Theta School Bowl
Test #832
Question #0

Theta School Bowl
Test #832
Question #0

#0 Theta School Bowl
MAΘ National Convention 2023

Factor the trinomial $35x^2 - 11x - 6$ into the form $(Ax + B)(Cx + D)$ where A, B, C, D are integers with $A, B > 0$.

What is the value of $A + B + C + D$?

#0 Theta School Bowl
MAΘ National Convention 2023

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What is the value of $A + B + C + D$?

Theta School Bowl
Test #832
Question #1

Theta School Bowl
Test #832
Question #1

#1 Theta School Bowl
MAΘ National Convention 2023

Let $A = \frac{(\log 10+\log 2+\log 5)\log 4}{\log 7}$

Let $B = e^{\ln\left(\frac{e^{\ln 28-\ln 2}}{e^{\ln 32-4\ln 2}}\right)}$

$f(x) = \frac{10(x^2-4)(x^3-125)}{(x^2+5x+6)(x^3-2x^2-25x+50)}$

Let $C =$ the sum of x-values of the removable discontinuities of $f(x)$

Let $D =$ the sum of the x-value(s) at which the vertical asymptote(s) of $f(x)$ occurs and the y-value(s) at which the horizontal asymptote(s) of $f(x)$ occurs

Find the value of $B^A + CD$

#1 Theta School Bowl
MAΘ National Convention 2023

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Find the value of $B^A + CD$

Theta School Bowl
Test #832
Question #2

Theta School Bowl
Test #832
Question #2

#2 Theta School Bowl
MAΘ National Convention 2023

Let A = the area of a regular hexagon with side length $3\sqrt{2}$

Let B = the area enclosed by an isosceles trapezoid with bases of lengths of 4 and 7 and base angle of 60°

Let C = the side length of the regular octagon formed by cutting isosceles right triangles from the corners of a square with perimeter 80

Let D = the side length of a cube with space diagonal $\sqrt{3} + \sqrt{6}$

What is the value of $\frac{ACD}{B}$?

#2 Theta School Bowl
MAΘ National Convention 2023

Let A = the area of a regular hexagon with side length $3\sqrt{2}$

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What is the value of $\frac{ACD}{B}$?

Theta School Bowl
Test #832
Question #3

Theta School Bowl
Test #832
Question #3

#3 Theta School Bowl
MAΘ National Convention 2023

Let $A =$ the length of the latus rectum of $4x^2 - 9y^2 - 16x - 54y - 101 = 0$

Let $B = a + b + c$ where the line perpendicular to $2x + 5y = 3$ and goes through the point $(-5, -8)$ is expressed in the form $ax + by = c$ with $a > 0$ and a, b, and c are relatively prime

Let $C =$ the sum of the x- and y-coordinates of all points that can be a possible fourth vertex of the parallelogram with vertices (6, 3), (10, 3), and (4, 4)

A quadrilateral is enclosed by the four lines tangent to the ellipse $4x^2 - 48x + 9y^2 - 90y + 333 = 0$ and perpendicular to its major and minor axes

Let $D =$ the volume of the solid formed by revolving this quadrilateral around the y-axis

Find the value of $ABC + D$

#3 Theta School Bowl
MAΘ National Convention 2023

Let $A =$ the length of the latus rectum of $4x^2 - 9y^2 - 16x - 54y - 101 = 0$

Let $B = a + b + c$ where the line perpendicular to $2x + 5y = 3$ and goes through the point $(-5, -8)$ is expressed in the form $ax + by = c$ with $a > 0$ and a, b, and c are relatively prime

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Find the value of $ABC + D$

Theta School Bowl
Test #832
Question #4

Theta School Bowl
Test #832
Question #4

#4 Theta School Bowl
MAΘ National Convention 2023

The repeating decimal $5.3\overline{49}$ can be expressed as a reduced fraction in the form $\frac{A}{B}$.

Let $C =$ the sum of the values of n that make $n+2$, n, and $2n-1$ a geometric sequence in that order

Let $D =$ the sum of all possible values of a if $a = 5 + \sqrt{a + \sqrt{a + \sqrt{a + \cdots}}}$

Let $E = \frac{(n+2)!}{7(n-1)!}$ if $n = 7$

Find the value of $A + B + C + D + E$

#4 Theta School Bowl
MAΘ National Convention 2023

The repeating decimal $5.3\overline{49}$ can be expressed as a reduced fraction in the form $\frac{A}{B}$.

Let $C =$ the sum of the values of n that make $n+2$, n, and $2n-1$ a geometric sequence in that order

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Let $E = \frac{(n+2)!}{7(n-1)!}$ if $n = 7$

Find the value of $A + B + C + D + E$

Theta School Bowl
Test #832
Question #5

Theta School Bowl
Test #832
Question #5

#5 Theta School Bowl
MAΘ National Convention 2023

Let $A =$ the sum of the slope and the y-intercept of the line formed by the set of points that are equidistant from the points $(3, 4)$ and $(-1, 2)$

Let $B = w$ where the line through the points $(1, w - 2)$ and $(3, 5)$ is parallel to the line through the points $(2, 5 - w)$ and $(3, 13)$

$f(x)$ and $g(x)$ are functions with inverses $f^{-1}(x)$ and $g^{-1}(x)$ respectively

x	0	1	2	3	4
$f(x)$	-4	0	3	4	11
$g(x)$	3	1	0	-4	-5

Let $C = (f \circ g)(1) - g((f \circ g)((f \circ g^{-1})(1)))$
Let $D = (g \circ f)(3) + g(3) + g^{-1}(-4)$

$h(3x + 1) = \frac{3x-2}{x+9}$
Let $E = h(7)$
Let $F = h\left(\frac{35}{2}\right)$

Find the value of $ABCDEF$

#5 Theta School Bowl
MAΘ National Convention 2023

Let $A =$ the sum of the slope and the y-intercept of the line formed by the set of points that are equidistant from the points $(3, 4)$ and $(-1, 2)$

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$h(3x + 1) = \frac{3x-2}{x+9}$
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Find the value of $ABCDEF$

Theta School Bowl
Test #832
Question #6

Theta School Bowl
Test #832
Question #6

#6 Theta School Bowl
MAΘ National Convention 2023

Let $A = x + y$ where (x, y) is the centroid of the triangle defined by $(2, 3)$, $(-4, 2)$, $(7, 5)$

Triangle XYZ has angle bisector YW, where W lies on side XZ. Its side lengths are $YX = 11$, $YZ = 10$, $ZX = 9$.
Let $B =$ the length of XW

Mr. Frazer is making a rectangular enclosure for his baby alligator. He has 100 feet of fencing to make three sides of the enclosure, and the fourth side of the enclosure will be the side of his house.
Let $C =$ the largest possible area of the region formed by the enclosure

Let $D =$ the positive difference between the measure of an interior angle of a regular decagon (in degrees) and the number of diagonals in an icosagon

Find the sum of the numerator and denominator of $\frac{BC}{AD}$ in simplest form

#6 Theta School Bowl
MAΘ National Convention 2023

Let $A = x + y$ where (x, y) is the centroid of the triangle defined by $(2, 3)$, $(-4, 2)$, $(7, 5)$

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Find the sum of the numerator and denominator of $\frac{BC}{AD}$ in simplest form

Theta School Bowl
Test #832
Question #7

Theta School Bowl
Test #832
Question #7

#7 Theta School Bowl
MAΘ National Convention 2023

Let $A = (10 + 5i)(3 + 6i)$

Let $B = \frac{10+5i}{3+6i}$

Let $C =$ the product of all possible positive integer values of x if $x < |3 - 4i|$

Let $D = i + 2i^2 + \cdots + 399i^{399} + 400i^{400}$

What is the value of $AB + CD$?

#7 Theta School Bowl
MAΘ National Convention 2023

Let $A = (10 + 5i)(3 + 6i)$

Let $B = \frac{10+5i}{3+6i}$

Let $C =$ the product of all possible positive integer values of x if $x < |3 - 4i|$

Let $D = i + 2i^2 + \cdots + 399i^{399} + 400i^{400}$

What is the value of $AB + CD$?

Theta School Bowl
Test #832
Question #8

Theta School Bowl
Test #832
Question #8

#8 Theta School Bowl
MAΘ National Convention 2023

Let $A =$ the area enclosed by the graph $x^2 + y^2 - 8x - 2y - 8 = 0$

Let $B =$ the area of the region bound by the line $4x - 7y + 15 = 0$ and the x and y-axes

Let $C =$ the eccentricity of the conic with the equation $16x^2 + 9y^2 - 32x + 18y - 119 = 0$

Let $D = p + q$ where (p, q) is the focus of the parabola $x = y^2 - 4$

What is the value of $\frac{ABC}{D}$?

#8 Theta School Bowl
MAΘ National Convention 2023

Let $A =$ the area enclosed by the graph $x^2 + y^2 - 8x - 2y - 8 = 0$

Let $B =$ the area of the region bound by the line $4x - 7y + 15 = 0$ and the x and y-axes

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Let $D = p + q$ where (p, q) is the focus of the parabola $x = y^2 - 4$

What is the value of $\frac{ABC}{D}$?

Theta School Bowl
Test #832
Question #9

Theta School Bowl
Test #832
Question #9

#9 Theta School Bowl
MAΘ National Convention 2023

Let $A = \sum_{n=-2}^{\infty} \left(\frac{3}{4}\right)^{n+1}$

Let $B = \prod_{n=0}^{\infty} 5^{(2^{-n})}$

Let $C = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \cdots$

Let $D = \sum_{n=3}^{\infty} \frac{2}{n^2 - 2n}$

Let $E =$ the sum of the squares of the first 30 positive integers

What is the value of $ABCD + E$?

#9 Theta School Bowl
MAΘ National Convention 2023

Let $A = \sum_{n=-2}^{\infty} \left(\frac{3}{4}\right)^{n+1}$

Let $B = \prod_{n=0}^{\infty} 5^{(2^{-n})}$

Let $C = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \cdots$

Let $D = \sum_{n=3}^{\infty} \frac{2}{n^2 - 2n}$

Let $E =$ the sum of the squares of the first 30 positive integers

What is the value of $ABCD + E$?

Theta School Bowl
Test #832
Question #10

Theta School Bowl
Test #832
Question #10

#10 Theta School Bowl
MAΘ National Convention 2023

Let A = the probability that the sum of the numbers generated by rolling three standard six-sided dice is a multiple of 3

Let B = the probability that product of two numbers selected at random from the interval [-5, 10] is negative

Let C = the number of distinct arrangements of MOSQUITO in which the first letter is a vowel

What is the value of $\frac{AC}{B}$?

#10 Theta School Bowl
MAΘ National Convention 2023

Let A = the probability that the sum of the numbers generated by rolling three standard six-sided dice is a multiple of 3

Let B = the probability that product of two numbers selected at random from the interval [-5, 10] is negative

Let C = the number of distinct arrangements of MOSQUITO in which the first letter is a vowel

What is the value of $\frac{AC}{B}$?

Theta School Bowl
Test #832
Question #11

Theta School Bowl
Test #832
Question #11

#11 Theta School Bowl
MAΘ National Convention 2023

The area of a segment of a circle with diameter 10 and central angle 120° can be expressed in the form $\frac{a\pi+b\sqrt{c}}{d}$, for integers a, b, c, d where a, b, and d are relatively prime, c is square-free, and $d > 0$.

Let $A = a + b + c + d$

Let $B =$ the volume of the cone formed by joining the cut edges of a 135° sector cut out of a circular piece of paper with radius 8

In triangle XYZ, the length of side XY is 7 and the length of side XZ is 8. The median from vertex Y is perpendicular to the median from vertex Z.
Let $C = YZ^2$

Let $D =$ the volume of a tetrahedron with side length 4

Let $E =$ the surface area of a tetrahedron with side length 1

Find the value of $AC + BDE$

#11 Theta School Bowl
MAΘ National Convention 2023

The area of a segment of a circle with diameter 10 and central angle 120° can be expressed in the form $\frac{a\pi+b\sqrt{c}}{d}$, for integers a, b, c, d where a, b, and d are relatively prime, c is square-free, and $d > 0$.

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Let $C = YZ^2$

Let $D =$ the volume of a tetrahedron with side length 4

Let $E =$ the surface area of a tetrahedron with side length 1

Find the value of $AC + BDE$

Theta School Bowl
Test #832
Question #12

Theta School Bowl
Test #832
Question #12

#12 Theta School Bowl
MAΘ National Convention 2023

Let $A =$ the product of all possible values of x if $\left|\left||x-6|-6\right|-6\right| = 8$

Let $B =$ the number of integer values of x that satisfy $\frac{x-2}{2x+2} > 2$

Let $C =$ the largest possible real value of n for which $x^2 + 2nx - n + 3$ has exactly one real root

Let $D =$ the sum of the real values of x that satisfy $(x^2 - 4x + 4)^{x^2+4x+3} = 1$

Find the value of $A + B + C + D$

#12 Theta School Bowl
MAΘ National Convention 2023

Let $A =$ the product of all possible values of x if $\left|\left||x-6|-6\right|-6\right| = 8$

Let $B =$ the number of integer values of x that satisfy $\frac{x-2}{2x+2} > 2$

Let $C =$ the largest possible real value of n for which $x^2 + 2nx - n + 3$ has exactly one real root

Let $D =$ the sum of the real values of x that satisfy $(x^2 - 4x + 4)^{x^2+4x+3} = 1$

Find the value of $A + B + C + D$

Theta School Bowl
Test #832
Question #13

Theta School Bowl
Test #832
Question #13

#13 Theta School Bowl
MAΘ National Convention 2023

Let A = the amount of 5% solution (in mL) needed to make 700 mL of 4% hydrochloric acid solution by mixing 2% and 5% hydrochloric acid solutions

Let B = the number of days it would take for Mr. Fayiga and Mr. Lu to paint a house together if it would take 6 days for Mr. Fayiga to paint the house alone and 8 days for Mr. Lu to paint the house alone

A bathtub (with the drain plugged) takes 30 minutes to fill. The bathtub (with the drain open) takes 45 minutes to empty.
Let C = the number of minutes it takes the bathtub to overflow if the water is turned fully on and the drain is left open in an empty bathtub

Find the value of ABC

#13 Theta School Bowl
MAΘ National Convention 2023

Let A = the amount of 5% solution (in mL) needed to make 700 mL of 4% hydrochloric acid solution by mixing 2% and 5% hydrochloric acid solutions

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Let C = the number of minutes it takes the bathtub to overflow if the water is turned fully on and the drain is left open in an empty bathtub

Find the value of ABC

Theta School Bowl
Test #832
Question #14

Theta School Bowl
Test #832
Question #14

#14 Theta School Bowl
MAΘ National Convention 2023

Let $A = \begin{vmatrix} 1 & 12 & -2 \\ -3 & 5 & 3 \\ 5 & 15 & 4 \end{vmatrix}$

Let $B =$ the sum of the elements of $\begin{bmatrix} 5 & 6 \\ -2 & 3 \end{bmatrix}^2$

Let $C =$ the determinant of $\begin{bmatrix} 4 & 16 \\ 2 & -6 \end{bmatrix}^T$

Let $D =$ the sum of the elements of $\begin{bmatrix} 3 & 4 \\ -9 & 5 \\ 7 & -2 \end{bmatrix} \begin{bmatrix} -1 & -3 \\ 3 & 2 \end{bmatrix}$

Find $\begin{vmatrix} A & B \\ C & D \end{vmatrix}$

#14 Theta School Bowl
MAΘ National Convention 2023

Let $A = \begin{vmatrix} 1 & 12 & -2 \\ -3 & 5 & 3 \\ 5 & 15 & 4 \end{vmatrix}$

Let $B =$ the sum of the elements of $\begin{bmatrix} 5 & 6 \\ -2 & 3 \end{bmatrix}^2$

Let $C =$ the determinant of $\begin{bmatrix} 4 & 16 \\ 2 & -6 \end{bmatrix}^T$

Let $D =$ the sum of the elements of $\begin{bmatrix} 3 & 4 \\ -9 & 5 \\ 7 & -2 \end{bmatrix} \begin{bmatrix} -1 & -3 \\ 3 & 2 \end{bmatrix}$

Find $\begin{vmatrix} A & B \\ C & D \end{vmatrix}$