is 17. What is the area of the union of the two squares?
- A. 57      B. 40      C. 17      D. 74      E. NOTA
10. In quadrilateral ABCD, point A is at  $(-5, 4, 0)$ , point B is at  $(3, 8, 0)$ , point C is at  $(7, 1, 0)$  and D is at  $(4, -2, 0)$ . What is the area of ABCD?
- A. 123      B.  $\frac{123}{2}$       C. 117      D.  $\frac{117}{2}$       E. NOTA
11. An octahedron contains ABCD (described in problem #10) as well as point E at  $(5, 8, 12)$  and point F at  $(6, 2, -5)$  such that there are 8 edges connecting E and F to each of A, B, C, and D, but no edge connecting E and F. What is the volume of this octahedron?
- A. 697      B.  $\frac{697}{2}$       C.  $\frac{663}{2}$       D.  $\frac{1989}{2}$       E. NOTA

12. In triangle KEJ,  $KE = 6$ ,  $EJ = 7$ , and  $JK = 8$ . Both the angle bisector and the median from E are drawn, which intersect JK at I and N respectively. Find the area of triangle EIN.
- A.  $\frac{8\sqrt{17}}{65}$       B.  $\frac{21\sqrt{130}}{26}$       C.  $\frac{5\sqrt{10}}{13}$       D.  $\frac{21\sqrt{15}}{104}$       E. NOTA
13. You are given two concentric circles and a chord of the outer circle that is tangent to the inner circle. If the chord has length 12, what is the area of the region between the two circles?
- A.  $24\pi$       B.  $36\pi$       C.  $108\pi$       D.  $144\pi$       E. NOTA
14. One of the angles in a rhombus of side length 8 is  $30^\circ$ . What is the area of a circle inscribed in this rhombus?
- A.  $4\pi$       B.  $16\pi$       C.  $64\pi$       D.  $256\pi$       E. NOTA
15. John is playing with some unusual building blocks. These blocks come in three shapes: circular frustum, cylinder, and cone. All blocks have a height of 4. John makes a stack of 3 blocks. On the bottom, he puts a frustum with bottom radius 10 and top radius 5. On top of that, he puts a cylinder with radius 5. Finally, on top, he puts a cone with radius 5. What is the total volume of the blocks in John's stack?
- A.  $600\pi$       B.  $\frac{800\pi}{3}$       C.  $400\pi$       D.  $\frac{500\pi}{3}$       E. NOTA
16. Erick is very bored, so he decides to start drawing circles and squares. Specifically, he draws a square with side length 12, then draws a circle inscribed in that square, then draws a square inscribed in that circle, then draws a circle inscribed in that square, and so on forever. What is the sum of the areas of the individual circles that Erick draws?
- A.  $144\pi$       B.  $96\pi$       C.  $72\pi$       D.  $64\pi$       E. NOTA

17. Points A, B, and C are selected on the circumference of a circle with diameter 12 such that  $AB = BC$  and  $m\angle ABC = 120^\circ$ . The area bounded by AB, BC and the major arc between points A and C is  $x$ . Find  $\frac{x}{6}$ .
- A.  $4\pi - 2\sqrt{3}$     B.  $3\pi + 2\sqrt{3}$     C.  $2\pi + 3\sqrt{3}$     D.  $4\pi + 3\sqrt{3}$     E. NOTA
18. Four points, C, O, L and E are chosen on the circumference of circle Y. Given that  $CO = 5$ ,  $OL = 7$ ,  $LE = 8$ , and  $EC = 10$ , what is the area of quadrilateral  $COLE$ ?
- A.  $20\sqrt{105}$     B.  $10\sqrt{105}$     C.  $20\sqrt{21}$     D.  $10\sqrt{21}$     E. NOTA
19. What is the lateral surface area of a cone that has base diameter  $d$  and a slant height that is 8 times the reciprocal of the base radius?
- A.  $16\pi$     B.  $8\pi$     C.  $16d\pi$     D.  $8d\pi$     E. NOTA
20. Given three mutually tangent circles of radius 6, the area of a circle placed in the middle of these circles such that the middle circle is tangent to all three of the circles of radius 6 is  $6k\pi$ . Find  $k$ .
- A.  $12 - 6\sqrt{3}$     B.  $10 - 4\sqrt{3}$     C.  $12 - 8\sqrt{3}$     D.  $14 - 8\sqrt{3}$     E. NOTA
21. A regular hexahedron with volume  $24\sqrt{3}$  is inscribed in a sphere. What is the volume of the sphere?
- A.  $48\pi$     B.  $36\pi$     C.  $32\sqrt{3}\pi$     D.  $288\pi$     E. NOTA
22. Mr. Moody has a square garden with a surrounding walkway. The outer edge of the walkway is always 1 unit away from the closest point on the edge of the garden. If the garden has area 48, what is the area of the walkway?
- A.  $4\sqrt{3} + \pi$     B.  $4\sqrt{3} + 4\pi$     C.  $16\sqrt{3} + \frac{\pi}{4}$     D.  $16\sqrt{3} + \pi$     E. NOTA

23. Sharay draws a Venn Diagram with two circles of radius 2 such that each circle passes through the center of the other. Find the area of the intersection of the two circles.
- A.  $\frac{2}{3}\pi + \sqrt{3}$     B.  $\frac{2}{3}\pi - \sqrt{3}$     C.  $\frac{4}{3}\pi - \sqrt{3}$     D.  $\frac{8}{3}\pi - 2\sqrt{3}$     E. NOTA
24. A rectangular prism with side lengths  $x$ ,  $y$ , and  $z$ . The space diagonal of the prism has length 14, and the sum of the lengths of all edges is 96. What is the surface area of the prism?
- A. 100    B. 292    C. 196    D. 380    E. NOTA
25. What is the area of an ellipse with major axis 7 and minor axis 4?
- A.  $28\pi$     B.  $7\pi$     C.  $\pi\sqrt{33}$     D.  $49\pi$     E. NOTA
26. In square ABCD of side length 20, a circle passes through A, B and the midpoint of C and D. What is the area of the circle?
- A.  $144\pi$     B.  $225\pi$     C.  $100\pi$     D.  $\frac{625}{4}\pi$     E. NOTA
27. Ignoring units, which of the following represents the surface area to volume ratio for a sphere with diameter  $x$ ?
- A.  $\frac{3}{2x}$     B.  $\frac{6}{x}$     C.  $\frac{3}{x}$     D.  $\frac{x}{3}$     E. NOTA
28. A cylindrical cup with radius 8 is filled up to a height of 12 with water. A cylindrical piece of ice with radius 4 and height 1 is placed in the cup so that one of the circular bases faces straight up, and 90% of the volume of the ice is submerged in the water. Assuming that the water does not spill over and the ice does not melt, what is the new height of the water (do not include the unsubmerged portion of the ice)?
- A.  $\frac{49}{4}$     B.  $\frac{249}{20}$     C.  $\frac{499}{40}$     D.  $\frac{489}{40}$     E. NOTA

29. A solid metal cone with radius 6 and height 1 is melted and recast into a cylinder with the same radius as the cone. What is the height of the cylinder?
- A. 9                      B. 3                      C.  $\frac{1}{3}$                       D.  $\frac{1}{9}$                       E. NOTA
30. The writer of this test apologizes for the fact that there are no pictures on this test. He hopes that you will accept this final easy question as compensation. What is the area of an equilateral triangle with side length 1?
- A. 1                      B.  $\frac{\sqrt{3}}{4}$                       C.  $\frac{\sqrt{3}}{2}$                       D.  $\sqrt{3}$                       E. NOTA