1. C The base of a tetrahedron is an equilateral triangle with area $\frac{a^2\sqrt{3}}{4}$ and the height of the tetrahedron is $\frac{a\sqrt{6}}{3}$ (this can be found using the Pythagorean theorem in 3D), so the volume is $\frac{1}{3} * \frac{a^2\sqrt{3}}{4} * \frac{a\sqrt{6}}{3} = \frac{a^3\sqrt{2}}{12}$.
2. A The hexagon and icosagon have the same area, so we can simply find the area of the hexagon. Since a hexagon's radius is equal to its side length, the area is $\frac{3\sqrt{3}*6^2}{2} = 54\sqrt{3}$.
3. C The water in the cup is in the shape of a cone that is similar to the entire cup. If the ratio of the volumes is $1:3$, then the ratio of the heights is $1:\sqrt[3]{3}$. Since the height of the whole cup is 12, the height of the water is $\frac{12}{\sqrt[3]{3}} = 4\sqrt[3]{9}$. Note that the radius of the cone is irrelevant.
4. D The pentagonal base can be divided into five congruent isosceles triangles at the center. Each triangle has a vertex angle of $\frac{360}{5} = 72°$, so they are similar to the triangle with area x. Since they have a base length of 8, their areas will be $8^2 * x = 64x$. Since there are five of them, and the height of the prism is 12, the volume of the prism is $64x * 5 * 12 = 3840x$.
5. A If we let point H be $(5,0)$, then the volume of the cylinder formed by rotating SEHN around the x-axis minus the volume of the cone formed by rotating AHN around the x-axis will give us the volume of the desired figure. The cylinder has a radius of 4 and a height of 8, so it has a volume of $4^2 * 8 * \pi = 128\pi$. The cone has a radius of 4 and a height of 2, so it has a volume of $\frac{1}{3} * 4^2 * 2 * \pi = \frac{32}{3}\pi$. Thus, the volume of our desired solid is $128\pi - \frac{32}{3}\pi = \frac{352}{3}\pi$.
6. A The slant height of both cones will be the radius of the paper circle, which is 6. For the $60°$ sector, the circumference of the cone will be $2\pi * 6 * \frac{60}{360} = 2\pi$, so the base radius of this cone will be $\frac{2\pi}{2\pi} = 1$ and the height is $\sqrt{6^2 - 1^2} = \sqrt{35}$, so the volume is $\frac{1}{3} * 1^2 * \sqrt{35} * \pi = \frac{\pi\sqrt{35}}{3}$. For the $120°$ sector, the circumference will be $2\pi * 6 * \frac{120}{360} = 4\pi$, so the radius will be $\frac{4\pi}{2\pi} = 2$, and the height will be $\sqrt{6^2 - 2^2} = 4\sqrt{2}$, so the volume is $\frac{1}{3} * 2^2 * 4\sqrt{2} * \pi = \frac{16\pi\sqrt{2}}{3}$. Thus our desired ratio is $\frac{\frac{\pi\sqrt{35}}{3}}{\frac{16\pi\sqrt{2}}{3}} = \frac{\sqrt{70}}{32}$.
7. A The rectangle will have one side length that is equal to the length of the latus rectum of the ellipse, which is $\frac{2b^2}{a}$, and another side length that is equal to the distance between the foci, which is $2c$. Thus the area is $\frac{4b^2c}{a}$. $a = 5\sqrt{2}, b = 4$, and $c = \sqrt{a^2 - b^2} = \sqrt{50 - 16} = \sqrt{34}$. Thus, our area is $\frac{4*16*\sqrt{34}}{5\sqrt{2}} = \frac{64\sqrt{17}}{5}$.
8. C The area of ABD will be $\frac{1}{6}$ of the area of the entire triangle since its base is $\frac{1}{6}$ of that of the entire triangle while its height is the same, and AFG will have an area that is $\frac{1}{4}$

of ABD since ABD is cut by the midsegment, which is $\frac{1}{24}$ of the area of the entire triangle. By the same logic, HICE will be $\frac{3}{4}$ of $\frac{3}{6}$ of the area of the entire triangle or $\frac{3}{8}$ of it. Thus the ratio of the areas of ABD and HICE is $\frac{\frac{1}{24}}{\frac{3}{8}} = \frac{1}{9}$.

9. A We can find the area of the union by adding the areas of the two squares and subtracting the area of the intersection. This gives us $25 + 49 - 17 = 57$.

10. D Since these points are all in the xy-plane, we can simply apply shoelace. This gives us $\frac{|-5(8) + 3(1) + 7(-2) + 4(4) - (4(3)+8(7)+1(4)+(-2)(-5))|}{2} = \frac{117}{2}$

11. C The octahedron can be split into two pyramids. Both pyramids will have base area $\frac{117}{2}$, and 1 will have a height of 5 while the other has a height of 12. Thus, the volume of the octahedron is $\frac{1}{3} * \frac{117}{2} * (12 + 5) = \frac{663}{2}$.

12. D First consider the angle bisector, EI. By the angle bisector theorem, $KI = \frac{48}{13}$ and $IJ = \frac{56}{13}$. The median, EN, will divide KJ such that $KN = NI = 4$. Thus $IN = \frac{56}{13} - 4 = \frac{4}{13}$, and the ratio of the area of EIN to EKJ is $\frac{\frac{4}{13}}{8} = \frac{1}{26}$. We can find the area of EKJ, using Heron's formula, to be $\sqrt{(\frac{21}{2})(\frac{9}{2})(\frac{7}{2})(\frac{5}{2})} = \frac{21\sqrt{15}}{4}$. Thus the area of EIN is $\frac{21\sqrt{15}}{4} * \frac{1}{26} = \frac{21\sqrt{15}}{104}$.

13. B Let R be the radius of the big circle and r be the radius of the small circle. If we draw a segment from the center of the circles to the point where the inner circle is tangent to the chord and a segment from the center of the circles to one of the points where the chord intersects the outer circle, we form a right triangle with hypotenuse R and legs of length 6 and r. By the Pythagorean theorem, $R^2 - r^2 = 36$. Since the area we desire is equal to $\pi R^2 - \pi r^2$, our answer is 36π.

14. A Let the rhombus be ABCD. If we draw a segment from A to a point on CD such that the segment is perpendicular to CD, we form a 30-60-90 triangle. The length of the segment from A to the CD is 4, which is the diameter of an inscribed circle. Thus, the radius is 2, and the area is 4π.

15. E We can rearrange these blocks so that the top and bottom block form a cone with radius 10 and height 8, which has volume $\frac{1}{3} * 10^2 * 8 * \pi = \frac{800\pi}{3}$. The cylinder has volume $5^2 * 4\pi = 100\pi$. Thus the total volume is the sum of these two or $\frac{1100}{3}\pi$.

16. C The initial square has an area of 144. The first circle that Erick inscribes will have a radius of 6 and an area of 36π. The next square has a diagonal equal to the diameter of the circle, which is 12, so its side length is $6\sqrt{2}$ and its area is 72. The radius of the next circle is $3\sqrt{2}$, so it has an area of 18π. The sum of all these areas is two infinite geometric series, one for the circles and one for the squares. Both have a common ratio of $\frac{1}{2}$. The squares have a first term of 144, so the sum is $\frac{144}{1-\frac{1}{2}} = 288$. The circles have a first term of 36π, so the sum is $\frac{36\pi}{1-\frac{1}{2}} = 72\pi$.

17. D This area is the area of the entire circle, which is $6^2\pi = 36\pi$, minus the area of the region bounded by segment AB and the minor arc between A and B minus the area bounded by segment BC and the minor arc between BC. Since AB = BC, these two areas are equal, so we will find the area of one and double it. By the inscribed angle theorem, the area of major arc AC is $240°$, so the measure of minor arc AC is $120°$ and the measure of minor arc AB is $60°$. The area of the region bounded by segment AB and minor arc AB is the difference between the $60°$ sector of the circle formed by points A, B, and the center and the equilateral triangle formed by A, B, and the center. The area of the sector is $6^2\pi * \frac{60}{360} = 6\pi$ and the area of the triangle is $\frac{6^2\sqrt{3}}{4} = 9\sqrt{3}$, so the area of this region is $6\pi - 9\sqrt{3}$. Doubling it gives us $12\pi - 18\sqrt{3}$, and subtracting from 36π gives us $36\pi - (12\pi - 18\sqrt{3}) = 24\pi + 18\sqrt{3}$. Thus our final answer is $\frac{24\pi+18\sqrt{3}}{6} = 4\pi + 3\sqrt{3}$

18. E Because the quadrilateral is inscribed in a circle, it is a cyclic quadrilateral, which means that we can use Brahmagupta's formula to find the area. The semiperimeter is 15, so the area is $\sqrt{(15-5)(15-8)(15-7)(15-10)} = 20\sqrt{7}$.

19. B The lateral surface area of a cone is $\pi r L$ where r is the base radius and L is the slant height. Thus, the lateral surface area is $\pi * r * \frac{8}{r} = 8\pi$. The fact that we called the diameter d is irrelevant.

20. D First, draw a triangle connecting the centers of these circles. This is an equilateral triangle with side length 12. The center of the middle circle is the center of the equilateral triangle. The distance from the center of the middle circle to one of the circles of radius 6 is $\frac{2}{3}$ of the length of the altitude of the triangle. The triangle as an altitude of $12 * \frac{\sqrt{3}}{2} = 6\sqrt{3}$, so the distance between the centers is $6\sqrt{3} * \frac{2}{3} = 4\sqrt{3}$.This distance is also equal to 6 plus the radius of the middle circle, meaning that the radius of the middle circle is $6 - 4\sqrt{3}$. Thus its area is $\pi(6 - 4\sqrt{3})^2 = \pi(84 - 48\sqrt{3})$. Dividing by 6π gives us $14 - 8\sqrt{3}$.

21. B A regular hexahedron is better known as a cube. If the cube has volume $24\sqrt{3}$, the side length is $\sqrt[3]{24\sqrt{3}} = 2\sqrt{3}$ and the diagonal of the cube is $2\sqrt{3} * \sqrt{3} = 6$, so the radius of the sphere is $\frac{6}{2} = 3$, and the volume of the sphere is $\frac{4}{3} * \pi * 3^3 = 36\pi$.

22. D The walkway will have straight edges of length $\sqrt{48} = 4\sqrt{3}$ parallel to the edges of the garden, a distance 1 away from them. At each corner, the walkway will be a quarter circle of radius 1. The quarter circles combine to form a full circle of radius 1 and area π. The straight edges form 4 rectangles, each with side lengths 1 and $4\sqrt{3}$ for a total area of $4 * 1 * 4\sqrt{3} = 16\sqrt{3}$. Thus the total area of the walkway is $16\sqrt{3} + \pi$.

23. D Let the centers of the circle be A and B and let the points of intersection between the circles be C and D. AB = 2, AC = 2, and BC = 2, so ABC is an equilateral triangle, meaning that $m\angle CAB = 60°$. ABD is also an equilateral triangle, so $m\angle DAB = 60°$. Thus, $m\angle CAD = 120°$. Now we can split the area of intersection up in the middle, giving us identical regions that we can find the area of. One of the areas is bounded by segment CD and minor arc CD (on circle B). The area of this region is the area of

the $120°$ sector defined by A, C, and D and triangle ACD. The area of the sector is $2^2\pi * \frac{120}{360} = \frac{4}{3}\pi$ and the area of the triangle is $\frac{base*height}{2}$. The base is segment CD, which has length $2\sqrt{3}$ (we can find this using 30-60-90 triangles) and the height is 1 (half of AB), so the area of the triangle is $\frac{2\sqrt{3}*1}{2} = \sqrt{3}$. Thus, the area of the region is $\frac{4}{3}\pi - \sqrt{3}$. Doubling this, we get $\frac{8}{3}\pi - 2\sqrt{3}$, which is our final answer.

24. D The space diagonal is length $\sqrt{x^2 + y^2 + z^2} = 14 \to x^2 + y^2 + z^2 = 196$. Since there are 4 edges with each length, the sum of the edges is $4x + 4y + 4z = 96 \to x + y + z = 24$. We seek the surface area, which is $2xy + 2yz + 2xz$, which happens to be $(x + y + z)^2 - (x^2 + y^2 + z^2) \to 24^2 - 196 = 380$.

25. B
$$a = \frac{7}{2}, b = \frac{4}{2} \to Area = \pi ab = 7\pi.$$

26. D Draw a segment from the midpoint of CD to the midpoint of AB and extend it until it intersects the circle. Let this intersection between the segment and the circle be point E. We can use power of a point. The distances from A to the midpoint of AB and B to the midpoint of AB both equal 10. The distance from the midpoint of CD to the midpoint of AB is 20, and the distance from the midpoint of AB to E is $2r - 20$ where r is the radius of the circle. Thus, by power of a point, we have $20(2r - 20) = 100 \to r = \frac{25}{2}$. So the area of the circle is $\frac{625}{4}\pi$.

27. B The surface area is $4\pi(\frac{r}{2})^2$ and the volume is $\frac{4}{3}\pi(\frac{r}{2})^3$. We seek the surface area-to-volume ratio, so we have $\frac{4\pi(\frac{r}{2})^2}{\frac{4}{3}\pi(\frac{r}{2})^3} = \frac{6}{r}$.

28. D The volume of the piece of ice $4^2 * 1 * \pi = 16\pi$. 90% of this volume is $16\pi * \frac{9}{10} = \frac{72}{5}\pi$. The added height to the water level that the ice gives is the volume of the ice submerged divided by the area of the surface of the water, which is $\frac{\frac{72}{5}\pi}{8^2\pi} = \frac{9}{40}$, so the new height of the water is $12 + \frac{9}{40} = \frac{489}{40}$.

29. C The volume formula for a cylinder is the same as that of a cone with the exception of the extra $\frac{1}{3}$ in the formula for the cone. The cylinder and cone in the problem have the same volume, so to compensate for the fact that the cone is becoming a cylinder, the height is multiplied by $\frac{1}{3}$. Thus the height of the cylinder is $\frac{1}{3}$.

30. B $\frac{1^2\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$.