Theta ciphering nationals 2023 solutions

0. $\frac{-4}{n}=\frac{3}{n-1}\rightarrow -4n+4=3n\rightarrow 7n=4\rightarrow n=\frac{4}{7}$ 7n=4

1. $100<\pi r^2<120\rightarrow \frac{100}{\pi}<r^2<\frac{120}{\pi}\rightarrow r=6\rightarrow d=12$

2. $\frac{k^2-4}{k^2-1}-\frac{k}{k+3}\le 0\rightarrow \frac{\left(k^2-4\right)\left(k+3\right)-k\left(k^2-1\right)}{\left(k-1\right)\left(k+1\right)\left(k+3\right)}\le 0$

$\frac{k^2-k-4}{\left(k-1\right)\left(k+1\right)\left(k+3\right)}\le 0\rightarrow \frac{1+\sqrt{17}}{2}$

3. $3^{2k-1}=2^{k+3}\rightarrow \frac{1}{3}\bullet 9^k=8\bullet 2^k\rightarrow \left(\frac{9}{2}\right)^k=24$

$\log_{\sqrt{B}}8+\log_B 9=\log_B 64+\log_B 9=\log_B 576$

$k=\log_{9/2}24=\log_{\frac{81}{4}}576$

4. $Dx+Ey+F=-\left(x^2+y^2\right)$

$-3D-4E+F=-25$

$5D+2E+F=-29$

$-3D+8E+F=-73\rightarrow 12E=-48\rightarrow E=-4$

$8D-24=-4\rightarrow 8D=20\rightarrow D=\frac{5}{2}\rightarrow \left(x+\frac{5}{4}\right)^2\rightarrow \frac{-5}{4}$

5. Draw a good picture and exploit some isosceles triangle situations. Call angle Y something and work your way around. Angle W is angle Y +10. 90-X+X+10+X+10=180 X=70 X+10=80

6. X,Y,X+Y,X+2Y,2X+3Y 2X+3Y=2023. If Y=1 X=1010

7. Draw a good picture. SW= $6\sqrt{3}$. By Pythagorean theorem RO= $3\sqrt{7}$. Triangles RKW and OKN are similar with a scale factor of 1:2. RK= $\sqrt{7}$

8. You can factor this or exploit Vieta's formula.

$f(x)=(x-r)(x-s)\rightarrow g(x)=(x-r-3)(x-s-3)$

sum of coefficients = $g(1)=(-r-2)(-s-2)=f(-2)=4-296\cdot 2+2023=1435$

9. $0 \le \sqrt{\frac{4-h}{2}} \le 3 \to 0 \le \frac{4-h}{2} \le 9 \to 0 \le 4-h \le 18$

$-4 \le -h \le 14 \to -14 \le h \le 4 \to -14-13-12\ldots-5 = -95$

10. Draw a picture and you get 10 for the diameter of the sphere and 6 for the diameter of the cylinder. This makes 8 the height of the cylinder. $\frac{4}{3}\pi(5)^3 - \pi(3^2)(8) = \frac{500\pi - 216\pi}{3} = \frac{284\pi}{3}$, so $L = \frac{284}{3}$

11. Temporarily flip the fraction to make it easy to work with and then flip your final answer.

$$\frac{2020k^2}{(k^2+1)^2} \to \frac{1}{2020}\left(\frac{k^4+2k^2+1}{k^2}\right) \to \frac{1}{2020}\left(k^2+2+\frac{1}{k^2}\right) = \frac{5}{2020} = \frac{1}{404} \to 404$$

12. $100 \le 36k^2 < 1000 \to 2 < k^2 < 28 \to k = 2,3,4,5$

$36(4+9+16+25) = 36 \bullet 54 = 1944$

Answers:

0. 4
1. 12
2. $\frac{1+\sqrt{17}}{2}$
3. $\frac{81}{4} = 20\frac{1}{4} = 20.25$
4. $\frac{-5}{4} = -1\frac{1}{4}$
5. 80
6. 1010
7. $\sqrt{7}$
8. 1435
9. -95
10. $\frac{284}{3}$
11. 404
12. 1944