Theta School Bowl
Test #832
Question #0

Theta School Bowl
Test #832
Question #0

#0 Theta School Bowl
MAΘ National Convention 2024

Let set S be the set of integers between 100 and 1000 inclusive.

$A =$ the number of integers in set S that have an odd number of factors

$B =$ the number of integers in set S that are divisible by 2

$C =$ the number of integers in set S that are divisible by 2 and 3

$D =$ the number of integers in set S that are divisible by 2 or 3

Find $A + B + C - D$.

#0 Theta School Bowl
MAΘ National Convention 2024

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Find $A + B + C - D$.

Theta School Bowl
Test #832
Question #1

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Question #1

#1 Theta School Bowl
MAΘ National Convention 2024

A: Kevin's house and Alan's house are 20 miles apart on a river. It takes Kevin 5 hours to swim from his house to Alan's. It takes Kevin 10 hours to swim from Alan's house back to his own. Assume that Kevin swims at a constant rate. $A =$ the rate of the current in the river in miles per hour.

B: Angela the cowgirl is travelling from her barn at point $(1,3)$ to a rodeo at point $(5,9)$. On the way, she must stop at a river which is $x = -1$. $B =$ the shortest distance of the trip she must take.

C: 10 printers can print 1000 sheets of paper in 10 minutes. $C =$ the amount of time in seconds required for 1 printer to print 1 sheet of paper.

D: Daniel and Nathan can build a house in $\frac{24}{7}$ days. Nathan and Will can build a house in $\frac{24}{5}$ days. Will and Daniel can build a house in 4 days. $D =$ the number of days it takes to build 30 houses if Nathan, Will, and Daniel work together.

Calculate $A + B + C + D$.

#1 Theta School Bowl
MAΘ National Convention 2024

A: Kevin's house and Alan's house are 20 miles apart on a river. It takes Kevin 5 hours to swim from his house to Alan's. It takes Kevin 10 hours to swim from Alan's house back to his own. Assume that Kevin swims at a constant rate. $A =$ the rate of the current in the river in miles per hour.

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Calculate $A + B + C + D$.

Theta School Bowl
Test #832
Question #2

Theta School Bowl
Test #832
Question #2

#2 Theta School Bowl
MAΘ National Convention 2024

For Parts A and B, let $H(x) = x^4 + 7$.

A: Let $f(x)$ be the remainder when $H(x)$ is divided by $x^2 - 5x + 6$. $A = f(4)$.

$B =$ the sum of the squares of all solutions to the equation $H(x) = 8x^2$

C: Suppose $G(2x - 2) = x^2 + 7$. $C =$ the sum of the value(s) of k that satisfy the equation $G(k) = 1943$

D: m and n are positive integers that satisfy the equation $mn = m + 4n + 3$. $D =$ the smallest possible value of $m + n$.

Calculate $A + B + C + D$.

#2 Theta School Bowl
MAΘ National Convention 2024

For Parts A and B, let $H(x) = x^4 + 7$.

A: Let $f(x)$ be the remainder when $H(x)$ is divided by $x^2 - 5x + 6$. $A = f(4)$.

$B =$ the sum of the squares of all solutions to the equation $H(x) = 8x^2$

C: Suppose $G(2x - 2) = x^2 + 7$. $C =$ the sum of the value(s) of k that satisfy the equation $G(k) = 1943$

D: m and n are positive integers that satisfy the equation $mn = m + 4n + 3$. $D =$ the smallest possible value of $m + n$.

Calculate $A + B + C + D$.

Theta School Bowl
Test #832
Question #3

Theta School Bowl
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Question #3

#3 Theta School Bowl
MAΘ National Convention 2024

A: A quadratic function $q(x)$ exists such that $q(0) = -1$, $q(3) = 2$, and $q(6) = 41$. $A = q(2)$.

B: $f(x) = 2x^3 - 9x^2 + kx + 12$. The three roots of $f(x)$ form an arithmetic sequence. $B = k$.

C: $g(x) = x^2 - x + 1$. $C = \sum_{i=0}^{5} g(i)$.

Calculate $A + B + C$.

#3 Theta School Bowl
MAΘ National Convention 2024

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Calculate $A + B + C$.

Theta School Bowl
Test #832
Question #4

Theta School Bowl
Test #832
Question #4

#4 Theta School Bowl
MAΘ National Convention 2024

Consider regular octagon $HIJKLMNO$ with side length 2.

A: Let A equal the area of octagon $HIJKLMNO$.

B: Let B equal the area of square $HJLN$ (Hint: $\sin a = \sin(180° - a)$).

C: Let C equal the perimeter of $HIJKLMNO$.

D: Let D equal the perimeter of $HJLN$.

Calculate $A + B + C + D^2$.

#4 Theta School Bowl
MAΘ National Convention 2024

Consider regular octagon $HIJKLMNO$ with side length 2.

A: Let A equal the area of octagon $HIJKLMNO$.

B: Let B equal the area of square $HJLN$ (Hint: $\sin a = \sin(180° - a)$).

C: Let C equal the perimeter of $HIJKLMNO$.

D: Let D equal the perimeter of $HJLN$.

Calculate $A + B + C + D^2$.

Theta School Bowl
Test #832
Question #5

Theta School Bowl
Test #832
Question #5

#5 Theta School Bowl
MAΘ National Convention 2024

A: A right circular cone is cut by a plane that is parallel to the base. The volume of the smaller cone that is cut from the top is $\frac{8}{19}$ of the volume of the leftover frustum. $A =$ the ratio of the smaller cone's radius to the original cone's radius.

B: A sphere with center W has radius $\frac{26}{3}$. A triangle with sides of length 13, 13, and 10 is situated in space so that each of its sides are tangent to the sphere (a cross section of the sphere through the plane determined by the triangle would be the incircle of the triangle). $B =$ the distance between W and the plane determined by the triangle.

Find $A + B$.

#5 Theta School Bowl
MAΘ National Convention 2024

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Find $A + B$.

Theta School Bowl
Test #832
Question #6

Theta School Bowl
Test #832
Question #6

#6 Theta School Bowl
MAΘ National Convention 2024

$$f(x) = 4x^4 + 16x^3 - 17x^2 - 66x + 63$$

Let $r_1, r_2, r_3,$ and r_4 be the distinct **rational** roots of $f(x)$ with $\mathrm{r}_1 < r_2 < r_3 < r_4$.

$A = (2 - r_1)(2 - r_2)(2 - r_3)(2 - r_4)$

$B = \frac{1}{r_1 r_2 r_3} + \frac{1}{r_2 r_3 r_4} + \frac{1}{r_3 r_4 r_1} + \frac{1}{r_4 r_1 r_2}$

$C = (r_1 + r_2 + r_3)(r_2 + r_3 + r_4)(r_3 + r_4 + r_1)(r_1 + r_2 + r_4)$

$D = r_1 r_2 r_3 r_4$

Calculate $\frac{ABD}{C}$.

#6 Theta School Bowl
MAΘ National Convention 2024

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$C = (r_1 + r_2 + r_3)(r_2 + r_3 + r_4)(r_3 + r_4 + r_1)(r_1 + r_2 + r_4)$

$D = r_1 r_2 r_3 r_4$

Calculate $\frac{ABD}{C}$.

Theta School Bowl
Test #832
Question #7

Theta School Bowl
Test #832
Question #7

#7 Theta School Bowl
MAΘ National Convention 2024

Let $g(x)$ be a function defined as follows

$$g(x) = \frac{(x^2 - 2x + 1)(x^2 - 9x + 20)}{x^3 - 2x^2 - 11x + 12}$$

$A =$ the sum of all integers less than 10 that satisfy $g(x) > 0$. If there are none, $A = 0$.

$B =$ the sum of the x values for any holes in the graph of $g(x)$. If there are none, $B = 0$.

$C =$ the sum of the x coordinates of the x-intercepts of all slant or horizontal asymptotes of $g(x)$. If there are none, $C = 0$.

$D =$ the sum of the x coordinates of the x-intercepts of all vertical asymptotes of $g(x)$. If there are none, $D = 0$.

Calculate $A + B + C + D$.

#7 Theta School Bowl
MAΘ National Convention 2024

Let $g(x)$ be a function defined as follows

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$D =$ the sum of the x coordinates of the x-intercepts of all vertical asymptotes of $g(x)$. If there are none, $D = 0$.

Calculate $A + B + C + D$.

Theta School Bowl
Test #832
Question #8

Theta School Bowl
Test #832
Question #8

#8 Theta School Bowl
MAΘ National Convention 2024

A: Let k be the sum of the solutions for x that satisfy the equation: $4^x - 9 \cdot 2^x = -18$. $A = \lfloor k \rfloor$.

B: The graph of $y = |x - 3| + 2|x + 1| + 3$ is composed of 3 linear segments of slopes a, b, and c with $a < b < c$. $B = |abc|$.

C: McKayla is building a rectangular play pen with 100 meters of fence. McKayla decides to build the play pen along the side of a 200 meters brick wall such that she only needs build three sides. $C =$ the maximum area of a play pen that McKayla can build.

Calculate $A + B + C$.

#8 Theta School Bowl
MAΘ National Convention 2024

A: Let k be the sum of the solutions for x that satisfy the equation: $4^x - 9 \cdot 2^x = -18$. $A = \lfloor k \rfloor$.

B: The graph of $y = |x - 3| + 2|x + 1| + 3$ is composed of 3 linear segments of slopes a, b, and c.
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Calculate $A + B + C$.

Theta School Bowl
Test #832
Question #9

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Test #832
Question #9

#9 Theta School Bowl
MAΘ National Convention 2024

Let matrix M be the 4-by-4 matrix $\begin{bmatrix} 1 & 3 & 2 & -1 \\ 1 & 3 & 1 & 2 \\ 5 & 3 & 0 & 1 \\ 3 & 2 & 0 & 2 \end{bmatrix}$ and matrix R be the 1-by-4 matrix [1 0 1 0].

$A =$ the determinant of M

$B =$ the sum of the elements of RM

For parts C and D, let K be the 4-by-4 matrix $K = M^2$.

$C =$ the determinant of K

$D = K_{11}$

Find $A + B + C + D$.

#9 Theta School Bowl
MAΘ National Convention 2024

Let matrix M be the 4-by-4 matrix $\begin{bmatrix} 1 & 3 & 2 & -1 \\ 1 & 3 & 1 & 2 \\ 5 & 3 & 0 & 1 \\ 3 & 2 & 0 & 2 \end{bmatrix}$ and matrix R be the 1-by-4 matrix [1 0 1 0].

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$B =$ the sum of the elements of RM

For parts C and D, let K be the 4-by-4 matrix $K = M^2$.

$C =$ the determinant of K

$D = K_{11}$

Find $A + B + C + D$.

Theta School Bowl
Test #832
Question #10

Theta School Bowl
Test #832
Question #10

#10 Theta School Bowl
MAΘ National Convention 2024

A: When terms are written in order of decreasing degree of x, the coefficients of the 7th and 15th terms of the binomial expansion of $(x-1)^N$ are equal. $A =$ the coefficient of the second term in the binomial expansion of $(x-1)^N$ when terms are written in order of decreasing degree of x.

B: The real solution to the cubic equation $x^3 - 18x - 30 = 0$ can be written as $\sqrt[3]{M} + \sqrt[3]{N}$, where M and N are distinct positive integers. $B = |M - N|$.

C: When simplified, the expression $2 + 3i + 4i^2 + 5i^3 + \cdots + 2024i^{2022} + 2025i^{2023}$ can be written in the form $R + Si$ for integers R and S. $C = |R - S|$.

Find $A + B + C$.

#10 Theta School Bowl
MAΘ National Convention 2024

A: When terms are written in order of decreasing degree of x, the coefficients of the 7th and 15th terms of the binomial expansion of $(x-1)^N$ are equal. $A =$ the coefficient of the second term in the binomial expansion of $(x-1)^N$ when terms are written in order of decreasing degree of x.

B: The real solution to the cubic equation $x^3 - 18x - 30 = 0$ can be written as $\sqrt[3]{M} + \sqrt[3]{N}$, where M and N are distinct positive integers. $B = |M - N|$.

C: When simplified, the expression $2 + 3i + 4i^2 + 5i^3 + \cdots + 2024i^{2022} + 2025i^{2023}$ can be written in the form $R + Si$ for integers R and S. $C = |R - S|$.

Find $A + B + C$.

Theta School Bowl
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Question #11

Theta School Bowl
Test #832
Question #11

#11 Theta School Bowl
MAΘ National Convention 2024

A: Consider triangle PQR with sides $PQ = 13$, $QR = 14$, and $RP = 15$. The midpoints of PQ, QR, and RP respectively are L, M, and O. $A =$ the ratio of the area of PMO to the area of PQR.

B: Consider a triangle BHS with $BH = 7$, $HS = 8$, and $BS = 9$. Line m is the angle bisector of angle HBS. Line m intersects HS at point K. $B =$ the length of HK.

C: Triangle XYZ has a right angle at X. Let the midpoint of YZ be point K. A line segment beginning at X that is perpendicular to YZ intersects YZ at W. It is also given that $YZ = 12$ and the area of XYZ is 24. $C =$ the length of KW.

Calculate ABC.

#11 Theta School Bowl
MAΘ National Convention 2024

A: Consider triangle PQR with sides $PQ = 13$, $QR = 14$, and $RP = 15$. The midpoints of PQ, QR, and RP respectively are L, M, and O. $A =$ the ratio of the area of PMO to the area of PQR.

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Calculate ABC.

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Question #12

#12 Theta School Bowl
MAΘ National Convention 2024

$A = \log_9 21 \cdot \log_{50} 100 \cdot \log_2 3 \cdot \log_{10} 64 \cdot \log_{441} 36 \cdot \log_{216} 50$

$B =$ the sum of the distinct values of x if $4^x - 2^{x+2} + 2^2 = 0$.

C: The three distinct, positive real roots of the equation $6x^3 - 80x^2 + 277x - 96 = 0$ are r, s, and t.
$C = \log_2 r + \log_2 s + \log_2 t$. If C cannot be computed, $C = 0$.

$D = \sqrt{7 + 2\sqrt{6}} + \sqrt{15 - 6\sqrt{6}}$

Calculate $A + B + C + D$.

#12 Theta School Bowl
MAΘ National Convention 2024

$A = \log_9 21 \cdot \log_{50} 100 \cdot \log_2 3 \cdot \log_{10} 64 \cdot \log_{441} 36 \cdot \log_{216} 50$

$B =$ the sum of the distinct values of x if $4^x - 2^{x+2} + 2^2 = 0$.

C: The three distinct, positive real roots of the equation $6x^3 - 80x^2 + 277x - 96 = 0$ are r, s, and t.
$C = \log_2 r + \log_2 s + \log_2 t$. If C cannot be computed, $C = 0$.

$D = \sqrt{7 + 2\sqrt{6}} + \sqrt{15 - 6\sqrt{6}}$

Calculate $A + B + C + D$.

Theta School Bowl
Test #832
Question #13

Theta School Bowl
Test #832
Question #13

#13 Theta School Bowl
MAΘ National Convention 2024

$A =$ the length of the latus rectum of the parabola $3y - 12 = 2x^2 + 8x + 10$.

$B\pi =$ the area of an ellipse with eccentricity of $\frac{1}{2}$ and latus rectum length of 3

$C =$ the number of lattice points on the graph of the equation $3x^2 - 6x + 5y^2 - 20y + 23 = 0$

$D =$ the shortest distance between the origin and the line $4x + 3y = 20$.

Calculate $ABCD$.

#13 Theta School Bowl
MAΘ National Convention 2024

$A =$ the length of the latus rectum of the parabola $3y - 12 = 2x^2 + 8x + 10$.

$B\pi =$ the area of an ellipse with eccentricity of $\frac{1}{2}$ and latus rectum length of 3

$C =$ the number of lattice points on the graph of the equation $3x^2 - 6x + 5y^2 - 20y + 23 = 0$

$D =$ the shortest distance between the origin and the line $4x + 3y = 20$.

Calculate $ABCD$.

Theta School Bowl
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Question #14

Theta School Bowl
Test #832
Question #14

#14 Theta School Bowl
MAΘ National Convention 2024

The equations of circles W and Z are listed below:

Circle W: $x^2 - 10x + y^2 + 6y + 9 = 0$
Circle Z: $x^2 - 20x + y^2 - 14y + 99 = 0$

Let point P be $(6,3)$. Circle W and Circle Z intersect at points R and S.

$A =$ the minimum distance between P and a point on Circle Z

B: The line that contains points R and S can be represented as $Mx + Ny + P = 0$, such that $M \geq 0$ and M, N, and P are relatively prime integers. $B = M + N + P$.

$C =$ the distance between the center of Circle W and the center of Circle Z.

$D\pi =$ the sum of the areas of circle W and circle Z

Find $A^2 + B + C^2 + D$.

#14 Theta School Bowl
MAΘ National Convention 2024

The equations of circles W and Z are listed below:

Circle W: $x^2 - 10x + y^2 + 6y + 9 = 0$
Circle Z: $x^2 - 20x + y^2 - 14y + 99 = 0$

Let point P be $(6,3)$. Circle W and Circle Z intersect at points R and S.

$A =$ the minimum distance between P and a point on Circle Z

B: The line that contains points R and S can be represented as $Mx + Ny + P = 0$, such that $M \geq 0$ and M, N, and P are relatively prime. $B = M + N + P$.

$C =$ the distance between the center of Circle W and the center of Circle Z.

$D\pi =$ the sum of the areas of circle W and circle Z

Find $A^2 + B + C^2 + D$.