1. A
2. C
3. D
4. B
5. C
6. E
7. B
8. B
9. A
10. A
11. C
12. E
13. B
14. B
15. A
16. C
17. A
18. E
19. D
20. B
21. A
22. C
23. C
24. A
25. C
26. A
27. D
28. D
29. B
30. A

1. A Angle bisector = median = altitude in an equilateral triangle. If an angle bisector is 6 inches, then the altitude must be as well. Drawing an altitude forms a 30-60-90 degree right triangle with the side opposite the 60 degree angle being 6 inches. Dividing by √3 gives us 2√3 as half of the side length. Then, side length must be 4√3, and we can plug it into the formula $\frac{s^2\sqrt{3}}{4}$ to solve for the area of the triangle. Area = $\left(4\sqrt{3}\right)^2 * \frac{\sqrt{3}}{4}$ = 12√3. Perimeter = 3*(4√3) = 12√3. Sum and Answer = 24√3.
2. C The figure formed is two identical cones with their bases attached to each other. The slant height of each of these cones is equal to the side length of the triangle, the radius of the cones is equal to the angle bisector of the triangle, and the height is equal to $\frac{1}{2} *$ side length of the triangle → r = 6, height = 2√3 → volume = $2\left(\frac{1}{3}\pi r^2 h\right) = 2\left(\frac{1}{3}\pi(36)\left(2\sqrt{3}\right)\right) = 48\pi\sqrt{3}$. Multiply by 2 because there are 2 cones that meet the description above.
3. D No calculations need to be made, we are told that B = A/3 so A/B = 3.
4. B The volume when the obelisk is completely summoned is $8x^2$. We can use the formula $\mathrm{r} * \mathrm{t} = \mathrm{w}$ to find the rate at which the obelisk is summoned in terms of x. If we plug our values in, we get $r * 6 = 8x^2$. Simplifying this gives us $r = \frac{4x^2}{3}$. Plugging this in when time = 3.25 should give us an equation to solve for x. $\frac{4x^2}{3} * \frac{13}{4} = 39$. Simplifying gives us $x^2 = 9$. Next we take the square root, but height must be a positive value, therefore the answer is 3 and not -3.
5. C First, the information that says PW = PM tells us that if a radius was drawn from P to the point where WM is tangent to circle P (let's call this point T), then WT = TM. Therefore, triangle WBM must be equilateral. To find the radius of the circle, we can draw a line segment connecting P to W (or M). This makes WPT a 30-60-90 degree right triangle. WT=TM, so WT = 4, which makes the radius $\frac{4\sqrt{3}}{3}$. Area inside the triangle but outside the circle = $\frac{8^2\sqrt{3}}{4} - \left(\frac{4\sqrt{3}}{3}\right)^2 \pi = 16\sqrt{3} - \frac{16\pi}{3}$.
6. E He only needs to travel along one face diagonal to reach the point opposite I, and take the space diagonal to I to reach Kyouko Hori. So that is one face diagonal + one space diagonal = $11\sqrt{2} + 11\sqrt{3}$ or E.
7. B Split up the area into two parts that need to be found: the area of the intersection between 2 circles (let us call this area A), and the weird crescent moon shaped thing near the outside of the larger circle (let us call this area B), so we are looking for A + B. Firstly, the face that each of the smaller circles passes through the center of the largest circle tells us that the radius of the smaller circles is 1. Then, we can find the value of A by drawing a 45-45-90 right triangle in the leftmost small circle such that the area A is cut in half by the hypotenuse. Find the area of the sector (90 degrees), and subtract the area of the triangle, which should give you half the area of A → $r = 1, \pi r^2 = \frac{\pi}{4} \to A = \left(\frac{\pi}{4} - \frac{1}{2}\right) * 2 = \frac{\pi}{2} - 1$. To find B, observe the larger circle. 4 regions with the area of B are in the larger circle and outside the smaller circles, so

we have to subtract out the area of the 4 smaller circles together to get 4 times B. So the area of small circles together = $4(\pi - 2A) + 4A = 4\pi - 4A$, accounting for the overlap between the smaller circles. Therefore, $4B = 4\pi - (4\pi - 4A) = 4A \rightarrow A = B \rightarrow A + B = 2A = 2B = \pi - 2$.

8. B The shape is a rectangle with semicircles attached in the top and the bottom. $40 \cdot (200 - 20) + 20^2\pi = 400\pi + 7200$

9. A The first piece of information, that any radius of the largest circle is trisected by the smaller circles tells us that the distance between each circle is the same. If we call the radius of the smallest circle r, then the largest circle will have radius = 3r, and the second largest will have radius = 2r. Area of smallest circle = πr^2, and area of the largest circle = $\pi * (3r)^2 = 9\pi r^2$. Ratio will then be $\frac{\pi r^2}{9\pi r^2} = \frac{1}{9}$.

10. A When spun, the figure that will be formed will be two cones with their bases placed together. But before that, we must find the radii/heights of the cones. They have the same radius because they share the same base. To figure out these measurements, we must find the long and short diagonals of the kite. The short diagonal is the diameter of the cone, and part of the long diagonal will be the height of each cone. Short diagonal is given, drawing the figure gives two 45-45-90 triangles, and two 30-60-90 triangles. The long diagonal can then be found to be split into two parts of 2 and 2√3, with the short diagonal splitting it into those pieces. Volume = $\frac{1}{3}\pi(4)(2) + \frac{1}{3}\pi(4)(2\sqrt{3}) = \frac{8\pi + 8\pi\sqrt{3}}{3}$.

11. C It's a torus. You can look it up to see what it looks like.

12. E Radius = side length of hexagon = 6. We need (area of circle – area of hexagon)/2 because that is the region that is described. Area of circle = 36π. Area of hexagon = $6 * \frac{6^2\sqrt{3}}{4} = 54\sqrt{3} \rightarrow \frac{36\pi - 54\sqrt{3}}{2} = 18\pi - 27\sqrt{3}$. Probability = this area/total area = $\frac{18\pi - 27\sqrt{3}}{36\pi} = \frac{1}{2} - \frac{3\sqrt{3}}{4\pi}$

13. B Drawing the figure shows us that the square cuts the larger triangle into 3 smaller triangles, which are all also 45-45-90. 2 of these triangles are congruent to each other, and the third is smaller than the other 2. Call the side length of the square x (which is also the side length of the two congruent 45-45-90 triangles), this makes the side length of the smallest triangle $\frac{x\sqrt{2}}{2}$. Side length of the original triangle now becomes $\frac{x\sqrt{2}}{2} + x\sqrt{2} = \frac{3x\sqrt{2}}{2}$. Area of original triangle = $\frac{9x^2}{2} * \frac{1}{2} = \frac{9x^2}{4}$. Ratio = $\frac{x^2}{\frac{9x^2}{4}} = \frac{4}{9}$

14. B When each vertex is connected to the center of the hexagon, it forms 6 equilateral triangles, so this is just a fancy way of saying the area of the hexagon minus the area of 6 equilateral triangles with side length 1. Area = $6 * \frac{4^2\sqrt{3}}{4} - 6 * \frac{1^2\sqrt{3}}{4} = 24\sqrt{3} - \frac{3\sqrt{3}}{2} = \frac{45\sqrt{3}}{2}$.

15. A Drawing the figure shows us that ADB is an equilateral triangle. If AB is 1 then AD = DB = AB = 1. We can solve for the hypotenuse BC, and we get BC = 2. If E is the midpoint, then BE = CE = 1, but DB = 1, therefore ED must be 0 because they are the same point.

16. C Using $A = ab\sin(c)$, the area of the dodecagon is $12 \cdot \frac{\sin(30)}{2}$, and the area of the icositetragon is $24 \cdot \frac{\sin(15)}{2}$. The ratio is $\frac{\sin(30)}{2\sin(15)} = \frac{\sqrt{6}+\sqrt{2}}{4}$

17. A The obscenely long, seemingly pointless fence keeps Razor from moving towards vertex D from vertex A. Next we just have to draw the picture, which forms several 90 degree sectors of circles with different radii, and add up the areas of those sectors. Area = $\frac{1600\pi}{4} + \frac{900\pi}{4} + \frac{400\pi}{4} + \frac{100\pi}{4} = 400\pi + 225\pi + 100\pi + 25\pi = 750\pi$

18. E We don't really have to know how much of the cube's surface area when disassembled is red, we just have to know how much is black. Then we can subtract that value from the total surface area of the cube when disassembled to get how much is red. If, while assembled, the cube is painted black, then it has $96\ cm^2$ of black. Total surface area of the cube when disassembled → There are 64 1x1x1 cubes, each of those has a surface area of 6 →64(6) = $384\ cm^2 \rightarrow 384 - 96 = 288\ cm^2\ is\ painted\ red$. This is $288 - 96 = 192 cm^2$ more than what is painted black.

19. D If we call the area of triangle ABC x, then the areas of triangles AMF, MFB, MDB, MCD, and AEM are all $\frac{1}{6}x$. Area of AEMF = AEM + AMF = $2 * \frac{1}{6}x = \frac{1}{3}x$. Area of ECBFM = CEM + CMD + MDB + FMB = $4 * \frac{1}{6}x = \frac{2}{3}x$. Ratio = $\frac{1}{2}$.

20. B Using shoelace, the area is 41.

21. A 3,4,5=6 6,8,10=24 5,12,13=30 9,12,15=54 6+24+30+54=114

22. C Radius of the cone and cylinder can be obtained by comparing the circumferences of the sector and of the base of the cone/cylinder. $2\pi r = \frac{240}{360} * 24\pi = 16\pi$. Radius of the cone/cylinder = 8. Height can be derived using the Pythagorean theorem. Height = $4\sqrt{5}$. Volume = $\pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h = \frac{2}{3}\pi(64)(4\sqrt{5}) = \frac{512\pi\sqrt{5}}{3}$.

23. C Radius of sphere = radius and height of cone. Great circle is the circle with the same radius as the sphere, where the sphere is cut in half. $4\pi r^2 = 1156\pi; \pi r^2 = 289\pi; r = 17$. Volume of cone = $\frac{1}{3}\pi(289)(17) = \frac{4913\pi}{3}$.

24. A Volume of largest = $6^3 \cdot \frac{\sqrt{2}}{12}$

Volume of rest = $2 \cdot \frac{\frac{V}{3}}{1-\frac{1}{3}} = V$

Sum $= 2V = 36\sqrt{2}$

25. C Apothem is the distance from the center of a regular polygon to the midpoint of any side. Drawing the picture, we have a 30-60-90 triangle, and the side length of the hexagon becomes 7. This makes the perimeter 7*6 = 42. $Area = \frac{1}{2}ap = \frac{1}{2}\left(\frac{7\sqrt{3}}{2}\right)(42) = \frac{147\sqrt{3}}{2}$.

26. A The first step should be to factor the given equation into standard form for an ellipse. When we do this, we get $\frac{(x+1)^2}{12} + \frac{(y-3)^2}{8} = 1$. $a = 2\sqrt{3}, b = 2\sqrt{2}, c = 2$. The height is the difference in the y values of the ellipses, and since we know they are parallel we don't care about what the x and z values are. y of the first ellipse is 3, the second

is 8, so the difference/height is 5. Volume of the prism = $Bh, B = area\ of\ base = \pi ab = 4\pi\sqrt{6}, h = 5. Answer = 20\pi\sqrt{6}$.

27. D Let's call the circles A and B, and the rhombus CEDF. Using arc lengths, we can prove that the largest angle of CDEF is also 120 degrees, which tells us that the circles pass through each other's centers, and CDEF is really ABCD if C and D are the points of intersection. We know the radius of the circle, so we basically know the side length of the rhombus. Using this, we can easily calculate for the lengths of the diagonals, which are $6\ and\ 6\sqrt{3}$. There are 4 congruent "sections" in the overlap that are outside the rhombus. Let's call the area of each of these "sections" a. If we take the area of the sector and subtract the area of the rhombus, we get 2a. Area of sector $= \frac{1}{3}\pi(36) = 12\pi$. Area of rhombus $= \frac{1}{2}(6)\left(6\sqrt{3}\right) = 18\sqrt{3}$. $2a = 12\pi - 18\sqrt{3}$. $a = 6\pi - 9\sqrt{3}$. Total area of overlap outside the rhombus = $4a = 24\pi - 36\sqrt{3}$.

28. D Draw the picture, label the measure of major arc AC as x and the measure of minor arc BC as y. We know that $\frac{x-y}{2} = 57 \rightarrow x - y = 114$. We also know that $x + y + 20 + 10 = 360 \rightarrow x + y = 330$. We just need to solve the system of equations to get x = 222 degrees.

29. B The scale factor between the smaller and larger cones is $\frac{2}{4} = \frac{1}{2}$. The slant height of the larger cone is 5, meaning its height must be 3, and the height of the smaller cone must then be $3 * \frac{1}{2} = \frac{3}{2}$. To get the height of the frustrum, we must subtract the height of the smaller cone from the height of the larger cone. $3 - \frac{3}{2} = \frac{3}{2}$.

30. A If volume is $\frac{500000\pi}{3}, \frac{4}{3}\pi r^3 = \frac{500000\pi}{3}, r = 50$. Surface area = $4\pi r^2 = 10000\pi$.